Prediction of Graupel Density in a Bulk Microphysics Scheme

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Abstract

A method to predict the bulk density of graupel $\rho_g$ has been added to the two-moment Milbrandt-Yau bulk microphysics scheme. The simulation of graupel using the modified scheme is illustrated through idealized simulations of a mesoscale convective system using a 2D kinematic model with a prescribed flow field and different peak updraft speeds. In order to examine the relative impact of the various approaches to represent rimed ice, simulations were run for various graupel-only and graupel-plus-hail configurations.

Due to the direct feedback of $\rho_g$ to terminal fall speeds, the modified scheme produces a much different spatial distribution of graupel, with more mass concentrated in the convective region resulting in changes to the surface precipitation at all locations. With a strong updraft, the model can now produce solid precipitation at the surface in the convective region without a separate hail category. It is shown that a single rimed-ice category is capable of representing a realistically wide range of graupel characteristics in various atmospheric conditions without the need for \textit{a priori} parameter settings.

Sensitivity tests were conducted to examine various aspects of the scheme that affect the simulated $\rho_g$. Specific parameterizations pertaining to other hydrometeor categories now have a direct impact on the simulation of graupel, including the assumed aerosol distribution for droplet nucleation, which affects the drop sizes of both cloud and rain, and the mass-size relation for snow, which affects its density and hence the embryo density of graupel converted from snow due to riming.
1. Introduction

With increasing computer power and model resolution in recent years, the representation of cloud and precipitation microphysics has moved to the forefront of physical parameterization development in atmospheric models. Many uncertainties remain including the parameterization of ice-phase microphysics which is particularly complex given the wide variety of ice particle shapes and types and a lack of observational constraints and physical understanding compared to liquid-phase microphysics. Improvements are greatly needed given the large impact of ice microphysical processes on quantitative precipitation forecasts (e.g., Rutledge and Hobbs 1983; Gilmore et al. 2004; Thompson et al. 2004), prediction of supercooled liquid water (e.g., Reisner et al. 1998; Thompson et al. 2004; Morrison and Pinto 2006), simulations of radiative transfer in clouds (e.g., Gu and Liou 2000; Wu 2002), and cloud dynamical interactions (e.g., Leary and Houze 1979; Lord et al. 1984; Zhang and Gao 1989; Van den Heever and Cotton 2004; McFarquhar et al. 2006; Morrison and Milbrandt 2011).

Bulk and bin microphysics schemes traditionally represent ice particles by separating them into predefined categories representing one or more “typical” particle types, with a priori specification of key parameters. Schemes generally use fixed bulk densities and have fall speeds within a particular range for each category. Graupel – heavily rimed crystals which have not undergone wet growth - is commonly parameterized as “medium density graupel” with a bulk density ($\rho_b$) of 400 kg m$^{-3}$ and corresponding fixed fall speed parameters (e.g. Rutledge and Hobbs 1983; Ferrier 1994; Meyers et al. 1997; Geresdi 1998; Thompson et al. 2004; Milbrandt and Yau 2005a; Seifert and Beheng 2006; Morrison et al. 2009). In nature, graupel has a large range of densities and fall speeds (Locatelli and Hobbs 1974; Pruppacher and Klett 1997). Most schemes model rimed ice with a single category with parameters set to represent either “graupe
or “hail” (dense ice that has undergone wet growth), though some schemes use two separate
categories (Ferri 1994; Meyers et al. 1997; Milbrandt and Yau 2005b).

There is considerable sensitivity of simulations to the values of the fixed density and fall speed
parameters. Gilmore et al. (2004) and Van Weeverberg et al. (2010) showed large sensitivity of
simulated supercell storm dynamics and precipitation to changes in the assumed ρg and fixed
intercept parameter N0g using single-moment bulk microphysics schemes (BMS). Similarly, van
storms to changes in specified mean graupel/hail particle size in a single-moment scheme.
Adams-Selin et al. (2012) found large sensitivity of a bowing squall line to ρg, N0g, and fall speed
parameters. Recent work has also shown considerable sensitivity of the structure and dynamics
of a supercell and squall line to graupel/hail fall speed parameters in two-moment BMSs, but
with less sensitivity to the density itself (Morrison and Milbrandt 2011; Bryan and Morrison
2012). Fall speeds should, however, depend explicitly on ρg, with higher values resulting in
greater fall speeds (Locatelli and Hobbs 1974; Mitchell and Heymsfield 2005). In most schemes,
the fall speed is not directly dependent on a varying density but varies only indirectly through
changes in the particle size distribution and mean size. Unless the fall speed parameters are
modified simultaneously with ρg, models will give an incorrect sensitivity of mass-weighted fall
speed to changes in density because increasing ρg results in a reduction of the mean particle size
and hence the mass-weighted fall speed.

The first work on modeling changes to ρg in a microphysical model is that of Farley (1987). In
this spectral ice-phase scheme, ρg was allowed to vary through predicted changes to particle
diameter (and hence volume) and mass. Changes to ρg from different physical processes were
determined, including changes due to riming during dry growth based on experimental results
Recent studies have sought to address the issues related to the representation of graupel in bulk schemes. Straka and Mansell (2005) developed a multi-category scheme that included several categories to represent low-density, medium-density, and high-density graupel, frozen drops, small hail, and large hail, each with fixed density and fall speed parameters. Connolly et al. (2006) added a third prognostic variable to the graupel category, the bulk volume mixing ratio, of their two-moment scheme. With this quantity, the scheme relaxes the assumption of a fixed $\rho_g$. This has advantages over the multi-category approaches because the particle density can evolve smoothly in time and space and there is no need for artificial conversion rates between categories. Two major limitations of this scheme, however, are the use of a fixed rime density (of 500 kg m$^{-3}$) and the use of fixed fall speed parameters. Laboratory studies have shown that the density of rime is in fact a function of the size of the accreted drops, the graupel surface temperature, and the impact speed, the latter of which depends on the graupel fall speed and hence its size and density (Macklin 1962). The value of rime density ranges from approximately 50-900 kg m$^{-3}$ (Macklin 1962; Heymsfield and Phlaum 1985). Mansell et al (2010; hereafter M2010) addressed these limitations by parameterizing rime density as a function of temperature, impact speed, and droplet size following Heymsfield and Pflaum (1985), similar to the approach of Farley (1987). They also explicitly relate particle fall speed to $\rho_g$ using the formulation of Wisner et al. (1972) and the assumption of a linear dependence of drag coefficient on density.

In this study, we continue to investigate the representation of rimed ice in BMSs by examining the relative impact of the various approaches to parameterize graupel and hail. A prognostic approach to model $\rho_g$ has been added to the two-moment version of the Milbrandt and Yau (2005a,b; hereafter MY2005a,b) multimoment scheme (hereafter referred to as the MY2
scheme). The approach is similar to that of M2010 but uses a different rime density parameterization, based on the laboratory experiments of Cober and List (1993; hereafter CL1993) and includes a physically-based treatment of graupel fall speeds with their dependence on density and size following the Reynolds number-Best number approach of Khvorostynov and Curry (2002) and Mitchell and Heymsfield (2005). The impacts of various approaches to representing rimed ice in a bulk model are examined, with focus on the prognostic-$\rho_g$ approach and the use of density-dependent terminal fall speeds. The modified BMS is tested in a two-dimensional (2D), kinematic, specified flow field model. This allows for the close examination of features of the scheme in a simple, controlled framework allowing consistent testing over a wide range of conditions without complications of feedbacks between the microphysics and dynamics. The sensitivity to various aspects in the scheme that ultimately affect the prediction of $\rho_g$ is also investigated.

The paper is organized as follows. In section 2, the prognostic-$\rho_g$ method applied to the two-moment scheme is described. An overview of the kinematic model is provided in section 3. Section 4 presents results from the control configuration of the modified scheme plus different combinations of one and two graupel/hail categories for weak and strong updrafts. In section 5, results from sensitivity tests on different aspects of the model that affect the prediction of $\rho_g$ are examined. A discussion and concluding remarks are given in section 6.

