Instrument Configuration for Dual-Doppler Lidar Coplanar scans: METCRAX II

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   - Weighting function
   - Equivalence of LS method with the average method ($LS = \text{Least squares}$)
3. Error analysis
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- Resolve the flow in a 2D cross plane and study the spatial and temporal evolution of downslope windstorm type flows.
- Need to determine the best possible scanning strategy and lidar parameters to perform a coplanar dual Doppler Lidar analysis for METCRAX II

Why Dual Doppler?

- A single Lidar measures the radial velocity.
- We can combine the radial velocities from different look directions to reconstruct the actual wind velocity.

\[
V = \sqrt{u^2 + v^2} \\
V_r = u \cos \theta + w \sin \theta
\]

L.O.S = Line of sight

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Objective

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\[
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L.O.S = Line of sight
Overview of the Technique

1. Set the lidars such that the angle between their LOS not close to 0 or 180 degrees.
2. Overlay a Cartesian grid over which the velocity components are retrieved.
Overview of the Technique

3. At each grid intersection point define a region.
4. Assign the range gates to the corresponding grid intersection point.

\[ u \cos \varphi_{1j} + w \sin \varphi_{1j} = v_{r1j} \]
\[ u \cos \varphi_{2k} + w \sin \varphi_{2k} = v_{r2k} \]
Overview of the Technique

Repeat for all points …
- Grid spacing must be greater than range gate length i.e, \( R \geq (\Delta p/\sqrt{2}) \)
- Total scan time should be:

\[
T_i = \left( \frac{\| \varphi_i^{\text{start}} - \varphi_i^{\text{end}} \|}{\pi D_i B_i} \right), i = 1, 2
\]

\[
d\varphi_i = \left( \frac{360R}{\pi D_i} \right)
\]

\[
B_i = \text{PRF/Pulse averaging used}
\]

(PRF= Pulse Repeatition Frequency)
The radial velocity measured at a range gate can be considered to be an average over a finite distance along the LOS. (Frehlich et al. 1998)

\[
v_r^{R_g}(t) = \int_{-\infty}^{\infty} W_{\Delta p}(r') v_r'(r', t) dr'
\]

\[
W_{\Delta p}^{R_g}(r') = \int_{-\infty}^{\infty} \left( \frac{2}{\Delta p \sqrt{\pi \tau c}} \right) \exp \left[ -4 \left( \frac{(r' - R_{tg}) - r''}{\tau^2 c^2} \right)^2 \right] dr''
\]

\[
= \frac{1}{2\Delta p} \text{erf} \left[ \frac{2(r' - R_{tg}) + \Delta p}{\tau c} \right] - \text{erf} \left[ \frac{2(r' - R_{tg})w - \Delta p}{\tau c} \right]
\]
- The weighting function relates to the portion of the range gate that falls within the circle of influence (R)

\[
\tilde{W}_{R_{rx}}^{R_{r}} = \int_{0}^{\infty} W_{\Delta p}^{R_{r}}(r') g(r') dr'
\]

\[
g(r') = \begin{cases} 
1, & -\frac{c'}{2} \leq (r' - R_{chord}) \leq \frac{c'}{2} \\
0, & \text{otherwise}
\end{cases}
\]

\[
R' = R + (\Delta p/2)
\]
Equivalence of LS method with the average method

1. From the LS method

\[ u \cos \varphi_{1j} + w \sin \varphi_{1j} = v_{r1j} \]
\[ u \cos \varphi_{2k} + w \sin \varphi_{2k} = v_{r2k} \]

\[ AU = R \]

\[ (A^TWA)U = A^TWR \]
Equivalence of LS method with the average method

3. Replacing the elevation angles with one single value corresponding to the grid centre:

\[
\begin{align*}
(m \bar{w}_1 \cos^2 \tilde{\varphi}_1 + n \bar{w}_2 \cos^2 \tilde{\varphi}_2)u + (m \bar{w}_1 \cos \tilde{\varphi}_1 \sin \tilde{\varphi}_1 + n \bar{w}_2 \cos \tilde{\varphi}_2 \sin \tilde{\varphi}_2)w &= m \bar{v}_{r1} \cos \tilde{\varphi}_1 + n \bar{v}_{r2} \cos \tilde{\varphi}_2 \\
(m \bar{w}_1 \cos \tilde{\varphi}_1 \sin \tilde{\varphi}_1 + n \bar{w}_2 \cos \tilde{\varphi}_2 \sin \tilde{\varphi}_2)u + (m \bar{w}_1 \sin^2 \tilde{\varphi}_1 + n \bar{w}_2 \sin^2 \tilde{\varphi}_2 \sin \tilde{\varphi}_1 + n \bar{v}_{r2} \sin \tilde{\varphi}_2
\end{align*}
\]

