An Improved Representation of Rimed Snow and Conversion to Graupel in a Multicomponent Bin Microphysics Scheme

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ABSTRACT

This paper describes the development of a new multicomponent detailed bin ice microphysics scheme that predicts the number concentration of ice as well as the rime mass mixing ratio in each mass bin. This allows for local prediction of the rime mass fraction. In this approach, the ice particle mass size, projected area size, and terminal velocity–size relationships vary as a function of particle mass and rimed mass fraction, based on a simple conceptual model of rime accumulation in the crystal interstices that leads to an increase in particle mass, but not in its maximum size, until a complete “filling in” with rime and conversion to graupel occurs. This approach allows a natural representation of the gradual transition from unrimed crystals to rimed crystals and graupel during riming. The new ice scheme is coupled with a detailed bin representation of the liquid hydrometeors and applied in an idealized 2D kinematic flow model representing the evolution of a mixed-phase precipitating cumulus. Results using the bin scheme are compared with simulations using a two-moment bulk scheme employing the same approach (i.e., separate prediction of bulk ice mixing ratio from vapor deposition and riming, allowing for local prediction of bulk rime mass fraction). The bin and bulk schemes produce similar results in terms of ice and liquid water paths and optical depths, as well as the timing of the onset and peak surface precipitation rate. However, the peak domain-average surface precipitation rate produced by the bulk scheme is about 4 times that in the bin simulation. The bin scheme is also compared with simulations that assume the ice particles consist entirely of either unrimed snow or graupel. While overall results are fairly similar, the onset and timing of the peak domain-average surface precipitation rate are substantially delayed in the simulations that treat the ice particles as either unrimed snow or graupel. These results suggest the importance of representing different ice types, including partially rimed crystals, for this case.

1. Introduction

The treatment of ice microphysics in models plays a key role in simulations of microphysical–dynamical interactions (e.g., Leary and Houze 1979; Lord et al. 1984; Zhang and Gao 1989; McFarquhar et al. 2006), radiative transfer in clouds (e.g., Gu and Liou 2000; Wu 2002), and the phase and intensity of precipitation at the surface (Rutledge and Hobbs 1984; Gilmore et al. 2004; Thompson et al. 2004; Milbrandt and Yau 2006a,b). Microphysics schemes are broadly classified into two types: bulk schemes and bin schemes. Bulk schemes predict one or more bulk quantities and assume some underlying form for the particle size distribution (PSD). Bin schemes predict the evolution of the PSD by discretizing it into multiple size (or mass) bins. They are less restrictive than bulk schemes because they allow the size distribution to evolve but are computationally demanding. However, with increasing computing power, bin schemes that treat both liquid and ice PSDs (Hall 1980; Khain and Sednev 1996; Reisen et al. 1996; Geresdi 1998; Ovtchinnikov and Kogan 2000; Fridlind et al. 2004; Khain et al. 2004; Fridlind et al. 2007) are being applied with increasing frequency in cloud models to improve understanding of microphysical processes. Bin models have also been utilized as benchmarks for developing and testing bulk schemes (e.g., Geresdi 1998; Morrison and Grabowski 2007, hereafter MG07; Seifert 2008).

Significant uncertainties remain in the treatment of ice microphysics in both bin and bulk schemes. One of the most difficult problems concerns the treatment of ice particle habit and type. A wide variety of habits can be
observed in the atmosphere (e.g., Magono and Lee 1966; Pruppacher and Klett 1997). These different habits are associated with a wide range of characteristic mass–dimension \( (m–D) \), projected area–dimension \( (A–D) \), and terminal fall speed–dimension relationships (here dimension refers to maximum length or size), which play a critical role in the ice microphysical characteristics and process rates. Most microphysics schemes separate these various types of ice into discrete categories (e.g., pristine ice/snow, aggregates, graupel, hail). Such a priori categorization requires conversion of mass and/or number concentration between the different species, with the conversion rates and thresholds poorly constrained and hence often used as tuning parameters. Bin microphysics schemes typically treat different types of ice by a priori classification of different species within the same size/mass distribution based on particle mass (e.g., Ovtchinnikov and Kogan 2000) or by using a single component approach that includes separate particle distributions for each species (e.g., Geresdi 1998; Khain et al. 2004). The difficulty in such schemes is determining how to parameterize the transfer of particles from one species to another, such as the conversion of snow to graupel via riming. For example, Khain et al. (2004) assume that riming of ice crystals and snow leads to graupel formation if the mass of the rimed fraction exceeds half of the particle mass and the melted radius of the resulting particle exceeds 100 \( \mu \text{m} \).