2. Description of Method

The prognostic-$\rho_g$ method described below is incorporated into MY2. A brief overview of the original MY2 scheme is provided in appendix A. In principle, this method can be used in any BMS, however it is most effective when applied to two-moment (or higher) schemes for reasons that will be discussed below.
a. The new prognostic variable

It is important that any prognostic state variable be a conserved quantity, as are the mass mixing ratio, \( q_x \) (kg kg\(^{-1}\)), and the total number mixing ratio, \( N_x \) (kg\(^{-1}\))\(^1\), for each category \( x \) (see appendix A). Although the objective is to predict \( \rho_g \), it is not possible to include directly a prognostic equation for \( \rho_g \) since it is not conserved during advection. Following the approach introduced by Connoly et al. (2006) and later used by M2010, the bulk graupel volume mixing ratio, \( B_g \) (m\(^{-3}\) kg\(^{-1}\)), has been introduced as a new variable into the scheme. The total volume of all particles is conserved since the quantity \( \rho B_g \) is advected and diffused (where \( \rho \) is the air density). At any point in time and space, \( \rho_g \) can be diagnosed from

\[
\rho_g = \frac{q_x}{B_g}. \tag{1}
\]

With this approach \( \rho_g \) evolves correctly during advection since errors produced during the advection of \( q_x \) and \( B_g \) tend to cancel when calculating their ratio, especially for monotonic advection schemes. This is analogous to accurate evolution of mean-mass particle diameter, \( D_x \), based on separate advection of \( q_x \) and \( N_x \) in two-moment schemes, where

\[
D_x = \left( \frac{\rho q_x}{c_x N_x} \right)^{\frac{1}{3}}. \tag{2}
\]

The conservation equation for \( B_g \) can be shown to be

\[
\frac{\partial B_g}{\partial t} = \left( \frac{\partial B_g}{\partial t} \right)_{\text{advection}} + \left( \frac{\partial B_g}{\partial t} \right)_{\text{diffusion}} + \left( \frac{\partial B_g}{\partial t} \right)_{\text{micro}} + \left( \frac{\partial B_g}{\partial t} \right)_{\text{sedimentation}}. \tag{3}
\]

\(^1\) In the MY2 scheme \( N_x \) is in concentration units (m\(^{-3}\)). However, the actual conservative quantity which is advected by the dynamical model is \( \rho N_x \) (where is \( \rho \) the air density), which is in mixing ratio units (kg\(^{-1}\)).
The first two terms on the right-hand side (RHS) of (3) are solved by the dynamics of the atmospheric model. The third term represents changes to $B_g$ due to microphysical sources and sinks, formulated through changes to $\rho_g$ by specific microphysical processes. The fourth term is the local change in $B_g$ due to sedimentation. In the following equations describing the algorithm in the scheme, the symbols ‘0’, ‘*’, and ‘+’ are used to denote the following time levels, respectively: the beginning of the time step (as seen by the BMS); the intermediate time level immediately after the microphysical source/sink terms have been added; and the end of the time step, after sedimentation.

Upon passing $q_g^0$, $B_g^0$, and $N_g^0$ to the microphysics scheme, $\rho_g^0$ is computed at each grid point from (1) in order to compute the mass-diameter coefficient,

$$c_g^0 = \frac{\pi}{6} \rho_g^0,$$  \hspace{1cm} (4)  

where,

$$m_g(D) = c_g D^{d_g},$$  \hspace{1cm} (5)  

is the mass of a single spherical particle with diameter $D$, implied by $d_g = 3$. The graupel size distribution parameters $\lambda_g$ and $N_0g$ are then computed. Note that $c_g$ is no longer a constant parameter, unlike in the original MY2. All variables and parameters pertaining to graupel in the new scheme are summarized in Table 1.

b. Fall speed parameters

The terminal fall speed for a graupel particle of diameter $D$ is represented by a power law velocity-diameter relation, given by

$$v_g = \left( \frac{\rho_0}{\rho} \right)^{0.5} a_g D^{b_g},$$  \hspace{1cm} (6)
where \( \rho_0 \) is the reference air density at mean sea level for a standard atmosphere. In most microphysics schemes, including the original MY2, the fall speed parameters \( a_g \) and \( b_g \) are constants, thus the \( v_g \) does not explicitly dependent on \( \rho_g \). M2010 improved this approach by allowing \( a_g \) to vary with \( \rho_g \) following Wisner et al. (1972) and a linearly-scaled, density-dependent drag coefficient. We further improve the physical basis for the dependence of \( v_g \) on \( \rho_g \) by using the power-law Reynolds number (\( Re \)) - Best number (\( X \)) approach of Khvorostyanov and Curry (2002) and Mitchell and Heymsfield (2005). This approach produces a \( Re-X \) relationship that closely matches data from the hail and graupel measurements of Knight and Heymsfield (1983) and Heymsfield and Kajikawa (1987); see Mitchell and Heymsfield (2005) for details. In turn, Best number is related to \( \rho_g \) following Mitchell (1996), assuming spherical graupel particles, giving

\[
X = \frac{4 \rho_g g \rho D^3}{3 \eta^2},
\]

(7)

where \( g \) is the acceleration of gravity and \( \eta \) is the dynamic viscosity.

This approach allows the fall speed to be expressed by a power law in \( D \) following (6) but with \( \rho_g \)-dependent \( a_g \) and \( b_g \). These parameters are given by MH 2005 as

\[
a_g = a_1 \nu^{2 - b_1} \left( \frac{4 \rho_g g}{3 \rho_a} \right)^{b_1},
\]

(8)

and

\[
b_g = 2b_1
\]

(9)

where \( \nu \) is the kinematic viscosity and

\[
a_1 = \frac{C_2 (1 + C_1 X^{1/2})^{1/2} - 1}{X^{b_g}}
\]

(10)
and

\[ b_1 = \frac{C_1 X^{1/2}}{[1 + C_1 X^{1/2}]^{1/2} - 1} b_0 X^{b_0} - \frac{a_0 b_0 X^{b_0}}{C_2 ([1 + C_1 X^{1/2}]^{1/2} - 1)^2}. \]

(11)

The values \( C_1 = 4/\delta_0^2 C_0^{d/2} \), \( C_2 = \delta_0^2 / 4 \), \( \delta_0 = 5.83 \), and \( C_0 = 0.292 \) are nondimensional surface roughness parameters (see Mitchell 1996) and \( a_0 \) and \( b_0 \) are coefficients in the \( Re-X \) relationship.

We set \( a_0 = b_0 = 0 \) since appropriate values are uncertain and Mitchell and Heymsfield (2005) state that for graupel and hail it is unclear whether the second terms in (10)-(11) involving \( a_0 \) and \( b_0 \) are needed.

Given the complicated dependence of \( a_g \) and \( b_g \) on \( D \), analytic integration of (6)-(11) over the particle size distribution to obtain the mass- and number-weighted terminal fall speeds is not possible. Instead, \( a_g(\rho_g) \) and \( b_g(\rho_g) \) are diagnosed from numerical solutions of (6)-(11) using the least-squares method in log-log space over the range \( 0.3 < D < 20 \) mm. For implementation into MY2, the fitted values of \( a_g(\rho_g) \) and \( b_g(\rho_g) \) are stored in a lookup table as a function of \( \rho_g \), with values ranging from 50 to 850 kg m\(^{-3}\) at intervals of 100 kg m\(^{-3}\) (Table 2), with linear interpolation to compute intermediate values. To approximate the dependence of \( v_g \) on \( \rho \) (air density), standardized surface values (temperature of 273.15 K, pressure of 1000 hPa) were used to calculate \( \rho \), \( \eta \), and \( v \) in (7)-(9) for derivation of the tabulated values of \( a_g \) and \( b_g \), and then scale \( v_g \) with the air density correction factor in (6).