Where,

\[
\begin{align*}
\bar{v}_{r1} &= (1/m) \sum_{j=1}^{m} (\bar{w}_j \nu_{r1}) \quad \bar{v}_{r2} = (1/n) \sum_{k=1}^{n} (\bar{w}_k \nu_{r2}) \quad \bar{w}_1 = (1/m) \sum_{j=1}^{m} (\bar{w}_j) \quad \bar{w}_2 = (1/n) \sum_{k=1}^{n} (\bar{w}_k)
\end{align*}
\]
Error Analysis (From Lidar Simulator)

- Temporal error

\[ E_{\Delta t}(i, j) = \sqrt{\frac{i}{K} \sum_{t=1}^{K} \frac{\sigma_{\Delta t}^2(i, j, t)}{\bar{v}(i, j)}} \]

\[ \bar{E}_{\Delta t} = \frac{1}{MN} \sum_{i=1}^{M} \sum_{j=1}^{N} E_{\Delta t}(i, j) \]

- Spatial error

\[ E_{\Delta s}(t) = \sqrt{\frac{1}{MN} \sum_{i=1}^{M} \sum_{j=1}^{N} \frac{\sigma_{\Delta s}^2(i, j, t)}{\bar{v}(i, j)}} \]

\[ \bar{E}_{\Delta s} = \frac{1}{K} \sum_{t=1}^{K} E_{\Delta s}(t) \]

- Retrieval error

Error due to the the cells with skewed LOS from Lidars. i.e., angle between the LOS close to 0 degrees or 180 degrees.  \cite{Lhermitte1970}

Errors are dependent on both averaging and range gate size
Error Analysis (From Lidar Simulator)

- Temporal errors increase with pulse averaging.
- Temporal errors increase with range gate length
- Spatial errors are independent of averaging used
- Spatial errors increase with range gate size

Red : Temporal errors
Blue : Spatial errors
(a) : errors in u
(b) : errors in w
## Lidars on site

<table>
<thead>
<tr>
<th>Lidar-1 parameters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>PRF</strong></td>
<td>15kHz</td>
</tr>
<tr>
<td><strong>Averaging</strong></td>
<td>10,000</td>
</tr>
<tr>
<td><strong>Range gate</strong></td>
<td>24 m</td>
</tr>
<tr>
<td><strong>Elevation angle at the beginning of the scan</strong></td>
<td>150</td>
</tr>
<tr>
<td><strong>Elevation angle at the end of the scan</strong></td>
<td>10</td>
</tr>
<tr>
<td><strong>Time taken for the one scan</strong></td>
<td>30 s</td>
</tr>
<tr>
<td><strong>Scan rate</strong></td>
<td>4.5 deg/s</td>
</tr>
<tr>
<td><strong>Sector</strong></td>
<td>140</td>
</tr>
</tbody>
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<table>
<thead>
<tr>
<th>Lidar-2 parameters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Accumulation time</strong></td>
<td>0.75 s</td>
</tr>
<tr>
<td><strong>Range gate</strong></td>
<td>25 m</td>
</tr>
<tr>
<td><strong>Elevation angle at the beginning of the scan</strong></td>
<td>-10</td>
</tr>
<tr>
<td><strong>Elevation angle at the end of the scan</strong></td>
<td>20</td>
</tr>
<tr>
<td><strong>Time taken for the one scan</strong></td>
<td>30s</td>
</tr>
<tr>
<td><strong>Scan rate</strong></td>
<td>1.0 deg/s</td>
</tr>
<tr>
<td><strong>Sector</strong></td>
<td>30</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Domain specifications</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Number of cells in X direction</strong></td>
<td>36</td>
</tr>
<tr>
<td><strong>Number of cells in Y direction</strong></td>
<td>19</td>
</tr>
<tr>
<td><strong>Cell dimensions</strong></td>
<td>26.8 m x 26.8m</td>
</tr>
</tbody>
</table>

Field Experiment results
Summary and Future Work

- Coplanar Dual Doppler Technique can be used to study complex flows.
- Care must be taken in choosing the lidar parameters to get the best temporal and spatial resolution of the phenomenon.
- Analysis can be simplified with certain assumptions and for small range gate lengths, the results are very close.
- Investigate the significance of weighting function for alternate approaches and larger range gate lengths.

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Questions