Snow Studies. Part II: Average Relationship between Mass of Snowflakes and Their Terminal Fall Velocity

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(Manuscript received 20 November 2009, in final form 25 March 2010)

ABSTRACT

This study uses a dataset of low-density snow aggregates measurements collected by a ground-based optical disdrometer that provides particle size and terminal fall speed for each size interval from which the velocity–size and area ratio–size relationships can be derived. From these relationships and relations between the Best and Reynolds numbers proposed in the literature, the mass power-law coefficients are obtained. Then, an approximate average relation between the coefficients in the experimentally determined velocity–size power law (with exponent fixed at 0.18) and the coefficients in the estimated mass power law (with exponent fixed at 2) is obtained. The validation of the retrieved relation is made by comparing, for each snowfall event, the time series of the reflectivity factor calculated from the derived mass–size relationship for a snowflake and from the size distribution measured by the optical disdrometer, with the reflectivity obtained from measurements. Using the measured snow size distribution and the retrieved mass–velocity relationship, a few useful relations between the bulk quantities of snow are derived. This study considers relations suitable for the microphysical modeling consistent with radar measurements of precipitating snow composed of unrimed or lightly rimed aggregate snowflakes.

1. Introduction

The retrieval of microphysics of precipitating snow from Doppler radar and other remote sensing measurements, as well as snow microphysical parameterizations, requires knowledge of the form of the size distribution that allows an accurate derivation of the distribution moments that are important for descriptions of microphysical processes. Moreover, characteristics of individual snowflakes such as representative dimensional relations of mass and velocity are needed. The study of Woods et al. (2007) demonstrated an important sensitivity of the precipitation redistribution on the changes in both mass and velocity dimensional relationships for snow in bulk microphysical schemes.

In Zawadzki et al. (2010, henceforth Part I), the study of the variability of the snowflake fall velocity, the environmental parameters controlling this variability, and the uncertainties related to the velocity measurement have been presented. In this work, based on hydrodynamic theory and the results presented in Part I, we derive an approximate relation between the mass and terminal velocity of snowflake.

According to observational, laboratory, and theoretical studies, mass has a major effect on the terminal velocity of particles. Empirical power-law relations obtained by Langleben (1954) express the snowflake fall speed in terms of its melted equivalent diameter representing the mass; the coefficients are dependent on the aggregate type. The dependence on the riming intensity related to the particle density has been introduced to other empirically derived formulas (Kajikawa 1998; Barthazy and Schefold 2006). Theoretical and laboratory work on the determination of the terminal velocity as a function of the particle mass, or density, has been mainly based on hydrodynamic theory using parameterized relationships between the Reynolds number and the Best (or Davies) number, the latter expressed in terms of mass and effective cross-sectional area. By inverting this procedure, the particle mass can be estimated from the observed terminal velocity (Hanesch 1999; Schefold 2004; Lee et al. 2008). The same approach is used here as a first step to estimate the mass relation from optical spectrograph measurements of terminal velocity as a function of snowflake size. In the next step, we determine the approximate average
relation between the experimentally obtained velocity–size relationship and the estimated mass–size relationship. In this way we obtain a consistent parameterization of velocity–size and mass–size relationships.

The cross-sectional area included in the calculation of the mass–velocity relation based on the Best number $X$ and Reynolds number $Re$ ($X$–$Re$) relationship is expected to be related the snowflake effective density as suggested by Cunningham (1978). More recently, an empirical average expression relating particle size, density, and effective area has been proposed by Heymsfield et al. (2004).

Two types of uncertainties contribute to the uncertainty of the derived relations. The first type represents fluctuations in the measured data, such as the velocity or the area ratio for a given size category, used as an important starting point for the calculations. The second type of uncertainty arises from the fact that the theoretical formulas used for the derivation of the resulting relation are not well known. These two types of uncertainties are combined when investigating the uncertainties of the derived relations.

The measurements that we use are from an optical disdrometer, which can give information not only on velocity and area for each size bin of snowflake but also on snowflake particle size distribution (PSD). Measured PSDs were used to evaluate the retrieved relation through the comparison of the expected and a measured radar reflectivity time series and also to derive some average relations between PSD bulk quantities.

We begin with the description of the experimental data used as the starting point of our calculations. In section 3, the theoretical basis and the main assumptions of our method to retrieve mass–velocity relations are described; the calculation results and uncertainties estimation are done in section 4. Section 5 is devoted to the validation of the proposed relation. Some useful applications can be found in section 6. Finally, section 7 is a summary of the results and a discussion of some of the limitations.

2. Experimental data used in the study

In this study we use a large dataset of snow measurements collected by a ground-based optical disdrometer [Hydrometeor Velocity and Shape Detector (HVSD)] (Barthazy et al. 2004). Measurements were taken at the Centre for Atmospheric Research Experiments (CARE) site (80 km north of Toronto) during winter 2005/06 as part of the Canadian CloudSat–Cloud-Aerosol Lidar and Infrared Pathfinder Satellite Observations (CALIPSO) Validation Project (C3VP) described by Hudak et al. (2006). The HVSD measurements provide particle size and terminal fall speed for each size class [assuming negligible vertical air velocity at the height of the HVSD measurement]. The study in Part I gives a detailed description of the measurements and investigates the variability in the velocity–size power-law coefficients. It shows that three environmental parameters—surface temperature, echo top temperature and, in particular, the depth of the precipitation system—determine the fall velocity fairly well. These factors express the impact of different active microphysical processes and of the time of growth.

In this study, nine snowfall events have been selected from the same database as in Part I by inspecting the vertically pointing X-band radar (VertiX) records and retaining only the snow systems uniform in time (for more details, see Part I). An additional requirement for the present study was the availability of well-sampled time series of the particle size distributions measured by the HVSD. For each snow event and each size bin, the mean values of velocity and area ratio, as well as their standard deviations, were obtained from Part I (see Table 1 for the summary of the analyzed events). We consider that during each event we have the same dominant type of particles characterized by the single mass and area relations. Values of the measured velocity and area ratio that deviate by more than 2 standard deviations from the average value for a given size class by the obtained relations were discarded as outliers.

The retrieved mass–velocity relationship is evaluated by comparing the time series of the reflectivity factor calculated from the mass–size relationship applied to the size distribution measured by the HVSD to the reflectivity obtained from the collocated small X-band bistatic Doppler radar providing measurements a few meters above ground [Precipitation Occurrence Sensor System (POSS); Sheppard 1990; Sheppard and Joe 2000]. The HVSD and POSS data are averaged over 6-min periods. The tests of the sensitivity of the results to the time averaging of the HVSD and POSS data have been done. Only very small differences in the results have been obtained for different averaging periods. The total number of analyzed spectra is 805.