The \( v_g \) values calculated explicitly using (6)-(11) for \( \rho_g \) between 50-850 kg m\(^{-3}\) are shown in Fig. 1 along with the parameterized \( v_g \) calculated from (6) but with diagnosed values of \( a_g(\rho_g) \) and \( b_g(\rho_g) \). Close correspondence is seen between the calculated and parameterized \( V_g \) across the range of \( D \) and \( \rho_g \). Figure 1 also shows curves for empirical \( v-D \) relationships based on
measurements of various related particle types. The calculated and parameterized $v_g$ values capture the full range of fall speeds associated with graupel of various densities compared to the empirical relations for conical, lump, and hexagonal graupel, graupel-like snow (Locatelli and Hobbs 1974), and to the original MY2 relations for hail and for medium density graupel.

c. Microphysical processes

After diagnosing the size distribution parameters (appendix A) and the fall speed parameters at time level ‘0’, the microphysical growth and decay rates for individual processes affecting $q_g$ and $N_g$ are then computed, as in the original scheme, as tendency terms for $q_g$ and $N_g$ for each relevant process which are then used to compute the updated values, $q_g^*$ and $N_g^*$. To compute $B_g^*$, changes to $\rho_g$ are computed and then the updated $B_g^*$ is determined from (1). The microphysical processes that change $\rho_g$ are conversion from snow to graupel, three-component freezing to graupel (either from rain-snow or rain-ice interaction or from rain collecting graupel), and riming (accretion of cloud and/or rain). All other processes that affect $q_g$ and or $N_g$ – including deposition, sublimation, melting, ice multiplication, and conversion to hail (see MY2005b for details) – are assumed to have negligible impact on $\rho_g$. The updated $\rho_g^*$ is computed as the mass-weighted sum of $\rho_g^0$ and the density of the new or depleted graupel mass from the individual processes:

$$\rho_g^* = [q_g^0 \rho_g^0 + \Delta q_{\text{CSGG}} \rho_s + \delta_{\text{rgg}} (\Delta q_{\text{CLrg}} + \Delta q_{\text{CLr}}) \rho_h + \delta_{\text{sr}} (\Delta q_{\text{CLsr}} + \Delta q_{\text{CLrs}}) \rho_{\text{sr}} + \delta_{\text{rs}} (\Delta q_{\text{CLrs}} + \Delta q_{\text{CLsr}}) \rho_{\text{rs}} + \delta_{\text{rg}} (\Delta q_{\text{CLrg}} + \Delta q_{\text{CLrg}}) \rho_{\text{rg}} + \delta_{\text{rg}} (\Delta q_{\text{CLrg}} + \Delta q_{\text{CLrg}}) \rho_{\text{rg}}] [q_g^0 + \Delta q_{\text{CSG}} + \delta_{\text{rgg}} (\Delta q_{\text{CLrg}} + \Delta q_{\text{CLr}}) + \delta_{\text{sr}} (\Delta q_{\text{CLsr}} + \Delta q_{\text{CLrs}}) + \delta_{\text{rs}} (\Delta q_{\text{CLrs}} + \Delta q_{\text{CLsr}})]$$

(12)

where the $\Delta q_{\text{AB}(c)}$ quantities in the terms on the RHS of (12) are the $q_g$ increments for a given process (i.e. the $q_g$-tendencies multiplied by the time step) for process “AB” affecting
hydrometeor categories $x$, $y$, and $z$). This approach is similar to that of Farley (1987). Details of the individual parameterized processes that affect $\rho_g$ are given in appendix B.

The updated values $q_g^*$ and $N_g^*$ are then computed. After applying clipping for negative values that may arise, the updated value of $B_g^*$ is then given by,

$$B_g^* = \frac{q_g^*}{\rho_g^*}.$$  (13)

At this point, all values of the prognostic state variables for graupel at the intermediate time level (*) have been computed.

d. Sedimentation

Immediately following the updates to the prognostic variables from the growth and decay rates, the diagnostic variables $a_g^*$, $b_g^*$, $c_g^*$, and the size distribution parameters at * are recomputed. The mass- and number-weighted bulk fall speeds, $V_q^*$ and $V_N^*$, and then computed in the standard ways (MY2005a) but using the density-dependent fall speed parameters from section 2b. The sedimentation of $q_g^*$ and $N_g^*$ at $V_q^*$ and $V_N^*$, respectively, are then computed to obtain the final $q_g^+$ and $N_g^+$. The sedimentation of $B_g^*$ is also computed at $V_q^*$ to obtain $B_g^+$. In 2-moment schemes, sedimentation can create problems due to uncontrolled size-sorting (Wacker and Seifert 2001; MY2005a). Size-sorting can be controlled by methods described in MY2005a and Milbrandt and McTaggart-Cowan (2010; hereafter MM2010). In most 2-moment schemes, size-sorting for graupel is not particularly problematic due to the relatively small range of fall speeds typical of low- or medium-density graupel (e.g. Locatelli and Hobbs, 1974).

However, with the prognostic-$\rho_g$ approach with the $\rho_g$-dependent fall speed parameters, the range of fall speeds is much larger (Fig. 2) so the potential problem of uncontrolled size-sorting is now greater. The treatment of this in the modified scheme is discussed in section 4d below.

3. Kinematic model description and setup
The modified MY2 scheme is tested in a two-dimensional (2D) kinematic model (Szumowski et al. 1998; Grabowski 1999; Morrison and Grabowski 2007; Slawinska et al. 2009). The kinematic framework uses a realistic, specified flow field which allows for testing of microphysics parameterizations in a simple, controlled framework. In addition to the conservation equations for the various hydrometeor variables, the kinematic model solves equations for the potential temperature and water vapor mixing ratio.

The specified flow field used in this study is similar to that of Slawinska et al. (2009), representative of a mature squall line with strong, deep ascent in the convective region and weak mesoscale upper-tropospheric ascent overlying weak lower-tropospheric descent in the stratiform region. The strength of the convective updraft $w_{peak}$ is increased sinusoidally from zero to its peak value over an initial 15 min spin-up period, remaining constant thereafter. Figure 2a shows the flow at steady state (after 15 min). The horizontal motion shown in the figure combines left-to-right 4 m s$^{-1}$ flow with weak (0.1 m s$^{-1}$ km$^{-1}$) left-to-right shear and perturbations (convergence and divergence) associated with a fixed convective updraft and stratiform updraft and downdraft. The resulting flow features a significantly sheared large-scale flow. The magnitude of the convective updraft is varied in different simulations as described below. The initial temperature and relative humidity profiles (Fig. 2b) are taken from the 0000 UTC 1 September, 1974 GATE [Global Atmospheric Research Programme (GARP) Atlantic Tropical Experiment] sounding (Slawinska et al. 2009), that were previously applied in the simulations described by Grabowski (1999).

The horizontal and vertical grid spacings used are 750 m and 250 m, respectively, over a 240 km $\times$ 12 km domain and the time step is 5 s. Variables are prescribed at the inflow boundary and taken from the initial sounding, whereas vanishing horizontal gradients are assumed at the
outflow boundary. The model is run until quasi-steady-state conditions are achieved, which occurs after several hours (see section 4a). Transport in the physical space is calculated using the 2D version of the multidimensional positive-definite advection transport algorithm (MPDATA) scheme (Smolarkiewicz 1984; Smolarkiewicz and Margolin 1998). An explicit treatment of subgrid-scale mixing is neglected, along with other physical processes besides microphysics.