Morrison and Grabowski (2008, hereafter MG08) applied the conceptual model of Heymsfield (1982) to shift away from the traditional approach of using predefined ice categories to treat different ice types, focusing on the rimed particles and conversion of snow to graupel during riming. This approach allows the crystal habit and type to evolve during the simulation based on the particle history. However, for simplicity this approach does not retain the history of differential axis growth due to vapor deposition or aggregation as in the approach of Chen and Lamb (1994) (as applied in the model of Hashino and Tripoli 2007), but it does allow the habit to evolve as a function of the rimed mass fraction of the crystal as well as size. The history of rime mass fraction is retained by predicting two ice mixing ratio variables: the mixing ratio due to vapor deposition and the mixing ratio due to riming. It follows that the rime mass fraction is derived locally from the ratio of the riming and total (deposition plus riming) mixing ratios. The \( m–D \) and \( A–D \) relationships evolve according to the predicted rimed mass fraction and particle dimension. All relevant microphysical processes and parameters are based on these \( m–D \) and \( A–D \) relationships for self-consistency. This approach removes the need for poorly constrained rates and thresholds for conversion of pristine ice/snow to graupel during riming.

In the current study, we extend the approach of MG08 to develop a new bin ice microphysics scheme. The new scheme is based on a multicomponent framework that predicts the number concentration of ice particles and the rime mass mixing ratio in each mass bin of the predicted ice particle mass spectrum. This allows the rime mass fraction to be predicted locally for each bin. The \( m–D \) and \( A–D \) relationships evolve according to the particle mass and rimed mass fraction. The new scheme is tested using a kinematic model with a specified, two-dimensional (2D), time-varying flow field corresponding to a precipitating mixed-phase cumulus. We compare results using the new bin scheme and bulk scheme of MG08 and also test the sensitivity of the bin scheme to a number of key parameters.

This paper is organized as follows. Section 2 gives a detailed description of the new scheme. Results using the 2D kinematic flow model are described in section 3. Section 4 contains discussion and conclusions.

2. Scheme description

The one-moment warm bin microphysics scheme of MG07 has been extended to the ice phase following the approach applied in MG08. The warm microphysics is detailed in MG07; the new ice component is described in this section.

The basic concept of the new ice scheme is to predict the number concentration of ice particles (i.e., concentration per kilogram of dry air) \( N_i \) in each mass bin, similar to the warm scheme for liquid drops (MG07). However, we extend this approach to a multicomponent framework by predicting an additional spectral variable corresponding to the rime mass mixing ratio \( q_r \). This allows for local diagnosis of the rime mass fraction \( F_r \) in each mass bin following

\[
F_r = \frac{q_r}{(N_i m)},
\]

where \( m \) is the particle mass associated with the given bin and \( N_i m \) is the total ice mass mixing ratio contained within the bin. Currently this approach assumes only a single value of rime mass fraction for a given bin at a given location and time. This is a simplification, especially when applied to cases such as deep, vigorous convection where frozen drops, hail, and rimed crystals may all be present in near proximity. The general multicomponent approach developed here could be applied to any conserved spectral microphysical variable, such as the mass mixing ratio of aerosol species embedded within the hydrometeors.

A more detailed description of the new approach is given below.
a. Spectral functions

The evolution of any generic conserved microphysical spectral variable \( \chi = \chi(m, x) \) (\( m \) is particle mass; \( x \) is the spatial location) is described by the following continuity equation:

\[
\frac{\partial \chi}{\partial t} + \frac{1}{\rho_a} \mathbf{V} \cdot [\rho_s (\mathbf{u} - V \mathbf{k}) \chi] + \frac{\partial}{\partial m} \left( \frac{dm}{dt} \right) \chi = S, \tag{2}
\]