The temperature at the ground during these events varied between $-17^\circ$ and $-2^\circ$C. The observed reflectivity factor varied between about $-10$ and $30$ dBZ.

3. Theoretical basis and main assumptions

a. Functional forms of the used dimensional relationships

The form of a terminal velocity–size and mass–size relationships adopted here is the most common power law:

$$u(D) = a_u D^{b_u},$$

(1a)
where $D$ represents a reference size of snowflake.

The coefficients $a_u$ and $b_u$ have been experimentally derived separately for each snow event. The range of the obtained $a_u$, $b_u$ values underlines the variability between the events as shown in Part I. For a given snow event, the derived $a_u$ and $b_u$ are assumed to be size independent. However, analytical relations such as those developed by Mitchell and Heymsfield (2005, henceforth MH05) or Khvorostyanov and Curry (2005, henceforth KC05) describe $a_u$ and $b_u$ for different size ranges. These variations with size are not very important for sizes that mostly contribute to the total mass for a given particle type having the same mass and area dimensional relationships. The calculated mean mass-weighted fall speed with constant coefficients used to characterize the particle sizes, where the major part of the mass is located, is only slightly different from the calculated fall speed with size-range-dependent coefficients. The calculations performed by McFarquhar and Black (2004) for different particle types also lead to the above conclusion.

The parameters $a_m$, $b_m$ in the mass–size relation (1b) are, in general, empirically derived from observations for different types of particles of different size ranges (e.g., Mitchell et al. 1990) and at different temperatures (Heymsfield et al. 2007). Our work aims at evaluating the coefficients $a_m$, $b_m$ and relating them to the $a_u$, $b_u$ coefficients in (1a) in the range of snowflake sizes that dominate the total snow mass or reflectivity factor.

Moreover, application of the hydrodynamic theory requires calculation of the cross-sectional area projected normally to the flow. In general, the cross-sectional area is directly related to the area ratio $A_r$, defined as the area of the particle image $A$ divided by that of a circle of diameter $D$:

$$A_r = A/(0.25\pi D^2) = (D_{eq}/D)^2,$$

where $D_{eq}$ is area equivalent diameter. Two forms have been proposed for the parameterization of $A_r$ for the side-view particle images, depending on the snowfall event:

$$A_r(D) = a_r(\exp(-b_rD) - 1),$$

$$A_r(D) = a_r D^{b_r},$$

for different types of particles of different size ranges (e.g., Mitchell et al. 1990) and at different temperatures (Heymsfield et al. 2007). Our work aims at evaluating the coefficients $a_m$, $b_m$ and relating them to the $a_u$, $b_u$ coefficients in (1a) in the range of snowflake sizes that dominate the total snow mass or reflectivity factor.

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where \( a_r, b_r, a_{re}, \) and \( b_{re} \) are constants determined empirically. The form selected to describe each snow event is the one with a greater correlation.

For nonspherical particles with complex structures such as ice crystals or snowflakes, the definition of size \( D \) is an issue in itself. This size is in general determined from particle image recorded with two-dimensional imaging probes; because it is noncircular, there are different possibilities to define the image size. Moreover, depending on the observation method, the image may refer to the side view when the image is projected on the vertical plane parallel to the flow (e.g., Barthazy et al. 2004) or the image projected on the horizontal plane normal to the flow (e.g., Mitchell et al. 1990). In investigations reported in the literature the dimension \( D \) has been considered in various ways—for example, as the diameter of an area-equivalent circle (i.e., the diameter that corresponds to the area as the particle shadow; Locatelli and Hobbs 1974; Francis et al. 1998), the equivalent spherical volume diameter (Brandes et al. 2007), the maximum diameter of the particle image (Kajikawa 1989; Mitchell et al. 1990), the mean of the two orthogonal extensions (Brown and Francis 1995), and the width of the enclosing box (Barthazy et al. 2004). For the mass relation a new approach is proposed by Baker and Lawson (2006), who introduce a size parameter that is a combination of the maximum dimension, width, area, and perimeter. A combination of parameters is also used by Hanesch (1999) in her study of the snowflake fall speed. In addition, the equivalent melted diameter is considered as a snowflake size parameter.

In the data collected by the HVSD the observed snowflake dimensions correspond to the two-dimensional side-view pattern. As a reference, the dimension \( D \) is chosen to be the maximum side-view size—that is, the maximum of the two perpendicular extensions: height of the image (vertical dimension as the snowflake falls) and width of the image (see Part I). This definition of the snowflake reference size is used in our dimensional relationships obtained from measurements of the velocity and area ratio and retrieved for mass. The dimensions normal to the flow required for the hydrodynamic calculations have to be estimated from the side-view projection.

b. Introduction of the Reynolds and Best (Davies) numbers

The general expression for terminal fall speed is given by equating the drag force \( F_D \) with the difference between the gravitational force \( mg \) and the buoyancy force. However, the problem with applying this expression is the dependence of the drag coefficient \( C_D \) on the fall speed itself. Thus, the common method for calculating the terminal velocity is from a determined relationship between the Reynolds number \( \text{Re} \) and the Best number \( X \) related to the drag coefficient but having no dependence on fall speed. The two numbers are defined as (e.g., List and Schemenauer 1970)

\[
\text{Re} = \frac{uD_s}{\nu} \quad \text{and} \quad \frac{X}{\nu} = \frac{2g(m/\rho_{\text{snow}})(\rho_{\text{snow}} - \rho_a)}{\rho_a \nu^2 A_{\perp}} \left( \frac{uD_s}{\nu} \right)^2; \tag{4}
\]

taking \( \rho_{\text{snow}} - \rho_a = \rho_{\text{snow}} \), the following expression for \( X \) is obtained:

\[
X = \frac{2g mD_s^2}{\rho_a \nu^2 A_{\perp}}. \tag{5}
\]

The index \( \perp \) describes the quantities related to the horizontal projection (perpendicular to the flow), while the index \( || \) (introduced in Eq. 7a below) represents the projection on the vertical plane (parallel to the flow).

