Chapter 5  Multiple Scattering

From now on neglect phase
i.e. incoherent light

Pile of Plates theory

Layer $i$ has properties $R_i$ and $T_i$:

\[ \begin{array}{c}
\text{Incident} \\
\hline
R_i, T_i \\
\hline
\text{Reflected} \\
F_i \\
\hline
\text{Transmitted} \ T_i
\end{array} \]

So

\[ F_i = T_i + F \uparrow R_i \]
\[ F \uparrow = F_i R_i \]

Note, same $R$ for $F_i$ and $F \uparrow$; principle of reciprocity

From above

\[ F \uparrow = T_i + F_i R_i \]

\[ F_i = \frac{T_i}{1 - R_i^2} \]

\[ F \uparrow = \frac{R_i T_i}{1 - R_i^2} \]

Note as $R_i \to \infty F \downarrow \to 0$
Two Plates

\[ \uparrow R_2 \]

\[ \downarrow V \]

\[ R_1 T_1 \]

\[ F \]

\[ R_1, T_1 \]

\[ \downarrow V F \]

\[ \Gamma_2 \]

\[ R_2 = R_1 + F \Gamma_1 \]

\[ T_2 = T_1 F \Gamma_1 \]

Include (1) and (2)

\[ R_2 = R_1 + \frac{R_1 T_1}{1-R_1^2} \]

\[ T_2 = \frac{T_1}{1-R_1^2} \]

Suppose no absorption

\[ R_1 + T_1 = 1 \implies R = 1 - T \]

\[ R_2 = R_1 + R_1 (1-R_1)^2 \]

\[ = R_1 + R_1 (1-R_1) \frac{(1+R_1)}{(1+R_1)} \]

\[ = \frac{2R_1}{(1+R_1)} \]
\[ \Gamma_2 = \frac{1 - R_1}{1 + R_1} \]

Double up, \( R_4 \)

\[ \frac{R_2}{1 - R_1} \]

\[ \frac{R_2}{1 + R_1} \]

\[ R_4 = \frac{4R_1}{1 + 3R_1} \]

Doubling - adding method

start: small and build up to any

arbitrary \( N \) of plates

By induction:

\[ R_n = \frac{NR_1}{1+(N-1)R_1} \]

If \( N \gg 1 \)

\[ R_n \to \infty = \frac{NR_1}{1 + NR_1} \]
\( B + C \) suggest

\[ N R_1 = \tau \quad \text{optical thickness} \]

\[ R_n = \frac{\tau}{2 + \tau} \quad \text{important equation} \]

Important equation

\[
\text{keep in mind that}
\text{this is not quite}
\text{same } \tau \text{ discussed}
\text{in rad. trans.}
\]

Note: for \( \tau \ll 1 \), \( R_n \propto \tau \)

But asymptotes as \( \tau \gg 1 \)