where the second term on the left-hand side (lhs) is the physical transport of \( \chi \) via advection and sedimentation, the third term on the lhs is the transport of \( \chi \) in mass space due to sublimation/evaporation or particle growth (vapor deposition/condensation, riming, and/or coalescence/aggregation), and \( S \) is the source/sink term for \( \chi \). Here, \( t \) is time, \( \rho_a \) is the air density, \( \mathbf{u} \) is the 3D wind vector, \( V \) is the particle fall speed, and \( \mathbf{k} \) is a unit vector in the vertical. Note that (2) applies to any conserved spectral variable including number concentration (in units of per kilogram of dry air) and mass mixing ratio. Condensational and riming growth conserve the total number concentration of particles (integrated over spectrum) and hence for number concentration this growth is treated as advection of \( \chi \) in the mass space without any additional source terms. However, for the mixing ratio the total mass is not conserved; hence, in addition to advection of \( \chi \) one must also include a source term on the right-hand side (rhs) of (2). This source has to ensure that the change of the integral of \( \chi \) is equal to the increase of the total mixing ratio. This source term is also required to conserve particle concentration; advection of the mixing ratio across bins with larger mass will effectively decrease the particle concentration since the concentration is equal to the mixing ratio divided by the bin mass. To compensate for this decrease, one needs to increase the mixing ratio (hence the source term) so that the total particle number is conserved.

The new scheme solves equations for three conserved microphysical variables, \( \phi, \zeta, \) and \( \psi \). Here, \( \phi(m, x) = dN(m, x)/dm \), where \( N(m, x) \) is the concentration of ice particles per unit mass of dry air smaller than \( m \) (i.e., cumulative concentration) at spatial location \( x \); \( \zeta(m, x) = dq_x(m, x)/dm \), where \( q_x(m, x) \) is the cumulative mass mixing ratio of rime ice smaller than \( m \) at \( x \); and \( \psi(m, x) = dN_w(m, x)/dm \), where \( N_w(m, x) \) is the cumulative number concentration of liquid drops smaller than \( m \) at \( x \). Note that here "rime ice" does not strictly refer only to ice grown by riming (accretion of liquid water) but also includes other processes that produce high-density ice (e.g., freezing of drops) since we do not separately treat frozen drops or hail. For simplicity, hereafter we will refer to all high-density ice as rime ice. The addition of particle concentration in a given bin due to drop freezing will increase the average \( F_r \) of that bin if \( F_r < 1 \). Extension of the scheme in the future to include a separate treatment of frozen drops/hail will address this issue.

Following (2), the kinetic equations for \( \phi(m, x) \), \( \zeta(m, x) \), and \( \psi(m, x) \) are

\[
\frac{\partial \phi}{\partial t} + \frac{1}{\rho_a} \mathbf{V} \cdot [\rho_s (\mathbf{u} - V \mathbf{k}) \phi] + \frac{\partial}{\partial m} \left( \frac{dm}{dt} \right) \phi = S_\phi = (\frac{\partial \phi}{\partial t})_{\text{act}} + (\frac{\partial \phi}{\partial t})_{\text{frz}} + (\frac{\partial \phi}{\partial t})_{\text{coag}} + (\frac{\partial \phi}{\partial t})_{\text{sub}}, \tag{3}
\]

\[
\frac{\partial \zeta}{\partial t} + \frac{1}{\rho_a} \mathbf{V} \cdot [\rho_s (\mathbf{u} - V \mathbf{k}) \zeta] + \frac{\partial}{\partial m} \left( \frac{dm}{dt} \right) \zeta = S_\zeta = (\frac{\partial \zeta}{\partial t})_{\text{trz}} + (\frac{\partial \zeta}{\partial t})_{\text{rim}} + (\frac{\partial \zeta}{\partial t})_{\text{sub}}, \tag{4}
\]

\[
\frac{\partial \psi}{\partial t} + \frac{1}{\rho_a} \mathbf{V} \cdot [\rho_s (\mathbf{u} - V \mathbf{k}) \psi] + \frac{\partial}{\partial m} \left( \frac{dm}{dt} \right) \psi = S_\psi = (\frac{\partial \psi}{\partial t})_{\text{act}} + (\frac{\partial \psi}{\partial t})_{\text{trz}} + (\frac{\partial \psi}{\partial t})_{\text{evap}} + (\frac{\partial \psi}{\partial t})_{\text{rim}} + (\frac{\partial \psi}{\partial t})_{\text{coal}}, \tag{5}
\]

where \( V_r \) and \( V_l \) are the terminal fall speeds for the ice particles and drops, respectively. The individual source/sink terms on the rhs of (3)–(5) correspond with various microphysical processes. The first two terms on the rhs of (3) are the source of \( \phi \) (ice particle number concentration) resulting from deposition and condensation/ freezing nucleation and droplet freezing. The third and fourth terms on the rhs represent the sink of \( \phi \) due to complete sublimation of small ice particles and aggregation. The first term on the rhs of (4) represents the source of \( \zeta \) (dense ice or rime ice mass mixing ratio) due to freezing of large drops, the second term is the source of \( \zeta \) due to riming, and the third term is the sink of \( \zeta \) due to sublimation. The liquid source/sink terms on the rhs of (5) for \( \psi \) (droplet number concentration) are activation of droplets, freezing, complete evaporation of small droplets, and collision/coalescence. These source/sink terms are discussed in more detail below.