The environmental conditions are included via the kinematic viscosity and air density, \( \nu \) and \( \rho_a \), respectively; \( g \) denotes gravitational acceleration. The expression for \( X \) contains the two primary variables that determine \( u \) (i.e., the mass \( m \) and the effective particle area projected normal to the flow \( A_{\perp} \), in addition to \( D_s \), which denotes the chosen characteristic dimension of the particle). Introducing the area ratio normal to the flow, \( A_{r\perp} \), and using (2), \( X \) can be expressed as

\[
X = \frac{8g}{\pi \rho_a \nu^2 A_{r\perp}} \left( \frac{D_s}{D_{\perp}} \right)^2, \tag{6}
\]

where \( D_{\perp} \) denotes the maximum diameter in the direction normal to the flow that is not measurable and therefore must be estimated. The final expression for \( X \) depends on the choice of the characteristic size \( D_s \). The symbols \( X_{||} \) and \( X_{\perp} \) below describe \( X \) calculated with \( D_s = D (\text{our measured maximum side-view diameter taken as reference}) \) and \( D_s = D_{\perp} \) (estimated maximum diameter normal to the flow), respectively:

\[
X_{||} = \frac{8g}{\pi \rho_a \nu^2 A_{r\perp}} \left( \frac{D}{D_{\perp}} \right)^2, \tag{7a}
\]

\[
X_{\perp} = \frac{8g}{\pi \rho_a \nu^2 A_{r\perp}}. \tag{7b}
\]

The corresponding Re numbers are given by

\[
\text{Re}_{||} = \frac{uD_s}{\nu} \quad \text{and} \quad \text{Re}_{\perp} = \frac{uD_{\perp}}{\nu}. \tag{8a}
\]
Assuming an idealized spheroidal shape for the snowflakes and taking $D_s$ equal to the maximum diameter projected on the flow $D_\perp$, Böhm (1992) modified the general expression for $X$ (7b) by introducing the fourth root of the inverse of area ratio and the axial ratio $\phi$ of this assumed oblate spheroid shape of snowflake:

$$X_\perp = \frac{2g}{\rho_d \nu^2} \left( \frac{m}{\max(\phi, 1)} \right).$$

### c. Relations between Reynolds and Best numbers

The equation relating the $X$ to the Re numbers can be obtained empirically from direct measurements of particle velocity, cross-sectional area, and size (e.g., Knight and Heymsfield 1983; Heymsfield and Kajikawa 1987; Redder and Fukuta 1991). However, important uncertainties are associated with these measurements, mainly those of the cross-sectional area that is normal to the flow.

A generalized theoretical relationship between Re and $X$ derived from the boundary layer theory was developed by Böhm (1989) and Mitchell (1996) on the basis of the previous work of Abraham (1970), giving

$$\text{Re}_\perp = \frac{u D_\perp}{\nu}.$$  \hspace{1cm} (8b)

The values of the two constants $\delta_0$ and $C_0$ characterizing the boundary layer shape and thickness are 5.83 and 0.6, respectively, for ice particles. For aggregates, MH05 modified slightly the relation by adding an empirical term $-a_0 X^{b_0}$ with $a_0 = 0.0017$, $b_0 = 0.8$. This additional term accounts for turbulent flow around larger aggregates so that there is better agreement with the measurements presented by Heymsfield et al. (2002b). KC05 proposed an alternate method to introduce this correction.

In this study, the particle mass is derived from the Best number that is calculated from Re. Therefore, we invert the proposed relations $Re = f(X)$ from MH05 and KC05 by fitting $\log(X)$ to an eighth-order polynomial of $\log(Re)$:

$$\log(X) = \sum_{i=1}^{8} C_i \left[ \log(Re) \right]^i,$$

with $C_i = C_{MH,i}$ for the MH05 relation, and $C_i = C_{KC,i}$ for the KC05 relation. The coefficients $C_i$ are given in Table 2. The $X$–Re relations given in (11) are applied for the pairs $(X_\perp, \text{Re}_\perp)$ and $(X_\parallel, \text{Re}_\parallel)$. The differences between the values of $X$ retrieved using different $X$–Re relationships are evident for the larger Re, which describe the snowflakes that have a larger contribution to the reflectivity values, as seen in Fig. 1, which gives a good estimate of the uncertainty associated with the relationship $X$–Re.

### d. Assumed relations between side-view snowflake projection and the section perpendicular to the flow

Snowflake size and area measured by the HVSD represent a side view of the particle. Because of aerodynamic forcing, the falling snowflakes have in general their maximum size oriented horizontally (e.g., Magono and Nakamura 1965). On average, their horizontal dimension is larger than the value measured when viewed on side projection. Since the horizontal (normal to flow) dimensions that are required for the velocity calculations cannot be measured, they must be estimated from the side projection. For this, 1) we assume that $A_{r\perp} \approx A_r$ (i.e., the area ratio is independent of the angle of observation, normal or parallel to the flow) and 2) we use the relation from Schefold (2004) giving the ratio of the area projected normally to the flow to the area from the side view, given as a function of the canting angle $\alpha$ and the side projected axial ratio $\epsilon$. The latter is the quotient of the side projected minor axis to the side projected major axis. Thus,

$$f_A = \frac{A_\perp}{A_\parallel} = 1 + \epsilon - \left( \frac{1}{\epsilon_{\text{min}}} \left( \frac{\alpha}{90^\circ} - 1 \right) + 1 \right),$$

where $\epsilon_{\text{min}}$ describes the minimal value of the axis ratio evaluated from the measurements as equal to 0.3. The last relation together with the assumption $A_{r\perp} \approx A_r$ is also used to calculate the maximum horizontal dimension $D_\perp$ from the maximum side dimension $D$ taken as the reference dimension:
Equations (12) and (13) give the ratio $D/D_\perp$ decreasing with $D$ on average. For smaller sizes the mean value of this ratio is around 0.85 and decreases progressively. Since the value of 0.8 is consistent with the results from the snowflake fractal analysis by Schmitt and Heymsfield (2010), the minimal value of 0.75 is imposed on this ratio and the corresponding maximal value on $f_A$ as calculated in (12).

Our first assumption about the area ratio being independent on the projection plane seems reasonable for snow aggregates for which the value of area ratio has been shown to be related to particle density (e.g., Heymsfield et al. 2004). Moreover, this postulate agrees with the results of the recent theoretical work on the fractal dimensions of aggregates (Schmitt and Heymsfield 2010).

4. Derivation of the mass–velocity relationship

An average relation between a snowflake mass and velocity has been derived in the following steps with the first four steps calculated separately for each snow event:

(i) Calculation of Re from the HVSD measurements of terminal velocity and area ratio from (8a) or (8b) and (13);

(ii) Estimation of the value of $X$ from Re using a polynomial fit (11) with constants $C_{MHf}$ for the MH05 relation, and with $C_{KCf}$ for the KC05 relation;

(iii) Best estimation of mass for $D$-size snowflake from the estimated $X$ and area ratio normal to the flow;

(iv) Determination by regression of the coefficients in the power-law relation $m-D$; and

(v) From all events, derivation by regression of an average relation between the power-law coefficients in the mass and velocity dimensional relationships.