To test the modified microphysics in a variety of conditions, $w_{peak}$ is varied between 1 and 40 m s$^{-1}$. Although updraft velocities less than $\sim$10 m s$^{-1}$ are not typical of mature squall lines (e.g., Houze 1993), weak updraft simulations are included to test the scheme in conditions supporting smaller amounts of liquid water and hence less riming. For simplicity, the mesoscale updraft and downdrafts and all other aspects of the flow field are kept the same for all runs.

4. Results

a. Demonstration of the prognostic-$\rho_g$ scheme

The effects of the new method are illustrated with the 2D simulation using the control configuration of the modified MY2 scheme used in this study, with prognostic $\rho_p$, diagnostic $a_g$ and $b_g$, and graupel only (i.e. the hail category shut off), and with $w_{peak} = 20$ m s$^{-1}$. This simulation is referred to as PD-20. All of the runs discussed in this paper are listed in Table 3. Figure 3 shows the model reflectivity$^2$ for PD-20 as it evolves. By 240 min, the simulation has reached a steady state and the overall features of the reflectivity pattern are very comparable to those of mature squall lines for observed cases, 3D mesoscale model simulations, and to conceptual models (e.g. Biggerstaff and Houze 1991; Bryan and Morrison 2012). Note that the purpose of this comparison is simply to demonstrate that the kinematic model with a prescribed

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$^2$ The model reflectivity, the total equivalent radar reflectivity factor, is computed as the sum of the sixth moments of the size distributions of each hydrometeor category (fourth moment for snow) assuming Rayleigh scattering (valid for large radar wavelengths), as described in Milbrandt et al. (2008).
flow field simulates a convective system with sufficient realism that we may proceed to examine the behavior of the microphysics scheme, not to claim that the 2D model accurately simulates a real squall line.

The evolution of the graupel fields in the early stages of the PD-20 run are shown in Fig. 4, which shows $q_g$, $\rho_g$, $D_g$, and mass-weighted mean fall speed ($V_g$) at 12, 16, and 20 min. At 12 min (Fig. 4, left column), elevated graupel is present near the updraft core (Fig. 4a; $x \sim 45$ km).

Most of the graupel in this simulation originates as freezing rain with an embryo density of that of the hail category, 900 kg m$^{-3}$ (note the location of this at $x \sim 45$ km and $z \sim 5$ km in Fig. 4d,e,f); $\rho_g$ then decreases as graupel grows quickly by riming. Note that in the graupel-only configuration, probabilistic freezing of rain goes to graupel, while in the full hail-on configuration it goes directly to hail. Rain can also freeze through collision with ice or snow (see appendix B). By 15 min (Fig. 4, middle column) the majority of the graupel mass is located above the updraft core (Fig. 4b), having relatively low densities ($\rho_g < 250$ kg m$^{-3}$; Fig. 4e), smaller sizes ($D_g < 1-2$ mm; Fig. 4h), and smaller fall speeds ($V_g \sim 1-3$ m s$^{-1}$; Fig. 4k). By 20 min a small amount of graupel reaches the surface, with the relatively high density ($\rho_g > 600$ kg m$^{-3}$; Fig. 4f), largest sizes ($D_g > 15$ mm; Fig. 4i), and high fall speeds ($V_g > 20$ m s$^{-1}$; Fig. 4l).

The low-density, slower-falling graupel is thus carried aloft while the higher-density, faster-falling graupel withstands the updraft, grows in the accumulation zone, and eventually spills over to the right of the updraft core. Note that with the large range of fall speeds now possible the effects of gravitational size-sorting are evident at all times, with the largest $D_g$ located at the lowest elevations in a column while the peak mass contents are located higher up.

Other fields at steady-state (240 min) are shown in Fig. 5. The high LWC in then convective region and the distinct stratiform region are clear from the $q_c$ and $q_r$ fields (Figs. 5a,b,
respectively). Similar to earlier times (Fig. 4), the majority of the graupel mass at this mature stage is concentrated above the updraft core, with small amounts reaching the surface in the convective region ($x \sim 40$-$70$ km) in the form of a high-density graupel “shaft” while lower-density graupel is transported far into the stratiform region ($x > 80$ km) which then melts to rain. Note that the very large graupel mass content in the convective region ($q_g > 10$ g kg$^{-1}$; Fig. 5c) is not of concern since in the kinematic framework there is no feedback to the flow field, which would otherwise react to buoyancy loading in a dynamical model (Szumowski et al. 1998; Morrison and Grabowski 2007).

The snow field is of some interest here since the inverse relation between $\rho_s$ (Fig. 5f) and $D_s$ (Fig. 5h) is apparent. The lowest snow density ($< 200$ kg m$^{-3}$) appears below the melting level ($z \sim 5$ km) well away from the convective region ($x > 100$ km) with the higher density snow (500 kg m$^{-3}$, the maximum allowable $\rho_s$) at higher elevations and closer to the convective region. For cases with weaker updrafts including winter cases, $\rho_s$ is more important in the simulation of graupel since graupel originating from snow has an embryo density equal to that of the snow.

b. Comparison to standard approaches of representing graupel/hail

Besides having a more conceptually sound and physically based representation of graupel overall, one potential advantage of the prognostic-$\rho_s$ approach is to enable a BMS to use a single rimed-ice category, for reduced computational cost, without the need to specify a priori if the category represents low-density graupel or high-density hail$^3$, depending on the weather system. To examine this, simulations with the control configuration used above (prognostic-$\rho_s$, graupel only) are compared to other graupel/hail configurations. The peak updraft speed is increased to

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$^3$ This is accomplished by the choice of constant density and fall speed-diameter parameters (Morrison and Milbrandt 2011).
40 m s$^{-1}$ for the following runs to ensure that one would expect the convective system to produce hail at the surface (Knight and Knight 2001).

1. GRAUPEL-ONLY

Figure 6 shows the steady-state solution for the $w_{\text{peak}} = 40$ m s$^{-1}$ control configuration run, PD-40. Except for larger condensate values, the simulation is qualitatively similar to PD-20 (Figs. 3-5). The simulation with the standard graupel parameterization – fixed $\rho_g = 400$ kg m$^{-3}$ and fixed fall speed parameters from the standard MY2 scheme (and hail off), FF-40, is shown in Fig. 7. There are two main differences in the results. First, due to the smaller fall speeds in FF-40 (Fig. 7d), the graupel mass is much less concentrated in the convective region as in PD-40 (Fig. 6a) and is more spread out into the stratiform region (Fig. 7a). As a result, there is much greater total precipitation in the stratiform region ($x > 100$ km; Fig. 7f). Second, despite the copious amounts of graupel mass in the convective region of FF-40, no graupel actually makes it to the surface without melting completely. Thus, for a single rimed-ice category scheme, the addition of the prognostic $\rho_g$ enables the simulation of high-density rimed-ice at the surface for a storm with a strong updraft. Note that in the PD configuration, with the hail category shut off, “hail” (high-density rimed ice) can still originate from frozen drop embryos, due to either collisional or probabilistic freezing of rain, even though it is represented by the graupel category.

It is important to recognize that differences between PD-40 and FF-40 are not due to the variable $\rho_g$ alone. To illustrate this, a graupel-only simulation was performed with prognostic $\rho_g$, as in PD-40, but with the $a_g$ and $b_g$ parameters fixed at their original values in the MY2 scheme. The results of this run, PF-40, are shown in Fig. 8. Despite the wide range of $\rho_g$ values in this run (Fig. 8b) with the distribution of values similar to that of PD-40 (Fig. 6b), all other aspects of the simulation are nearly identical to FF-40 (Fig. 7). Thus, to benefit from the prognostic $\rho_g$, it is
crucial that the fall speed parameters vary correctly as a function of $\rho_g$, as described in section 2.

Otherwise there is little apparent improvement in the overall parameterization of graupel.