Here we neglect melting since the focus in this paper is on the treatment of riming and conversion of snow to graupel and the scheme is applied to subfreezing conditions only. Future work will include a detailed treatment of the melting. This could be done, for example, by including an additional spectral function to account for liquid water mixing ratio accumulated on particles of given mass, similar to Khain et al. (2004). Such an approach would also be able to more realistically treat the "wet growth" regime, which may be important for simulating hail formation (Pflaum 1980).
Following the discussion below (2), the same microphysical processes are represented by different source/sink terms when one predicts either particle number concentration or mass mixing ratio (in our case, the rime mass mixing ratio). For example, riming is a source for rime mass mixing ratio and a sink for liquid particle number concentration, but it does not impact total ice particle number. Thus, it appears as a source term on the rhs of (4) and (5) but not (3). Similarly, aggregation is a sink for ice particle number concentration in (3) but not the rime mass mixing ratio in (4); however, it results in transport of $\zeta$ in mass space. Growth of ice due to vapor deposition is not a source for either particle concentration or rime mass mixing ratio; it is instead represented by the transport of $\phi$ and $\zeta$ in mass space, which is included in the third term on the lhs of (3) and (4). The growth of liquid drops by condensation is similarly represented by the transport of $\psi$ in mass space and included in the third term on the lhs of (5).

More detailed description of the formulations of the various microphysical source/sink terms is given in section 2c, along with idealized tests demonstrating the ability of the scheme to produce physically consistent results for growth due to vapor deposition, aggregation, or riming, with the different growth processes calculated in isolation. The liquid microphysical processes (condensation/evaporation, droplet activation, collision/coalescence) are detailed in MG07.

In the discrete system used in the scheme consisting of $N$ bins (or classes) of mass $m$, the spectral functions for each bin $i$ are defined as $\phi^{(i)} = N^{(i)}/\Delta m^{(i)}$, where $N^{(i)}$ is the concentration of ice particles per unit mass of dry air in the bin $i$ and $\Delta m^{(i)}$ is the width of this bin; $\bar{\zeta}^{(i)} = q^{(i)}/\Delta m^{(i)}$, where $q^{(i)}$ is the mass mixing ratio of rime ice in $\Delta m^{(i)}$; and $\psi^{(i)} = N_{\text{w}}^{(i)}/\Delta m^{(i)}$, where $N_{\text{w}}^{(i)}$ is the number concentration of liquid drops per unit mass of dry air in $\Delta m^{(i)}$. This transforms the continuous (3)–(5) into a system of $3N$ coupled equations:

$$
\frac{\partial \phi^{(i)}}{\partial t} + \frac{1}{\rho_a} \mathbf{v} \cdot \left[ \rho_a \left( \mathbf{u} - V^{(i)} \right) k \right] \phi^{(i)} = \left( \frac{\partial \phi^{(i)}}{\partial t} \right)_{\text{dep}} + \left( \frac{\partial \phi^{(i)}}{\partial t} \right)_{\text{rim}} + \left( \frac{\partial \phi^{(i)}}{\partial t} \right)_{\text{nuc}} + \left( \frac{\partial \phi^{(i)}}{\partial t} \right)_{\text{trz}} + \left( \frac{\partial \phi^{(i)}}{\partial t} \right)_{\text{sub}} + \left( \frac{\partial \phi^{(i)}}{\partial t} \right)_{\text{agg}},
$$

for $i = 1, \ldots, N$,

$$
\frac{\partial \zeta^{(i)}}{\partial t} + \frac{1}{\rho_a} \mathbf{v} \cdot \left[ \rho_a \left( \mathbf{u} - V^{(i)} \right) k \right] \zeta^{(i)} = \left( \frac{\partial \zeta^{(i)}}{\partial t} \right)_{\text{dep}} + \left( \frac{\partial \zeta^{(i)}}{\partial t} \right)_{\text{agg}} + \left( \frac{\partial \zeta^{(i)}}{\partial t} \right)_{\text{trz}} + \left( \frac{\partial \zeta^{(i)}}{\partial t} \right)_{\text{rim}} + \left( \frac{\partial \zeta^{(i)}}{\partial t} \right)_{\text{sub}},
$$