In what follows steps iii–v are described in some detail.

a. Best estimation of mass for $D$-size snowflake and its uncertainty

Using the value of $X$ estimated from Re, and with the help of the relations (12) and (13) and the approximation $A_\perp \approx A_r$, the following three equations for mass are obtained from (7a), (7b), and (9):

$$m = \frac{\pi \rho_d v^2}{8g} X_{\parallel} A_r f_A,' \quad (14a)$$

$$m = \frac{\pi \rho_d v^2}{8g} X_{\perp} A_r,' \quad (14b)$$

$$m = \frac{\rho_d v^2}{2g} X_{\parallel} (A_r)^{1/4} \phi.' \quad (14c)$$

For each size bin, using (14a)–(14c) 24 different values of mass, $m_k$ with $k = 1, \ldots, 24$, are calculated from different combinations of the $X$–Re relationships, smoothed and unsmoothed measured $u$ and $A_r$, and two values of axial ratio of 0.7 and 0.8 put in (14c). Each value of mass is considered as the “measured” value of mass using different methods and interrelated through the common parameters. The geometric weighted mean is taken as the best estimate of the snowflake mass with reference diameter $D$:

$$\log[\bar{m}(D)] = \frac{n_{k=24}}{2} \log(m_k) \left/ \sum_{k=1}^{n_{k=24}} \frac{1}{(\Delta \log m_k)^2} \right. \sum_{k=1}^{n_{k=24}} \frac{1}{(\Delta \log m_k)^2}. (15)$$

The weight of each measured mass is the inverse of the square of the corresponding uncertainty $(\Delta \log m_k)^2$ obtained by combining the contribution of the uncertainty related to the different quantities involved. To unify the calculation of $(\Delta \log m_k)^2$, we take the following single logarithmic form of Eqs. (14a)–(14c):

$$\log(m_k) = \log(C_E) + \delta_e \log(X_{\parallel}) + \delta_b \log(X_{\perp}) + (1 - 0.75\delta_e) \log(A_r) + \delta_a \log(f_A) + \delta_c \log(\phi), (16)$$
with \( \delta_a = 1 \) if (14a), otherwise 0,
\( \delta_c = 1 \) if (14c), otherwise 0, and
\( \delta_{b,c} = 1 \) if (14b) or (14c), otherwise 0.

Here \( C_E \) is a parameter describing the environmental conditions, and is different by a factor of \( 4/\pi \) in (14c) relative to (14a) and (14b). The uncertainty of \( C_E \) will be neglected here.

In general, the uncertainty related to the calculated \( X \) value can be decomposed into two terms. The first represents the uncertainty of the used \( X \)--Re relationship (11) for different coefficients \( C_i \). The second describes the contribution to the uncertainty of the computed Re value:

\[
(\Delta \log X)^2 \approx (\Delta \log X)_{C,i}^2 + (\Delta \log X)_{Re,c}. \tag{17}
\]

Assuming that the uncertainty in the \( X \)--Re relation is 30% (as in Field et al. 2008), we have \((\Delta \log X)_{C,i}^2 \approx 0.017\).

The same potential bias of \( \log(\cdot) \) evaluated from (11) as calculated via (10). The second term on the rhs of (17) is calculated via (10). The uncertainty of \( \log(\cdot) \) is different for \( \text{CE} \) relative to (14a) and (14b). The uncertainty of \( \log(\cdot) \) is different for \( \text{CE} \) and (14a), (14b), and (14c), respectively. This means that the relative error in our mass estimate is between 40% and 50% and is the same for all sizes \( D \) since the relative uncertainties were taken as constant.

To evaluate the uncertainty included in the mass estimate \( \log[m(D)] \) given by (15), the effect of the existing correlations between the many components of every couple of \( m_k \) has to be taken into account. To do this, the uncertainty of \( \log(m_k) \) has to be separated into uncorrelated and fully correlated uncertainties for each combination of two values of \( m_k \). Then the propagation error law has to be applied. To avoid these complex calculations we only estimate the maximum uncertainty for which all \( m_k \) are fully correlated between them. In this case, the total uncertainty is simply the linear sum of the weighted uncertainties of an individual \( m_k \):

\[
(\Delta \log m) = (1/W_{\text{tot}}) \sum_{k=1}^{n_k} (1/\Delta \log m_k), \tag{21}
\]

with \( W_{\text{tot}} = \sum_{k=1}^{n_k} 1/(\Delta \log m_k)^2 \). The value of \( (\Delta \log m) \) computed from (21) provides the average uncertainty of \( m_k \) calculated from the relations (20) and is equal to 0.193.

\[ b. \ \text{Determination by regression of the coefficients}
\]

in the power-law relation \( m--D \)

Assuming that the calculated value of \( \tilde{m}(D) \) represents the best evaluation of the snowflake mass with reference diameter \( D \), we look for a dimensional relation \( m--D \) of the power-law form given by (1b) for a snowflake of a given size \( D \) for each snow event. Taking the logarithm of (1b) yields the linear equation
\[
\log(\tilde{m}) = \log(a_m) + b_m \log(D). \tag{22}
\]

The best estimate of the two coefficients is obtained by a least squares fit. Two examples are shown in Fig. 2, where the solid line shows the best-fit power law. For the nine snow events studied, the obtained coefficients \(a_m\) and \(b_m\) and their values are given in Table 1 and shown in Fig. 3. For comparison, the mean value obtained by Brandes et al. (2007) from a very large dataset is also superposed. However, because \(D\) in their study was taken as equivalent diameter, their relation has been recalculated using the average relation \(D_{eq} = D\) obtained from our data: \(D_{eq} = 0.69D^{0.95}\) in CGS units (very close to the average relation in Part I: \(0.71D^{0.92}\) obtained from the larger dataset). Moreover, the sets of the parameters in the relations of mass and maximum size are also superposed after recalculating using our relation (13) between maximum side-view size and maximum horizontal dimension, taking the value for larger particles of \(1/\sqrt{f_A} = 0.75\). The relations shown are the average mass relation from Mitchell et al. (1990) for the 1986–87 field season, from Kingsmill et al. [2004; based on the data by Heymsfield et al. (2002b)], and from Matrosov and Heymsfield (2008) obtained for precipitating clouds.

For some of the cases studied, the investigation of the regression that gives the best set \((a_m, b_m)\) shows that a better description of the \(m–D\) relationship is obtained when the analysis is performed separately for two size regimes: for \(D\) smaller than 2–3 mm, and for \(D\) larger than 2–3 mm. This separation can be important in all microphysical studies where the lower-order moments of the size distribution become important.