2. GRAUPEL + HAIL

Given that the PD control configuration of the scheme appears to be capable of simulating solid precipitation at the surface using a single rimed-ice category, one must consider if indeed there is a need for a separate hail category at all. The PD-40 and FF-40 simulations are re-run but with the separate hail category switched on. The results of these runs, PD-HAIL-40 and FF-HAIL-40, are shown in Figs. 9 and 10, respectively. The overall differences in the graupel fields are quite similar to those between PD-40 (Fig. 6) and FF-40 (Fig. 7). PD-HAIL-40 has much less graupel transported into the stratiform region than FF-HAIL-40, and thus has much less surface precipitation there, and the graupel in PD-HAIL-40 is concentrated in the convective region with some reaching the surface. However, both PD-HAIL-40 and FF-HAIL-40 have a distinct hail shaft near the convective core, with hail at the surface having $D_h > 20$ mm, bulk fall speeds $> 30$ m s$^{-1}$, and a corresponding concentrated region with a large solid precipitation rate (Figs. 9i and 10i). Although PD-40 does produce a non-zero solid precipitation rate at the surface, the realistic narrow region of large, high-density ice that is simulated in the hail-on runs is not captured in the graupel field of PD-40.

Depending on the operational or research requirements of the modeling system and the degree of detail that is needed, a BMS with a single rimed-ice category and prognostic density may be sufficient, given that it is capable of producing solid precipitation at the ground for very strong convection. Moreover, there is a similar distribution of hydrometeors aloft and surface precipitation rate in the graupel-only and graupel-plus-hail configurations when $\rho_g$ is prognosed. However, testing this in a 3D dynamical model with correct feedbacks from latent heat release
and hydrometeor mass loading will be required to determine conclusively if the simulated storm structure and evolution is indeed adequate using a single-category prognostic-density scheme. On the other hand, if one wishes to model hail to a more realistic degree, separate categories for graupel and hail are clearly still required. Even with a two rimed-ice category scheme, the prognostic-$\rho_g$ treatment of graupel is of clear benefit due to the more realistic range of densities and fall speeds which impacts the spatial distribution within the storm and ultimately on the surface precipitation outside of the convective region.

c. Weak updraft cases

Although the results with the prognostic-$\rho_g$ method are encouraging in that it can produce solid precipitation at the surface for strong updraft cases, the scheme would be of limited use if it accomplishes this by simply forcing graupel to the ground regardless of the updraft. To be robust the scheme must produce solid precipitation when it should (strong updrafts or winter cases) but not to allow graupel to reach the surface otherwise. To test this, the PD and FF configurations were run with $w_{\text{peak}} = 3 \, \text{m s}^{-1}$. The results of these simulations, PD-3 and FF-3, are shown in Figs. 11 and 12, respectively. The spatial distribution of graupel mass is very similar over much of the domain, but with the notable exception that a small amount of graupel nearly reaches the surface in PD-3 near the main updraft (Fig. 11a; $x \sim 50 \, \text{km}$), with $D_g > 13 \, \text{mm}$ and $V_g > 12 \, \text{m s}^{-1}$. PD-3 thus simulates relatively large rimed ice at the surface despite having a very weak updraft. In contrast, FF-3 (Fig. 12) simulates a more realistic graupel field, with bulk fall speeds $< 2 \, \text{m s}^{-1}$ and which entirely melts before reaching the surface.

Thus, the prognostic-$\rho_g$ modification has introduced a new problem into the graupel-only configuration of the scheme. Due to the now large range of possible fall speeds, graupel is susceptible to uncontrolled sizes-sorting that can occur in two-moment sedimentation,
particular with an inverse-exponential size distribution (A1), as used in the control configuration (MY2005a; MM2010). Despite the weak forcing and low mass contents (Fig. 11a), size-sorting in the PD-3 simulation is clearly producing large $D_g$ and large $V_g$ values which means graupel reaches the surface and the run exhibits higher reflectivity in that region (Fig. 11e) compared to FF-3 (Fig. 12e). In the standard configuration (FF), size-sorting of graupel still exists, as is evident in Fig. 12c, but it never creates problems since the range of fall speeds, and thus rate of size-sorting, is small. With the prognostic-$\rho_g$ approach, size-sorting of graupel must therefore be controlled.

d. Controlling size-sorting: Diagnostic shape parameter for graupel

Excessive size-sorting occurs in two-moment schemes since the hydrometeor mass sediments at a faster rate than the number concentration. The result is that the ratio between the mass and number, which is proportional to the mean-mass diameter, increases in the lower part of the column. MY2005a discussed this problem and proposed an effective solution by which the shape parameter $[\mu_x$ in (A1)] varies as an increasing function of the mean particle size. As $D_x$ increases, the size spectrum narrows and the ratio of the bulk fall speeds for mass and number tends towards unity, thereby reducing the rate of size sorting.

With the prognostic-$\rho_g$ approach, the fall speeds are much different than for the standard graupel parameterization in the MY2 scheme (see Fig. 1), thus the specific diagnostic function for the graupel shape parameter proposed in MY2005a is not appropriate. To test the approach with the current prognostic-$\rho_g$ scheme, the following simple diagnostic relation was tested:

$$\mu_g = \left(1000D_g\right)^{0.075} \quad (14)$$

This equation was selected rather arbitrarily, through minor trial-and-error, and is used only to illustrate that this approach can indeed be used to treat the size-sorting problem that arises with a
prognostic $\rho_g$. The $w_{\text{peak}} = 3 \text{ m s}^{-1}$ simulation PD-MU-3, identical to PD-3 but using (14) to diagnose $\mu_g$, is shown in Fig. 13. The values of $D_g$, $V_g$, and $Z$ in the lower region near the main updraft ($x \sim 50 \text{ km}$; Figs. 13c,d,e, respectively) are now greatly reduced compared to PD-3 (Figs. 11c,d,e), graupel no longer reaches the surface, and there are no noticeable effects anywhere else in the domain. To ensure that the use of (14) does not overcompensate for the problems exhibited in PD-3, the strong updraft case was re-run using the diagnostic equation for $\mu_g$. This run, PD-MU-40 (Fig. 14), is very similar to PD-40 (Fig. 6) for all of the fields. Thus, the application of the diagnostic-$\mu_g$ relation (14) appears to have at least partially corrected the size-sorting problem with the prognostic-$\rho_g$ graupel scheme for the weak updraft case without doing anything detrimental for the strong updraft case.

Certainly equation (14) can be improved upon; PD-MU-3 still appears to have unrealistically large graupel sizes ($D_g \sim 10 \text{ mm}$; Fig. 13c). Ideally, this should be done by parameterizing $\mu_g = f(D_g)$ from results from a prognostic-$\mu_g$ (three-moment) scheme, as proposed by MY2005a and MM2010.

5. Sensitivity to the computation of $\rho_g$

A series of sensitivity experiments was performed using the control configuration with $w_{\text{peak}} = 3 \text{ m s}^{-1}$ and $w_{\text{peak}} = 40 \text{ m s}^{-1}$ to study the effects of specific aspects of the BMS that affect the calculation of the changes in $\rho_g$ through (12). Results for the various runs are summarized in Fig. 15 which shows the liquid precipitation rates $R_{\text{liq}}$, solid precipitation rates $R_{\text{sol}}$, column-integrated graupel mass, and the 1-km model reflectivity at the steady-state (240 min). The control runs for the weak and strong updrafts are the PD-3 and PD-40 simulations, respectively, discussed above.
The “oldSnow” runs (blue curves in Fig. 15) use the original MY2 mass-diameter parameters which assume spherical snow with a constant bulk density of 100 kg m\(^{-3}\). This becomes the embryo graupel density for graupel converted from snow. For \(w_{\text{peak}} = 3\) m s\(^{-1}\), this ultimately leads to a considerable increase in graupel in the convective region (Fig. 15e) and thus to increased \(R_{\text{iq}}\) and \(R_{\text{sol}}\) (Fig. 15a,c). In the strong updraft case, the old snow parameters result in a noticeable shift in graupel mass towards the stratiform region with corresponding changes to the surface precipitation, apparently due to the reduction in the initial \(\rho_g\) and hence fall speeds.

