Assignment 4, 5140
Due February 9, 2018

1. Read Chapter section 4.4

2. Read Chapter section 4.5.1 including inset 4.1

3. Read Problem 4.44. Sketch the temperature profile in the stratosphere (using your choice of source) and comment about the relevance of the solution for explaining how ozone absorption of UV radiation determines the temperature profile.

   In problem 4.44, it is shown that the strongest absorption per unit volume, i.e., where \( d (dI_\lambda/dz)/dz = 0 \), is found at the location where \( \tau_\lambda = 1 \). Temperature in the stratosphere increases with height up to the stratopause where it is a maximum but the link to the optical depth is less clear when the derivation is based on the assumption that the mixing ratio of ozone is constant with height, which it is not.

4. Take and print out a photo of one of a rainbow, glory, corona, or halo. Give the date and location, and describe the basic physics of the phenomenon you photographed using either the class notes or Wallace and Hobbs as a reference. The photo must be yours and recent.

5. Make an argument for why the return from radar is proportional to the sixth power of the size of the particle. Hint: consider that \( \sigma_s = K_s \sigma \), and consider that you are trying to show that \( \sigma_s \) goes as the sixth power of the particle size.

   The radar wavelength \( \lambda \) is about 1 cm – much larger than the size of most precipitation particles which are of order mm. Thus, we are in the Rayleigh (or dipole scattering) regime where \( x = 2\pi r/\lambda \ll 1 \), in which case

   \[ K_s \sim x^4 \sim r^4 \]

   where \( a \) is the particle radius and the size parameter \( x = 2\pi r/\lambda \). The geometric cross section of a particle goes as

   \[ \sigma \sim r^2 \]

   Therefore

   \[ \sigma_s = K_s \sigma \sim r^2 r^4 \sim r^6 \]

6. The return from lidar, which uses a He-Ne red laser to determine extinction by cloud particles, is proportional to the second power of the size of the particle. With reference to \( K_S \), why is this?

   The same argument as above, expect the lidar wavelength is much smaller than typical cloud particle sizes, so we are in the geometric optics regime where \( x \gg 1 \) and \( K_s \sim 2 \), thus, \( \sigma_s \sim r^2 \)

7. Clouds are white, but this is not because droplets are large compared to the wavelength, contrary to what many people think. Rather clouds are white for the same reason that milk is white. Cloud droplets scatter all wavelengths equally, while individual milk particles are small enough to scatter blue. What matters though for making both things white is that they
both have a high optical depth. That is that multiple scattering leads to a washout of the color dependence. With reference to the equation

\[ R = \frac{\tau}{1 + \tau} \]

show that if even if \( \tau = \tau(\lambda) \), if \( \tau \) is small then the wavelength dependence of \( \tau(\lambda) \) (i.e. \( d\tau/d\lambda \)) can be seen in the reflectivity, but if \( \tau \) is large, then it can’t. You can show this either by graphing, or (perhaps more easily) by using calculus to find \( dR/d\lambda \) as a function of \( \tau \) for a small \( \tau \) and a large \( \tau \) scenario.

If \( \tau \ll 1 \) then

\[ R \simeq \tau \]

so

\[ \frac{dR}{d\lambda} = \frac{d\tau}{d\lambda} \]

If

\[ \tau \gg 1 \]

then

\[ \frac{dR}{d\lambda} = \frac{1}{1 + \tau} \frac{d\tau}{d\lambda} - \frac{\tau}{(1 + \tau)^2} \frac{d\tau}{d\lambda} \]

\[ \frac{dR}{d\lambda} = \frac{1 + \tau - \tau}{(1 + \tau)^2} \frac{d\tau}{d\lambda} \]

\[ \frac{dR}{d\lambda} = \frac{1}{(1 + \tau)^2} \frac{d\tau}{d\lambda} \]

If \( \tau \to \infty \) then \( dR/d\lambda \to 0 \) and it is not possible to see the wavelength dependence of \( \tau \).

8.

This image shows streaks of bright cloud off the coast of France that are caused by the particulate emissions from large container ships as they pass beneath a clouds deck. To first order, the ships don’t actually change the liquid water content in the cloud, but the added particulate matter does change the number concentration of particles
(a) Suppose that in a shiptrack, the LWC and cloud depth is unchanged but the droplet number concentration increases by about a factor of 2. Show that the mean volume droplet radius decreases by only about 26%. Note that \( LWC = 4\rho_l N \pi r^3 / 3 \), where \( N \) is the droplet number concentration, \( r \) the droplet radius and \( \rho_l \) the bulk liquid water density.

If volume is conserved:

\[ N_1 r_1^3 = N_2 r_2^3 \]

Therefore,

\[ \frac{r_2}{r_1} = \left( \frac{N_1}{N_2} \right)^{1/3} \]

So, the droplet radius decreases by a factor of \( 2^{1/3} = 1.26 \) or 26%.

(b) By what factor does the optical depth increase in the visible where \( \tau = N K_\lambda \sigma \Delta z \), where \( N \) is the number concentration of droplets, \( \sigma \) is their cross-section, \( K_\lambda \) is the scattering efficiency and \( \Delta z \) is the cloud depth? In general

\[ \tau = N K_\lambda \sigma \Delta z \]

So

\[ \frac{\tau_2}{\tau_1} = \frac{N_2 r_2^2}{N_1 r_1^2} = 2 \times (1/1.26)^2 = 1.25 \]

or the optical depth increases by 26%.

(c) The albedo of a cloud can be given by the approximate formula

\[ \alpha = \frac{\tau}{7.7 + \tau} \]

If the initial optical depth is 1, show that the albedo inside the shiptrack is 0.14 whereas outside of the shiptrack it is 0.11. From the equation for albedo

\[ \alpha = \frac{\tau}{7.7 + \tau} \]

Outside the shiptrack

\[ \alpha = \frac{1}{7.7 + 1} = \frac{1}{8.7} = 0.11 \]

Inside the shiptrack

\[ \alpha = \frac{1.25}{7.7 + 1.25} = \frac{1.25}{8.95} = 0.14 \]