As shown in Fig. 3, the obtained exponents \(b_m\) through the mass power law for different snow events cluster between values of 1.8 and 2.10, with the mean value close to 1.9. The value of 2.0 corresponds to an effective snowflake density decreasing with size as \(D^{-1}\) and is in good agreement with numerous observational studies of snow at the surface (Mitchell et al. 1990 from the overall observed particle types; Brandes et al. 2007) and aircraft observations (Heymsfield et al. 2002a). Moreover, most of the snowfall mass reaching the ground is generally associated with aggregates of ice crystals and theoretical studies (Westbrook et al. 2004) predict a value of 2 for the exponent in the mass–dimension power law for snow aggregates. Certainly, individual crystals with forms other than aggregates may be less well represented by this exponent; however, they contribute only a small fraction of the overall snowfall characteristics. On the other hand, an exponent equal to 2 probably does not describe correctly the snowflakes larger than about 1.5 cm because of the rather small number of the observed particles with these sizes. Contribution of these particles to the reflectivity is significantly reduced because of the non-Rayleigh effect. For example, for a snowflake with diameter of 2 cm, the reduction of backscattering cross section with respect to Rayleigh regime is about two orders of magnitude, as shown from the results of the T-matrix calculations in Matrosov et al. (2009).

Taking into account the general uncertainty in the mass–size relationship, in our attempt to develop a relatively simple snow parameterization we set the exponent \(b_m\) to 2 for the power-law relation representing the precipitating snow. The relation (22) becomes

\[
\log(\tilde{m}) = \log(a_m) + 2 \log(D), \tag{22'}
\]
with only the coefficient \(a_{\text{mf}}\) to be determined in the mass–size relationship (the symbol \(a_{\text{mf}}\) denotes \(a_{m}\) with fixed \(b_{m} = 2\)). All symbols used here are listed in Table 4.

On the other hand, the velocity power laws of the form (1a) obtained by Part I for different homogenous snow events show the relatively small variability of the exponent \(b_{u}\). They showed that the snowflake velocity can be modeled with good accuracy with a fixed exponent \(b_{u} = 0.18\) and a varying coefficient \(a_{u}\). This fixed value of \(b_{u}\) is adopted here for further calculations and the corresponding value of \(a_{u}\) is represented by \(a_{\text{uf}}\). Subsequently, all the calculations that lead to the best set of mass–size relationship based on the measured snowflake velocities and area ratios are repeated but with an imposed value of \(b_{u} = 0.18\) in the fitted velocity–size relationship and \(b_{m} = 2\) in the final mass–size power law. In Table 1 the obtained values of \(a_{\text{mf}}\) and \(a_{\text{uf}}\) are given. In Fig. 2 the dashed line show the result with imposed \(b_{m} = 2\) using (22'), while the solid line has been obtained through regression with no restriction on the \(b_{m}\) value (22). The difference is within the bounds of uncertainty.

To evaluate the impact of imposing the value of 2 for \(b_{m}\) we calculate, from the observed PSD for the time series of snow events, the snow ice mass content (IWC) in two ways: first, from the \(a_{m}\) and \(b_{m}\) set that was obtained by regression with \(b_{m}\) varying, named IWC_{am}, and second, using the \(a_{\text{mf}}\) value obtained for \(b_{m} = 2\), named IWC_{amf}. The root-mean-square error RMSE = \(\left< (\text{IWC}_{\text{am}} - \text{IWC}_{\text{amf}})^2 \right>^{1/2} = 0.004 \text{ g m}^{-3}\) while the root mean of fractional error RMFE = \(\left< (\text{IWC}_{\text{am}} - \text{IWC}_{\text{amf}})/\text{IWC}_{\text{am}} \right>^{1/2} = 0.07\) for the entire length of the time series. The same calculation performed for the reflectivity factor gives an RMSE value of 0.35 dB; for reflectivity-weighted velocity (using fixed \(b_{u} = 0.18\)), the calculated RMSE and RMFE are 0.006 m s\(^{-1}\) and 0.007, respectively. The particles’ radar backscatter cross section is calculated based on the Mie theory assuming the mixing rule of Maxwell–Garnett for ice–air mixture [for more details, see model 5 in Fabry and Szyrmer (1999)]. We discuss the inaccuracy of the scattering calculations arising from the Mie calculations in section 5.

The estimates of the uncertainties in the coefficient \(a_{\text{mf}}\) and in the value of \(m\) predicted by (22') for each \(D\) must take into account the effect of the correlation between calculated values of \(\tilde{m}\) resulting from the contribution to their uncertainty of the correlated components of \(m_{k}\). With the assumption of a positive correlation, the magnitude of the uncertainties rises and the analytical calculations become very complex. To simplify the calculations we use the uncertainty estimate for each log \(m\) given by the error bars in Fig. 2. By imposing a slope equal to 2, the uncertainty in \(a_{\text{mf}}\) is estimated by finding the range of intercept values for which the regression line covers most of the error bars representing \(\Delta(\log \tilde{m})\). The obtained relative uncertainties in \(a_{\text{mf}}\), denoted by \(\Delta a_{\text{mf}}/a_{\text{mf}}\) in the last column of Table 1 for each snow event, are included in the regression calculation of the approximate relation between \(a_{\text{mf}}\) and \(a_{\text{uf}}\) in (23) below and shown by error bars in Fig. 4.

c. Derivation of average mass–velocity relationship

To reduce the number of parameters describing an individual snowflake with size \(D\), we search for an approximate relation between its mass and velocity, assuming that other factors (such as the shape of the individual crystals) introduce negligible correction to the average mass–velocity relationship. This relation is obtained for the mass and velocity exponents fixed at 2.0 and 0.18, respectively.

The atmospheric conditions have some influence on the snowflake mass calculated from (14a)–(14c), and hence on the value of \(a_{\text{mf}}\) derived from experimental values of \(a_{\text{uf}}\). This influence is mainly through the kinematic viscosity of the air \(\nu\), which is equal to the dynamic viscosity, a function of temperature, divided by the air density. The air density variation can be neglected here because all the observations were made at the ground and to a first approximation the air density changes with height (the temperature correction is evaluated at less than 2% with respect to the average temperature of –10°C). Besides the explicit dependence of \(m\) on the
magnitude of \( \nu \) given in (14), the values of the calculated \( \text{Re} \) and \( X \) also change with \( \nu \). Therefore, prior to the search of the mass–velocity relationship, the influence of temperature variations on the estimated value of \( a_{mf} \) has to be investigated.

The snow measurements considered here were done at temperatures between \(-2^\circ \text{C}\) and \(-17^\circ \text{C}\). The range in dynamic viscosity corresponding to this temperature range is from \(1.72 \times 10^{-5}\) to \(1.63 \times 10^{-5}\) kg m\(^{-1}\) s\(^{-1}\). The correction multiplication factor that normalizes the estimated mass, and therefore \( a_{mf} \), was calculated with respect to calculations done for an average value of \(-10^\circ \text{C}\).