Thus, the value of \(\rho_g\) that is prescribed upon conversion of snow to graupel has a notable affect on the simulated graupel, even for the strong updraft case where riming dominates changes to \(\rho_g\).

The calculation of \(\rho_{\text{rime}}\) itself depends on several microphysical fields. Since the size of accreted drops affects \(\rho_{\text{rime}}\) through (B1) and (B2), the cloud droplet number concentration \((N_c)\) is now expected to have a more direct affect on the simulation of graupel. The PD control configuration used a continental cloud condensation nuclei (CCN) distribution, giving \(N_c\) on the order of \(2 \times 10^8\) m\(^{-3}\) (MY2005b). Results for the runs with maritime CCN (giving \(N_c \sim 0.8 \times 10^8\) m\(^{-3}\)) are depicted by the green curves in Fig. 15. The values of \(N_c\) upon cloud nucleation and thus the droplet sizes ultimately have a notable impact on the simulation of graupel for the weak-updraft case, with much more suspended graupel mass (Fig. 15e). This is most likely due to fewer and thus larger drops with the maritime CCN which results in larger graupel density, either from larger rime density and/or an increased likelihood of freezing to high-density graupel, giving larger graupel fall speeds and hence accretion rates.

While the rime density \((\rho_{\text{rime}})\) parameterization of CL1993 was used in this study, the formulation of Rasmussen and Heymsfied (1985) was used in M2010. The sensitivity test with the RH85 \(\rho_{\text{rime}}\) formulation is shown in red curves in Fig. 15. That parameterization gives larger
rime densities and leads to greater fall speeds. For the strong updraft this results in more graupel mass (Fig. 15f) and greater liquid and solid precipitation (Figs 15b,d) in the convective region.

In both the CL1993 and Rasmussen and Heymsfied (1985) parameterizations $\rho_{rime}$ depends on the surface temperature $T_{sfc}$ of the graupel particle. For simplicity, we assume that $T_{sfc}$ is equal to the ambient air temperature, $T$, though due to latent heat release $T_{sfc}$ can be warmer than $T$, depending on the ambient conditions and the amount of liquid water. The grey curves in Fig. 15 shows a run similar to control configuration (CTR, with the CL1993 parameterization) but with a prescribed $T_{sfc} = T + 2^\circ$C (the maximum surface temperature elevation found in the CL1993 experiments). Except for some slight changes in the stratiform region of the weak updraft case, there is very little effect on the results. Thus, the simple $T_{sfc} = T$ assumption in the control configuration is reasonable.

In (12), the change in $\rho_g$ due to the so-called 3-component freezing mechanism, such as the collection between ice and rain to produce graupel, is computed by assuming a mass-weighted average of the densities of the component categories (see appendix B). While this is a reasonable assumption, its sensitivity is tested with the simulation “3comp”, depicted by the brown curves in Fig. 15, which simply assumes that the $\rho_g$ does not change during this growth processes. Except for the slight changes in the stratiform region of the weak updraft case, there is virtually no sensitivity to the change in assumption for this process.

6. Conclusions

A method to predict the bulk graupel density $\rho_g$ has been described and added to the two-moment version of the Milbrandt-Yau bulk microphysics scheme. The approach involves the addition of a new prognostic variable, the bulk graupel volume mixing ratio $B_g$, to the scheme. Changes to $B_g$ are predicted through changes in $\rho_g$ for various microphysical processes including
riming, which uses a laboratory-based rime density parameterization that exploits the
independent prediction of mass and number concentration of several hydrometeor categories in
the two-moment scheme. Graupel fall speeds are now explicitly dependent on the predicted $\rho_g$
through diagnostically varying fall speed parameters. Idealized 2D simulations were performed
using a kinematic model with a prescribed flow field typical for a mesoscale convective system
and with different prescribed peak updraft speeds in order to illustrate the behavior of the
modified scheme and to examine the sensitivity to various configurations.

Through direct feedback to the fall speeds, the prognostic $\rho_g$ significantly changes the
simulation of graupel compared to the standard MY2 scheme, producing a much different spatial
distribution with a reduction of the large mass in the stratiform region. It also results in solid
precipitation at the surface in the convective region for the strong updraft simulations. These
results indicate that a BMS with a single rimed ice category is capable of representing a wide
range of graupel types, from lightly-rimmed, low-density, slowly-falling ice to high-density, fast-falling graupel – small or large – provided that $\rho_g$ is prognostic and is correctly linked to the
calculation fall speeds. This approach thus constitutes a significant improvement in the
representation of graupel in microphysics schemes. This is part of a paradigm shift for
microphysics models (bulk or bin) towards adding physical degrees of freedom for a given
hydrometeor type rather than including more categories with different prescribed physical
characteristics (Hashino and Tripoli 2007; Morrison and Grabowski 2008; Morrison and
Grabowski 2010; M2010; Harrington et al. 2012).

The above comments on the benefits of the prognostic-$\rho_g$ method apply equally to schemes
with or without a separate category for hail. A further benefit of the method is that it may reduce
the need for a separate hail category (and hence reduce computational cost), depending on the
specific requirements of the experimental or operational modeling system. If one is content to
simulate properly the bulk effects that come from the inclusion of hail, including the dynamical
feedbacks and the distribution of precipitation at the ground, a single rimed-ice category may be
sufficient. Ultimately, of course, a single category is incapable of representing the simultaneous
existence of both high-density and lower-density rimed ice in the same point in space and time,
which is probably necessary to simulate hail processes realistically in a cloud-resolving model,
as suggested by the results presented.

In general, adding complexity to a model solves some problems but simultaneously introduces
new ones. In the prognostic-$\rho_g$ scheme, the need to prescribe certain graupel characteristics is
relaxed, and there is now a wide range of possible densities and fall speeds, absent in standard
graupel schemes. Thus, there is a need to control size sorting with the prognostic-$\rho_g$ approach. A
variable relative spectral dispersion (shape) parameter $\mu_g$ dependent on the mean-particle size is
an effective means to treat this. Alternatively, though at increased computational cost, the
addition of a third\(^4\) prognostic moment such as reflectivity, thereby making a four-prognostic-
variable graupel scheme, would allow for the independent evolution of three size distribution
parameters plus the bulk density. Such a scheme would likely benefit from all of the aspects of
the prognostic-$\rho_g$ discussed above and would also control size-sorting while retaining changes to
the spectral dispersion. Further, simulation results could be used to determine the optimum
diagnostic relation for $\mu_g$ for a two-moment prognostic-$\rho_g$ scheme. Another aspect that merits
attention is a reexamination of the conversion of graupel to hail for schemes with separate rimed-
ice categories. These will be topics of further research.

\(^4\) Note that while the modified MY2 scheme now has three prognostic graupel variables, only the 0\(^{th}\) and 3\(^{rd}\)
moments are predicted. $B_g$ is equal to the 3\(^{rd}\) moment of the distribution with $q_g$ proportional to this through $\rho_g$. 
While the modified scheme has been examined in this study in the context of warm conditions, another area of potential benefit with the prognostic-\(\rho_g\) method is for winter storms with potential for riming and surface snowfall. In principle, the now large range of \(\rho_g\) should facilitate a smoother transition from snow to graupel and a conversion parameterization that is not based on arbitrary thresholds. Improvements to the conversion to and from graupel will be reported in future work as well examination of the behavior of the new approach for winter weather situations and for real cases in full 3D model simulations.