for $i = 1, \ldots, N$, and

$$
\frac{\partial \psi^{(i)}}{\partial t} + \frac{1}{\rho_a} \mathbf{v} \cdot \left[ \rho_a \left( \mathbf{u} - V^{(i)} \right) k \right] \psi^{(i)} = \left( \frac{\partial \psi^{(i)}}{\partial t} \right)_{\text{cond}} + \left( \frac{\partial \psi^{(i)}}{\partial t} \right)_{\text{act}} + \left( \frac{\partial \psi^{(i)}}{\partial t} \right)_{\text{trz}} + \left( \frac{\partial \psi^{(i)}}{\partial t} \right)_{\text{evap}} + \left( \frac{\partial \psi^{(i)}}{\partial t} \right)_{\text{rim}} + \left( \frac{\partial \psi^{(i)}}{\partial t} \right)_{\text{coal}},
$$

for $i = 1, \ldots, N$,

where the first terms on the rhs of (6), (7), and (8) represent the transport across bins due to condensation/deposition and evaporation/sublimation; the second term on the rhs of (6) represents transport of $\phi^{(i)}$ across bins due to riming; the second term on the rhs of (7) represents transport of $\zeta^{(i)}$ across bins due to aggregation; and the other source/sink terms are as described for (3)–(5).

For application here, 54 mass bins between $10^{-14}$ and $1.3 \times 10^{-3}$ kg are employed for both liquid and ice. Note, however, that this scheme does not require the same mass bins for liquid and ice; it was chosen here for simplicity. The grid in mass space is linear–exponential, with the mean mass $m_i$ (kg) for each bin $i$ given by

$$
m_i = 5 \times 10^{-14}(i - 1) + 10^{-14} \times 10^{0.21(i - 1)},
$$

with linear spacing important for small masses (less than about $10^{-11}$ kg) and exponential spacing dominant for larger masses to provide a stretched grid incorporating precipitation-sized particles. We tested sensitivity to the bin spacing by doubling the number of bins over the same mass range. Ice microphysical quantities are similar, whereas the liquid microphysical quantities show more sensitivity to the bin resolution. Time-average values of the liquid water path, ice water path, and surface precipitation rate for the runs with 54 and 108 bins applied to the case study described in the next section differ by 17%, 1%, and 11%, respectively. Despite the fairly large sensitivity of liquid microphysical quantities to the bin spacing, the impact of this sensitivity on key ice microphysical quantities (riming rate and rime mass fraction) is minimal. This result likely reflects the fact that most of the change in LWP appears...
to be due to changes in the small droplet part of the spectrum, which has small (or zero) collision efficiencies for collection by ice [see section 2c(5)]. The impact of bin resolution on various microphysical process rates is further discussed in section 2c.

The system (6)–(8) is integrated on split time steps, with advective and gravitational transport together with coalescence, riming, and aggregation calculated over the primary time step, and condensation/evaporation and deposition/sublimation treated as advection in mass space over variable subtime steps (to ensure numerical stability in terms of the Courant–Friedrichs–Levy criterion) using the 1D advection scheme of Smolarkiewicz (1984). The substep is determined such that the advective Courant number for diffusional growth in mass space cannot exceed unity.

b. Representation of particle dimension and projected area

The size (hereafter particle size will be referred to in terms of its maximum dimension $D$) and projected area of the crystals, which are key in determining the various microphysical process rates, are derived as follows for each bin. Two cases are considered for each bin $i$: one with no rime mass present, $q_r^{(i)} = 0$, and one with rime mass present, $q_r^{(i)} > 0$.