The obtained values of the correction factor are 1.027 and 0.983 at \(-2^\circ \text{C}\) and \(-17^\circ \text{C}\), respectively. This range is very

### Table 4. List of symbols.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A )</td>
<td>Area of the particle image (here side-view image)</td>
</tr>
<tr>
<td>( A_\perp )</td>
<td>Cross-sectional area projected normal to the flow</td>
</tr>
<tr>
<td>( a_m )</td>
<td>Constant in mass–size power law (1b)</td>
</tr>
<tr>
<td>( a_{mf} )</td>
<td>Constant in mass–size power law (1b) with imposed ( b_m = 2 )</td>
</tr>
<tr>
<td>( A_t )</td>
<td>Ratio of the area of the particle image to the circle of diameter ( D )</td>
</tr>
<tr>
<td>( A_{r,\perp} )</td>
<td>Ratio of ( A_\perp ) to ( (\pi/4)D^2 )</td>
</tr>
<tr>
<td>( a_r )</td>
<td>Constant in area ratio–size relation (3a)</td>
</tr>
<tr>
<td>( a_{re} )</td>
<td>Constant in area ratio–size relation (3b)</td>
</tr>
<tr>
<td>( a_u )</td>
<td>Constant in velocity–size relation (1a)</td>
</tr>
<tr>
<td>( a_{uf} )</td>
<td>Constant in velocity–size relation (1a) with imposed ( b_u = 0.18 )</td>
</tr>
<tr>
<td>( b_m )</td>
<td>Exponent in mass–size power law (1b)</td>
</tr>
<tr>
<td>( b_r )</td>
<td>Constant in area ratio–size relation (3a)</td>
</tr>
<tr>
<td>( b_{re} )</td>
<td>Exponent in area ratio–size relation (3b)</td>
</tr>
<tr>
<td>( b_u )</td>
<td>Exponent in velocity–size relation (1a)</td>
</tr>
<tr>
<td>( C_D )</td>
<td>Drag coefficient</td>
</tr>
<tr>
<td>( C_F )</td>
<td>Parameter describing the environmental conditions in (16)</td>
</tr>
<tr>
<td>( C_I )</td>
<td>Coefficients in the polynomial fitting in (11) to the relation ( \text{Re}–X ) from MH05 ((C_{MH})) and from KC05 ((C_{KC})), given in Table 2</td>
</tr>
<tr>
<td>( C_0 )</td>
<td>Constant in the relation ( \text{Re}–X ) (10) from Böhm (1989) and Mitchell (1996)</td>
</tr>
<tr>
<td>( D )</td>
<td>Snowflake reference size (taken here as maximum side view diameter)</td>
</tr>
<tr>
<td>( D_{eq} )</td>
<td>Area equivalent diameter</td>
</tr>
<tr>
<td>( D_\perp )</td>
<td>Estimated maximum diameter in the horizontal plane (i.e., normal to the flow)</td>
</tr>
<tr>
<td>( D_\alpha )</td>
<td>Characteristic dimension of the particle (in general)</td>
</tr>
<tr>
<td>( f_A )</td>
<td>Ratio of the effective area projected normally to the flow to the area from the side view</td>
</tr>
<tr>
<td>( g )</td>
<td>Gravitational acceleration</td>
</tr>
<tr>
<td>( \text{IWC} )</td>
<td>Snow ice water content</td>
</tr>
<tr>
<td>( \text{IWC}_{am} )</td>
<td>IWC calculated using the derived ( a_m ) and ( b_m ) in (1b)</td>
</tr>
<tr>
<td>( \text{IWC}_{amf} )</td>
<td>IWC calculated using the derived ( a_{mf} ) with imposed ( b_m = 2 ) in (1b)</td>
</tr>
<tr>
<td>( m )</td>
<td>Particle mass</td>
</tr>
<tr>
<td>( \text{Re} )</td>
<td>Reynolds number</td>
</tr>
<tr>
<td>( \text{Re}_s )</td>
<td>Reynolds number calculated with ( D ) (maximum side-view diameter)</td>
</tr>
<tr>
<td>( \text{Re}_\perp )</td>
<td>Reynolds number calculated with ( D_\perp )</td>
</tr>
<tr>
<td>( S )</td>
<td>Liquid equivalent snowfall precipitation rate</td>
</tr>
<tr>
<td>( U_Z )</td>
<td>Mean reflectivity-weighted fall speed</td>
</tr>
<tr>
<td>( U_M )</td>
<td>Mean mass-weighted fall speed</td>
</tr>
<tr>
<td>( X )</td>
<td>Best number</td>
</tr>
<tr>
<td>( X_{ih} )</td>
<td>Best number calculated with ( D ) (maximum side view diameter)</td>
</tr>
<tr>
<td>( X_{i,\perp} )</td>
<td>Best number calculated with ( D_\perp )</td>
</tr>
<tr>
<td>( Z_e )</td>
<td>Equivalent reflectivity factor (mm(^6) m(^{-3}))</td>
</tr>
<tr>
<td>( Z_{e,\text{dBZ}} )</td>
<td>Equivalent reflectivity factor (dBZ)</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>Canting angle from the side projection</td>
</tr>
<tr>
<td>( \chi )</td>
<td>Coefficient in ( Z–S ) power law</td>
</tr>
<tr>
<td>( \delta_\alpha )</td>
<td>Constant in the relation ( X–\text{Re}–(10) ) from Böhm (1989) and Mitchell (1996)</td>
</tr>
<tr>
<td>( \nu )</td>
<td>Side projected axial ratio</td>
</tr>
<tr>
<td>( v_{min} )</td>
<td>Minimal value of ( \nu )</td>
</tr>
<tr>
<td>( \nu )</td>
<td>Kinematic viscosity of air</td>
</tr>
<tr>
<td>( \rho_a )</td>
<td>Air density</td>
</tr>
<tr>
<td>( \rho_{a0} )</td>
<td>Air density at the ground level</td>
</tr>
<tr>
<td>( \rho_{snow} )</td>
<td>Density of snowflakes</td>
</tr>
<tr>
<td>( \phi )</td>
<td>Axial ratio of snowflakes</td>
</tr>
<tr>
<td>( \omega )</td>
<td>Exponent in ( Z–S ) power law</td>
</tr>
</tbody>
</table>
small and therefore suggests that the impact of the atmospheric conditions during the measurements on the derived amf–auf relation can be neglected.