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Appendix A – Overview of the MY2 Bulk Microphysics Scheme

The BMS used is the multimoment scheme described in Milbrandt and Yau (2005a,b) and references therein, with modifications discussed in Milbrandt et al. (2010) and Milbrandt et al. (2012). The scheme has prognostic equations for moments of six hydrometeor categories: cloud (liquid droplets), rain (precipitating drops), ice (pristine crystals), snow (large crystals/aggregates), graupel (rimed ice), and hail (high density rimed ice and/or frozen drops),
where the particle size distribution (PSD) of each category $x$ is represented by a complete gamma function of the form:

$$N_x(D) = N_{0x} D^\mu_x e^{-\lambda_x D},$$  \hspace{1cm} (A1)

with $N_{0x}$, $\lambda_x$, and $\mu_x$ are the intercept, slope, and shape parameters, respectively, and $D$ is the particle diameter. In this study, the two-moment (with prognostic mixing ratios, $q_x$, and total number concentrations, $N_x$) fixed-$\mu_x$ version (MY2) is used, with $\mu_x = 0$ for $x = i, r, s, g, h$ and $\mu_x = 3$ for cloud droplets.

All particles were originally assumed to be spherical with constant bulk densities, $\rho_x$. Thus, for each category $c_x = (\pi/6 \rho_x)$ and $d_x = 3$, where the mass of a particle with diameter $D$ is given by

$$m_x(D) = c_x D^{d_x}.$$ \hspace{1cm} (A2)

The snow category now uses $c_s = 0.1597$ and $d_s = 2.078$ (mks units). With the modification described in this paper, $c_g$ is now a diagnostic function of $\rho_g$ (see Milbrandt et al. 2012).

**Appendix B – Parameterization Details**

*a. Graupel initiation processes*

Terms 2-5 in the numerator on the RHS of (12) represent graupel initiation processes, which can occur even if graupel is already present. For conversion from snow to graupel ($CN_{sg}$), which occurs when the riming rate of snow is at least triple the depositional growth rate, the density of the new graupel mass is given by the density of the snow. However, the MY2 scheme no longer assumes a constant bulk snow density $\rho_s$, which is now a diagnostic variable (Milbrandt et al. 2012). Thus, the graupel density for the mass converted from snow is given by $\rho_s = f(D_s)$.

*b. Three-component freezing*
For interactions rain and the ice-phase categories ice, snow, and graupel, the so-called “three-component freezing” occurs, resulting in either snow, graupel, or hail, depending on the density of the destination category (Ferrier, 1994; Milbrandt and Yau, 2005b). For interactions between ice and rain (i.e. rain collecting ice [\(CL_{ri}\]), and ice collecting rain [\(CL_{ir}\)]) that result in graupel (with \(\delta_{irg} = 1\); see Milbrandt and Yau, 2005b) the resulting mass will be high density, thus the density of hail, \(\rho_h = 900 \text{ kg m}^{-3}\), is assumed. A similar treatment is assumed for interactions between snow and rain, except that the density of the resulting mass (in the case that the destination category is graupel), is given by the mass-weighted mean density of the snow and the rain. The same is true for interactions between rain and graupel resulting in graupel, except that only collection of graupel by rain is considered in this three-component freezing mechanism; collection of rain by graupel is treated as riming (below).

c. Riming

The density of new rime, \(\rho_{rime}\), due to accretion of cloud and rain water (\(CL_{cg}\) and \(CL_{rg}\), respectively) is computed following the laboratory-based parameterization of CL1993. In a given set of ambient conditions, \(\rho_{rime}\) is determined as

\[
\rho_{rime} = 0.078 + 0.184R_i - 0.015R_i^2
\]  

(B1)

where

\[
R_i = \frac{0.5D_{\text{drop}}V_{\text{impact}}}{T_{\text{sfc}}},
\]  

(B2)

as in Macklin (1962), where \(D_{\text{drop}}\) is the diameter of the accreted liquid drop, \(V_{\text{impact}}\) is the impact velocity, equal to the difference in fall speeds between the graupel particle and the accreted drop, and \(T_{\text{sfc}}\) equal to the surface temperature of the graupel particle. Lower and upper boundaries on \(\rho_{rime}\) (and \(\rho_h\)) of 50 and 900 kg m\(^{-3}\), respectively, are imposed.
For implementation into MY2, the accreted drops taken to be the mean-mass drops of the sum of the cloud and rain distributions, giving

\[
D_{\text{drop}} = \left[ \frac{6 \rho(q_c + q_r)}{\pi \rho_r (N_c + N_r)} \right]^{\frac{1}{3}}.
\]  

(B3)

The densities from rime from cloud and from rain could be computed individually and summed, but it is done as above in MY2 for simplicity. Strictly speaking \( \rho_{\text{rime}} \) should be computed by integrating (B1) over the entire particle size spectra. Since this cannot be done analytically, the simpler application of (B3) is used. Future work will include the strict computation \( \rho_{\text{rime}} \) and application into the scheme using a look-up table.

Using (B3), the impact velocity between the graupel and liquid particles is determined from

\[
V_{\text{impact}} = \left| V(D_g) - V(D_{\text{drop}}) \right|, 
\]  

(B4)

where \( V(D_g) \) is computed as described in section 2b, with \( D_g \) the mean-mass diameter of graupel, and the fall speed parameter for \( V(D_{\text{drop}}) \) is the fall speed of the mean-mass drop. If \( D_{\text{drop}} < 1.2 \) mm, the fall speed parameter used, \( a_{\text{drop}} = 4000 \) and \( b_{\text{drop}} = 1 \) (appropriate for the intermediate drop size range; Rogers and Yau 1987); if \( D_{\text{drop}} > 1.2 \) mm the liquid spectrum is assumed to be dominated by rain and thus the standard rain fall speed parameters are used (\( a_{\text{drop}} = a_r = 149.1 \) and \( b_{\text{drop}} = b_r = 0.5 \); Tripoli and Cotton 1980). Note that (B4) is an overestimation of the impact velocity since it does not account for stagnation pressure. In the future, this effect will be accounted for in the scheme by applying the \( V_{\text{impact}} \) parameterization of Rasmussen and Heymsfield (1985).

For simplicity, it is assumed that \( T_{\text{sfc}} \) is equal to the ambient air temperature, \( T \). For real graupel, \( T_{\text{sfc}} \) is slightly warmer than the \( T \) during riming due to the latent heat release from the freezing accreted liquid, thus \( T_{\text{sfc}} \) should strictly be computed from a heat balance equation.
However, sensitivity test in the model indicated very little sensitivity to increases in $T_{sfc}$ within realistic ranges.
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Table 1. Variables and parameters pertaining to graupel in the prognostic-$\rho_g$ scheme.

<table>
<thead>
<tr>
<th>Prognostic variables</th>
<th>Diagnostic parameters</th>
<th>Constant parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_g$ mass mixing ratio</td>
<td>$a_g$ velocity-diameter coefficient parameter</td>
<td>$d_g$ mass-diameter exponent</td>
</tr>
<tr>
<td>$B_g$ bulk volume mixing ratio</td>
<td>$b_g$ velocity-diameter exponent parameter</td>
<td>3</td>
</tr>
<tr>
<td>$N_g$ total number mixing ratio</td>
<td>$\rho_g$ bulk graupel density</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$c_g$ mass-diameter coefficient</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$N_{0g}$ intercept parameter</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\lambda_g$ slope parameter</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\mu_g$ relative dispersion (shape) parameter</td>
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</tbody>
</table>

Table 2. Fitted fall speed parameters $a_g$ and $b_g$ for various $\rho_g$.

<table>
<thead>
<tr>
<th>$\rho_g$ (kg m$^{-3}$)</th>
<th>$a_g$ (m$^{1-b}$ s$^{-1}$)</th>
<th>$b_g$</th>
</tr>
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<tbody>
<tr>
<td>50</td>
<td>62.923</td>
<td>0.67819</td>
</tr>
<tr>
<td>150</td>
<td>94.122</td>
<td>0.63789</td>
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<td>250</td>
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<td>550</td>
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<tr>
<td>650</td>
<td>168.98</td>
<td>0.59663</td>
</tr>
<tr>
<td>750</td>
<td>179.36</td>
<td>0.59330</td>
</tr>
<tr>
<td>850</td>
<td>189.02</td>
<td>0.59048</td>
</tr>
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</table>
Table 3. List of simulations. Simulation names include hyphenated numbers denoting the peak updraft velocity ($w_{peak}$) used.