The case for bins with no rime ice (i.e., ice grown only by vapor diffusion and aggregation) is described first. For a given particle mass associated with bin $i$, $m^{(i)}$, we derive particle size in each bin following

$$D^{(i)} = \max \left( \left( \frac{m^{(i)}}{\alpha_u} \right)^{1/\beta_u}, \left( \frac{6m^{(i)}}{\pi \rho_i} \right)^{1/3} \right),$$

where $\rho_i$ is the bulk density of solid ice and $m = \alpha_u D^{\beta_u}$ is the power-law $m$–$D$ relationship for unrimed ice. Since empirical $m$–$D$ relationships apply only to a limited size range (e.g., Mitchell 1996), at small masses these relationships can give a particle size smaller than that of a sphere, which is physically impossible. Thus, if the $m$–$D$ relationship for a particular value of $m$ produces $D$ smaller than that of a sphere, we use $D$ calculated for an ice sphere of mass $m$ according to (10). A similar approach is used to derive projected area. Here we apply the same power-law $m$–$D$ and $A$–$D$ relationships for unrimed crystals used in MG08 and shown in Table 1. These relationships are for plate-like crystals with sector branches (Mitchell 1996) and are a reasonable assumption in moderately supercooled conditions near water saturation.

Next, we describe the case for bins with rime ice. The $m$–$D$ relationship for these crystals is derived following the conceptual model of Heymsfield (1982). Based on this model, rime accumulation in the crystal interstices and on the faces increases the particle mass but not the particle dimension $D$, and such a picture is valid up to the point of a complete “filling in” of crystal interstices. This conceptual model of riming is likely to be valid for all nonspherical crystal types that exhibit a preferential orientation of the long axis that is perpendicular to the direction of their settling, leading to rime growth mostly along the short axis and hence an increase of $m$ but not of $D$. Thus, the process of riming in this conceptual model increases the effective particle density, where effective density is defined as particle mass divided by the volume of a sphere with the same $D$. In the example of a plate crystal in which riming occurs on the leading face, the effective density increases with riming because of thickening of the plate.

Once the particle is filled in with rime, it becomes graupel and further growth (by either riming or vapor deposition) increases both particle size and mass. Traditionally, graupel is defined as those particles for which the riming has progressed to the point that the underlying habit is no longer identifiable. Since we do not currently treat frozen drops as a separate hydrometeor species in this scheme, it is assumed that freezing of large liquid drops may also produce graupel without the intermediary step of riming. A more complete approach would be to separately treat these frozen drops, but the development of such a scheme is beyond the scope of this paper.

Prior to the complete filling in of the interstices of a partially rimed crystal, $D^{(i)}$ is determined by the crystal mass grown by only diffusion of water vapor and aggregation in bin $i$, $m_d^{(i)}$, using the relation

$$m_d^{(i)} = m^{(i)} - q_r^{(i)} N_r^{(i)} = \alpha_u D^{(i) \beta_u} \geq 0,$$

Next, we derive projected area $A^{(i)}$ for graupel and further growth (by either riming or vapor deposition) increases both particle size and mass. Traditionally, graupel is defined as those particles for which the riming has progressed to the point that the underlying habit is no longer identifiable. Since we do not currently treat frozen drops as a separate hydrometeor species in this scheme, it is assumed that freezing of large liquid drops may also produce graupel without the intermediary step of riming. A more complete approach would be to separately treat these frozen drops, but the development of such a scheme is beyond the scope of this paper.

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$$m_d^{(i)} = m^{(i)} - q_r^{(i)} N_r^{(i)} = \alpha_u D^{(i) \beta_u} \geq 0.$$
which logically follows from the assumption that rime increases particle mass but not size up to the point of complete filling in. Since this conceptual model provides no direct information on the evolution of projected area during the riming, the projected area of a partially rimed crystal is given by interpolation based on particle mass and graupel for the given \( D^{(i)} \) (graupel mass is derived from the \( m-D \) relationship described below).

We assume complete filling in of the crystal interstices and conversion to graupel occurs when the \( D^{(i)} \) found by rearranging and solving (11) is smaller than the \( D^{(i)} \) obtained from the \( m-D \) relationship for graupel for the same \( m^{(i)} \). In this instance \( D^{(i)} \) is given by

\[
D^{(i)} = \max \left[ \left( \frac{m^{(i)}}{\alpha_g} \right)^{1/\beta_g}, \left( \frac{m^{(i)}}{\alpha_n} \right)^{1/\beta_n}, \left( \frac{6m^{(i)}}{\pi \rho_i} \right)^{1/3} \right], \tag{12}
\]

where \( m = \alpha_g D^{\beta_g} \) is the \( m-D \) relationship for graupel. Since the empirical \( m-D \) relationship for graupel applies to a limited size range, we ensure that the \( D^{(i)} \) for graupel must be larger than that of unrimed ice or solid ice spheres in (12). A similar approach is applied to derive the projected area. Here, the parameters \( \alpha_g \) and \( \beta_g \) are for lump graupel following Heymsfield and Kajikawa (1987) (see Table 1). Since the \( A-D \) for graupel is not given by Heymsfield and Kajikawa (1987), we use the power-law formulation for hail following Matson and Huggins (1980) (see Table 1). Note that for a given mass we assume constant effective density once the particles are considered to be graupel. However, laboratory and field measurements suggest that graupel densities can vary significantly as a function of particle temperature, fall speed, and size of the riming drops (e.g., Pflaum and Pruppacher 1979; Rasmussen and Heymsfield 1985).