In Fig. 4 the values of amf derived for bm = 2 and bu = 0.18 are plotted as a function of the velocity coefficient auf obtained from the data. Each point represents one of the nine events. The analytical approximate relation between amf and auf is assumed to have the form log(amf) = a1 + b2 auf. Coefficients a1 and b2 have been determined by a weighted total least squares (WTLS) regression described in Krystek and Anton (2007). For each [auf, log(amf)] pair, the corresponding uncertainties are D auf = 0.2 auf from Part I and D(log(amf)) obtained from regression (22). These uncertainties are shown in Fig. 4. The solid line in Fig. 4 represents the least squares fitting with a1 = 3.02 and b2 = 0.0065 in CGS units, giving

\[
\log(a_{mf}) = -(3.02 \pm 0.04) + (6.5 \pm 0.5) \times 10^{-3} a_{uf}. \tag{23}
\]

The uncertainties associated with the two coefficients are valid when the effect of correlation between the values of amf is not taken into account. The magnitudes of amf obtained from (23) are shown in Table 1. The weighted average value of amf for the nine analyzed events is shown in Fig. 4 by a dotted line. Its value is equal to 0.0044 g cm\(^{-2}\) and is very close to the average from observations of Mitchell et al. (1990) and Kingsmill et al. (2004) recalculated using (13) as in Fig. 3. The value of a_{uf} corresponding to this weighted average is equal to 102 cm\(^{0.82}\) s\(^{-1}\).

Using (23), snowflake velocities are calculated as a function of melted equivalent diameter, and the results are shown in Fig. 5. In the upper graph, the derived relations between snowflake velocity and melted diameter for each of the nine events are compared with the empirically obtained relations of Langleben (1954) for the different type of snowflakes indicated by the various dashed lines described in the top of the figure. (bottom) Comparison of snowflake velocity calculated by our relation (23) for average amf with some other relations published previously. [LH 1974 indicates Locatelli and Hobbs (1974).]

All presented results are derived from measurements at the surface. Therefore, when applied at other pressure levels the velocity coefficients (a_{uf}, a_{uf}) have to be adjusted for a change in altitude using, as a first approximation, the adjustment given in Heymsfield et al. (2007): [a_{uf}(p)/a_{uf}(1000 hpa)] = [\rho_{a0}/\rho_{a0}(p)]^{0.54}, where \rho_{a0} and \rho_{a0}(p) are the air density at the ground level and at pressure p.
5. Evaluation

In this study the measured reflectivity is used to evaluate the procedure for the retrieval of mass/density as a function of particle size. For each snowfall event, the mass coefficient $a_{amf}$ is estimated from the velocity coefficient $a_v$ using (23). Knowing the mass–size relationship, the backscatter cross section of each size bin is calculated using the Mie theory from model 5 described in Fabry and Szyrmer (1999). For particles smaller than about 5 mm, the scattering at X-band is in the Rayleigh regime (i.e., that the reflectivity is proportional to mass squared, independent of the particle shape; e.g., Matrosov et al. 2009). For larger particles the non-Rayleigh effect leads to the reduced backscattering that decreases with size for particles larger than about 1.5 cm (Matrosov et al. 2009).

The other question to consider is the effect of nonsphericity. According to Ishimoto (2008), using the finite difference time domain (FDTD) method for fractal-shaped snowflakes, the sensitivity to the particle shape become important for snowflakes with a diameter of the ice-mass-equivalent sphere greater than about 2.4 mm (which corresponds on the average to $D = 1.2$ cm). The same result has been obtained with T-matrix method in Wang et al. (2005). The inaccuracy of the Mie calculations assuming spherical form depends on the choice of mapping of particle properties into sphere parameters. By choosing in our model the area-equivalent diameter (i.e., between ice-mass-equivalent sphere diameter and maximum dimension) as diameter of the equivalent sphere, the inaccuracy of the scattering calculations arising from the Mie calculations for snowflakes not larger than 2 cm is relatively low. Moreover, as shown in Heymsfield et al. (2008), the contribution to the X-band reflectivity of the particles larger than about 1 cm (assuming the exponential size distribution with slope equal to 6 cm$^{-1}$ that is representative for our dataset) is negligible. Furthermore, Matrosov et al. (2009) concluded that at X-band the spherical model for low-density snowflakes provides the necessary accuracy in the reflectivity calculations if the non-Rayleigh effects are not too important.

But perhaps the most direct way of considering the effect of nonsphericity on reflectivity is by noting that observed values of differential reflectivity in precipitating snow are a few decibels, indicating that the strong effect of low density of particles decreases the influence of shape on backscattering, which confirms the results of Matrosov et al. (2009).

The expected reflectivity factor is computed from the snowflake size distributions measured by the HVSD. The time series of the calculated reflectivity are compared with the reflectivity derived from the collocated POSS. The HVSD and POSS data are averaged over 6-min intervals. The scatterplot of the reflectivity calculated for all nine events versus the POSS measured reflectivity is presented in Fig. 6. The root mean standard error is equal to 2.71 dB. Each symbol represents a 6-min average.

Figure 7 presents an example of the reflectivity time series. The solid line gives the POSS-measured reflectivity; the dotted–dashed line is the reflectivity calculated from the mass relationship derived for the given event of 6 January 2006. This particular event is characterized by the lowest retrieved value of $a_{amf}$ (see Table 1). To provide a limit on the importance of the mass–size relation on the uncertainty in the reflectivity calculations, the dashed line in Fig. 7 shows the reflectivity calculated for the PSDs of 6 January 2006 but using the mass–size relation retrieved for the event of 9 January 2006 where the estimated $a_{amf}$ was the greatest (see Table 1). The relative difference in $a_{amf}$ between these two events is about 1.5. Under the Rayleigh regime, the effective backscattering cross section is proportional to the square of the mass. Then, we have $\Delta mZ_e/Z_e \approx (1 + 1.5)^2 - 1$ for $Z_e$ in mm$^6$ m$^{-3}$, giving the contribution of the mass uncertainty to the calculated dBZ of $\Delta mZ_e dBZ = 10 \log(1 + \Delta mZ_e/Z_e) \approx 8$ dB, as can be noted in Fig. 7. For a smaller relative uncertainty in $\Delta a_{amf}/a_{amf}$ we can write $\Delta mZ_e/Z_e \approx 2\Delta a_{amf}/a_{amf}$ from which we can deduce that $\Delta mZ_e dBZ = (10/\ln10)\Delta mZ_e/Z_e$, equal to about 8.7 $\Delta a_{amf}/a_{amf}$ valid under Rayleigh regime.
6. Applications

The retrieved relation (23) used in conjunction with the PSD information makes it possible to derive useful relations between different bulk properties of snow PSD such as equivalent reflectivity factor $Z_e$, snow ice water content (IWC), liquid equivalent snowfall precipitation rate $S$, or reflectivity- and mass-weighted velocity $U_Z$ and $U_M$. The time series of the snowflake PSDs are provided for each analyzed snowfall case by the optical imager HVSD.