<table>
<thead>
<tr>
<th>Simulation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>PD-$[w_{peak}]$</td>
<td>Prognostic-$\rho_g$, diagnostic-$a_g$, $b_g$; graupel only (control configuration)</td>
</tr>
<tr>
<td>FF-$[w_{peak}]$</td>
<td>Fixed-$\rho_g$, fixed-$a_g$, $b_g$; graupel only</td>
</tr>
<tr>
<td>PF-$[w_{peak}]$</td>
<td>Prognostic-$\rho_g$, fixed-$a_g$, $b_g$; graupel only</td>
</tr>
<tr>
<td>PD-HAIL-$[w_{peak}]$</td>
<td>Prognostic-$\rho_g$, diagnostic-$a_g$, $b_g$; graupel + hail</td>
</tr>
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<td>Fixed-$\rho_g$, fixed-$a_g$, $b_g$; graupel + hail</td>
</tr>
<tr>
<td>PD-MU-$[w_{peak}]$</td>
<td>Prognostic-$\rho_g$, diagnostic-$a_g$, $b_g$; diagnostic-$\mu_g$; graupel only</td>
</tr>
<tr>
<td>OldSnow-$[w_{peak}]$</td>
<td>As control, but with spherical, constant-$\rho$, $m$-$D$ parameter for snow</td>
</tr>
<tr>
<td>Maritime-$[w_{peak}]$</td>
<td>As control, but with maritime CCN for cloud droplet nucleation</td>
</tr>
<tr>
<td>RH_rime-$[w_{peak}]$</td>
<td>As control, but with RH85 formulation for $\rho_{rime}$</td>
</tr>
<tr>
<td>3comp-$[w_{peak}]$</td>
<td>As control, but with constant $\rho_g$ for 3-component freezing</td>
</tr>
<tr>
<td>delT2-$[w_{peak}]$</td>
<td>As control, but with $T_{sfc} = T + 2^\circ C$ in calculation of $\rho_{rime}$</td>
</tr>
</tbody>
</table>
Figure Captions

Fig. 1 Terminal graupel fall speeds $v$ as a function of diameter $D$, for values of density $\rho_g$ ranging from 50 to 850 kg m$^{-3}$ with increments of 100 kg m$^{-3}$ (with $v$ increasing with $\rho_g$), computed explicitly (dotted grey) and computed using fitted falls speed parameters $a_g(\rho_g)$ and $b_g(\rho_g)$ (solid grey). Also shown are $v$ using fall speed parameters for the original MY2 hail (blue), MY2 graupel (red), and for various types of graupel from Locatelli and Hobbs (1974) (green).

Fig. 2 Prescribed flow field (left) and initial temperature and humidity profiles (right) used in the 2D kinematic model. The colored shading denotes vertical velocity (variable) in the convective region, with the peak updraft speed in the middle. The dark and light grey shading denotes the vertical velocities (fixed) for the mesoscale updraft and downdraft, respectively, with shading regions of -1, -0.25, -0.1, 0.1, 0.5, and 1 m s$^{-1}$. Arrows denote wind vectors.

Fig. 3 Model reflectivity at indicated times for the PD-20 (prognostic $\rho_g$; diagnostic $a_g$, $b_g$; $w_{peak} = 20$ m s$^{-1}$) simulation.

Fig. 4 Graupel mixing ratio ($q_g$), density ($\rho_g$), mean-mass diameter ($D_g$), and mass-weighted bulk fall speed ($V_g$) at 12 min (left), 15 min (middle), and 20 min (left) for PD-20 simulation. The mixing ratio shading thresholds are 0, 0.001, 0.01, 0.1, 0.5, 1.0, … g kg$^{-1}$.

Fig. 5 a) Cloud mixing ratio ($q_c$), b) rain mixing ratio ($q_r$), c) graupel mixing ratio ($q_g$), d) snow mixing ratio ($q_s$), e) graupel density ($\rho_g$), f) snow density ($\rho_s$), g) graupel mean-mass diameter ($D_g$), h) snow mean-mass diameter ($D_s$), i) graupel mass-weighted fall speed ($V_g$), and j) snow mass-weighted fall speed ($V_s$) for PD-20 simulation at steady state (240 min). The mixing ratio shading thresholds are 0, 0.001, 0.01, 0.1, 0.5, 1.0, ... g kg$^{-1}$.
Fig. 6  a) Graupel mixing ratio \( (q_g) \), b) graupel density \( (\rho_g) \), c) graupel mean-mass diameter \( (D_g) \),
d) graupel mass-weighted fall speed \( (V_g) \), e) model reflectivity \( (Z) \), and f) liquid and solid
precipitation rates for PD-40 (prognostic \( \rho_g \); diagnostic \( a_g, b_g; w_{peak} = 40 \text{ m s}^{-1} \) ) simulation at
steady state (240 min). The mixing ratio shading thresholds are 0, 0.001, 0.01, 0.1, 0.5, 1.0, ...
g kg\(^{-1}\).

Fig. 7  As in Fig. 6 but for FF-40 (fixed \( \rho_g \); fixed \( a_g, b_g; w_{peak} = 40 \text{ m s}^{-1} \) ) simulation.

Fig. 8  As in Fig. 6 but for PF-40 (prognostic \( \rho_g \); fixed \( a_g, b_g; w_{peak} = 40 \text{ m s}^{-1} \) ) simulation.

Fig. 9  a) Graupel mixing ratio \( (q_g) \), b) graupel density \( (\rho_g) \), c) graupel mean-mass diameter \( (D_g) \),
d) graupel mass-weighted fall speed \( (V_g) \), e) hail mixing ratio \( (q_h) \), f) hail mean-mass diameter
\( (D_h) \), g) hail mass-weighted fall speed \( (V_h) \), h) model reflectivity \( (Z) \), and i) liquid and solid
precipitation rates for PD-HAIL-40 (prognostic \( \rho_g \); diagnostic \( a_g, b_g; \text{hail on}; w_{peak} = 40 \text{ m s}^{-1} \) )
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g kg\(^{-1}\).

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Fig. 11  As in Fig. 6 but for PD-3 (prognostic \( \rho_g \); diagnostic \( a_g, b_g; w_{peak} = 3 \text{ m s}^{-1} \) ) simulation.

Fig. 12  As in Fig. 6 but for FF-3 (fixed \( \rho_g \); fixed \( a_g, b_g; w_{peak} = 3 \text{ m s}^{-1} \) ) simulation.

Fig. 13  As in Fig. 6 but for PD-MU-3 (prognostic \( \rho_g \); diagnostic \( a_g, b_g; \text{diagnostic } \mu_g; w_{peak} = 3 \text{ m s}^{-1} \) ) simulation.

Fig. 14  As in Fig. 6 but for PD-MU-40 3 (prognostic \( \rho_g \); diagnostic \( a_g, b_g; \text{diagnostic } \mu_g; w_{peak} = 40 \text{ m s}^{-1} \) ) simulation.

Fig. 15  Liquid precipitation rate [a) and b)], solid precipitation rate [c) and d)], column-
integrated graupel mass content [e) and f)], and 1-km model reflectivity [g) and h)] for
simulations with peak updraft velocities of 3 m s$^{-1}$ (left) and 40 m s$^{-1}$ (right) for the PD (prognostic $\rho_g$; diagnostic $a_g, b_g$) control configuration (CTR) and various sensitivity tests, indicated in the legend in panel b) and summarized in Table 3.
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