In Fig. 8 the reflectivity-weighted velocity at the ground level is plotted against the reflectivity as obtained from the calculations for all snow events. The solid lines are obtained through regression for the following linear relation between $U_Z$ and $Z_e$ in dBZ and $\log(amf)$:

$$U_Z = 326.0 + 0.559Z_e + 104.3\log(amf).$$

(24)

Note that $U_Z$ and $amf$ are in CGS units. Introducing a weighted mean value for $amf$ equal to 0.0044 in CGS units as shown in Fig. 4, the average $U_Z-Z_e$ relation becomes

$$U_Z = 80.2 + 0.559Z_e$$

(24')

for precipitating low-density snow. The last term in (24) introduces the estimated sensitivity of $U_Z$ to the error in mass-dimensional relationship. The uncertainty in $U_Z$ resulting from the uncertainty in the value of $amf$ may be estimated as $\Delta U_Z = 50\Delta amf/amf$. The relation (24) when $amf$ is given is expected to provide a more accurate $U_Z-Z_e$ relation compared to (24'). For higher altitudes, the air density correction for $U_Z$ has to be introduced as given at the end of section 4. The mean ratio of calculated reflectivity-weighted velocity to mass-weighted velocity is equal to 0.93.

The second useful relationship between two bulk quantities is the power law relating the equivalent reflectivity factor $Z_e$ and the snowfall precipitation rate $S$:

$$Z_e = \chi S^\omega,$$

with $Z_e$ in mm$^6$ m$^{-3}$ and $S$ in mm h$^{-1}$. In Fig. 9, the calculated snow precipitation rate $S$ is plotted as a function of the calculated $Z_e$ in dBZ. The solid line shows the best fit obtained by the WTLS method applied to the logarithmic form of the $Z_e-S$ power law. The uncertainties taken into account for $Z_e$ and $S$ in WTLS calculations describe the contributions from the uncertainties in snowflake fall speed and mass represented by the uncertainties in $auf$ and $amf$, and the correlation between them. The obtained values of the constants are $\chi = 494$ and $\omega = 1.44$.

Our $Z_e-S$ relationship is in a general agreement with some of relations previously derived empirically, semi-empirically, or theoretically that are drawn in Fig. 9 [Ohtake and Henmi (1970) for snowflake consisting of spatial dendrites; Fujiyoshi et al. (1990) for 1-min and 30-min average; Matrosov (1992) for snow density of 0.02 g cm$^{-3}$; Zawadzki et al. (1993) recalculated using our average relation $S-IWC$: $S = 3.3IWC^{1.03}$, with IWC
in g m\(^{-3}\); and seven relations retrieved by Huang et al. (2010). However, compared with the recent work of Matrosov et al. (2009), our value of \(x\) is much larger for similar values of the exponent \(\omega\). This difference may be partly explained by the applied snow velocity relationship. Our measured velocity for low-density snowflakes as shown in Part I is in general lower than that obtained from the relation used in Matrosov’s work.

The increase of snowflake density (i.e., the value of \(amf\) in our parameterization) is related to the increasing velocity described by \(auf\). For the same snowflake spectrum, this increase results in the increase of both \(Z_e\) (proportional to \(a_{mf}^2\) under Rayleigh regime) and \(S\) (proportional to the product \(a_{mf} \times auf\)). To estimate the impact of these changes on the coefficients in the \(Z_e-S\) relation, we present in Fig. 10 the calculations of \(Z_e\) and \(S\) repeated 3 times for the time series of PSDs on 9 January 2006 using 1) \(amf\) and \(auf\) obtained for this event (solid circle), 2) these two coefficients retrieved for a different event, 6 January 2006 (open circle), and 3) weighted average values of these coefficients as shown in Fig. 4 (symbol “X”). The calculation results presented in Fig. 10 show that the coefficients of the power law \(Z_e-S\) for low-density snowfall do not exhibit a significant dependence on the assumed mass–velocity relationship if (23) is satisfied. Matrosov et al. (2009) show the slight dependence of the coefficient \(\chi\) on the assumed mass dimensional relationship. However, they used the density independent expression for snowflake velocity. Our result suggests that for the unrimed snowfall of our study, the observed spread in the \(Z_e-S\) power law is mainly due to the PSDs differences, as shown in Matrosov et al. (2009), and to a lesser degree to the assumed mass relation.

7. Summary and discussion

Both modeling and remote sensing studies require the parameterizations of bulk quantities. Generally for snow the accuracy of these calculated variables and the relations between them are dependent on the parameterization of the PSDs used and on the mass and velocity dimensional relationships as well. The variability of snowflake velocity discussed in Part I between different snowfall events is related here to the variability in the mass dimensional relationship based on the idea that the reliable estimators of snowflake velocity are its density together with its size. Even if only light riming (or no riming) cases were used here, we see appreciable differences in (23) and all the derived relationships.

Radar observables provide an alternative verification of microphysical description of snow. Our ultimate goal is to derive the microphysical relations that are consistent with radar measurements and suitable for modeling
values of “X” symbols represent the results obtained using the average mass expression retrieved from a different event (9 Jan 2006). The retrieved for this event. The open circles are obtained using the solid circle gives the results obtained with the mass relationship developed in MH05 and KC05. In our relationship the size–velocity relationship that is a simpler expression of the general relations of mass, area, size, and velocity selected in the microphysics scheme have to be related to each other to ensure consistency between calculated quantities. The requirement of the self-consistency between snowflake mass and velocity may be partly satisfied by incorporating our retrieved relation (23) into microphysical calculations. Heavy rimed snow is excluded from our analysis, and consequently the average relationships derived in this study may not be valid for this type of snow, as in the cases of reflectivity higher than about 25–30 dBZ. Our relationships may as well not be representative of other geographical conditions. The sensitivity of the proposed relations to the PSDs parameterization has to be taken into account when applying to higher levels in the atmosphere.

In Part III of our study, we will present the third important element of the bulk snow microphysics...
Acknowledgments. The authors are grateful to EunSil Jung for providing data used in this study. The editing of the manuscript by Aldo Bellon is greatly appreciated. The authors also acknowledge the comments of Andy Heymsfield and anonymous reviewer in improving and clarifying certain portions of the manuscript.

REFERENCES

Lee, G. W., and Coauthors, 2008: Snow microphysical processes and variation of effective density–diameter relationships. Proc. Fifth European Conf. on Radar in Meteorology and Hydrology, Helsinki, Finland, ERAD, 7.2.