c. Description of the microphysical process rates

1) PRIMARY AND SECONDARY ICE INITIATION

The number concentration of ice nuclei acting in deposition and condensation freezing nucleation modes, \( N_{NIN} \), is given by Meyers et al. (1992) as a function of ice supersaturation. Nucleation is allowed in conditions with 5% or greater ice supersaturation at temperatures less than 268.15 K, following MG08. The initial radius of a new crystal is assumed to be 1 \( \mu m \). At present we assume that ice nuclei are always available, so that

\[
\left( \frac{\partial m^{(i)}}{\partial t} \right)_{nuc} = \max \left( \frac{N_{NIN} - N}{\Delta t}, 0 \right), \tag{13}
\]

where \( N \) is the total number concentration of existing ice and \( \Delta t \) is the time step. Ice supersaturation is calculated from the predicted temperature and water vapor mixing ratio fields. An alternative approach proposed by Khain et al. (2004) is to use the semi-Lagrangian differential supersaturation to determine the number of nucleated crystals within a time step [see their Eq. (4)]. A more complete approach is to predict the concentration of ice nuclei, which allows for their depletion, regeneration, and transport (Fridlind et al. 2007).

The freezing of cloud droplets and drizzle/rain drops occurs through immersion freezing following the probabilistic formulation of Bigg (1953). Immersion freezing rates are a function of drop volume, with the number concentration of new ice particles interpolated (using a mass-conserving scheme) from the number concentration of frozen drops. At temperatures below 233.15 K, liquid cloud and rain drops are assumed to freeze homogeneously within a single model time step.

Ice multiplication occurs by rime splintering in the temperature range from \(-3^\circ \) to \(-8^\circ \)C following Hallett and Mossop (1974). Splintering is based on the number of droplets with diameter exceeding 24 \( \mu m \) that are collected by graupel particles (i.e., rimed particles that are “filled in” with rime; see section 2b).

2) PARTICLE FALL SPEED

Terminal particle fall velocity is given by Mitchell and Heymsfield (2005) as function of the nondimensional Best (or Davies) number (related to mass divided by the projected area), which is similar to the approach of Khvorostyanov and Curry (2002). This formula gives the particle Reynolds number as a power-law function of the Best number to account for the dependence on drag coefficient. Terminal fall speed is then calculated from these power-law coefficients, along with the viscosity of air, air density, and the coefficients in the \( m-D \) and \( A-D \) relationships, via Eqs. (10)–(12) in Mitchell and Heymsfield (2005). Here we neglect the correction factor proposed by Mitchell and Heymsfield (2005) to account for the effects of wake turbulence associated with large aggregates. The same fall speed is applied to both the number concentration and rime mass mixing ratio in each bin for calculation of sedimentation in (6) and (7).

3) DEPOSITION/SUBLIMATION

Deposition/sublimation is given by diffusional mass and heat balance neglecting surface kinetic effects but including ventilation effects. Crystal capacitance is assumed to be that of a sphere of diameter \( D \) for graupel particles, and 0.48\( D \) for unrimed nonspherical ice particles
following Field et al. (2008). For partially rimed crystals, capacitance is found by linear interpolation between the values for graupel and unrimed crystals based on particle mass for a given $D$. Ventilation is treated following Hall and Pruppacher (1976) for $X \geq 1$, where $X(D) = \frac{N_{Sc}^{1/3}N_{Re}^{1/2}}{N_{Sc}^{1/3}}$, $N_{Sc}$ and $N_{Re}(D)$ are the Schmidt and Reynolds numbers associated with the falling particle for the given environmental conditions, respectively. Ventilation effects are neglected for $X(D) < 1$.

Numerically, deposition/sublimation is treated as 1D advective transport of $\phi$ in $m$ space using the multidimensional positive definite advection transport algorithm (MPDATA) scheme (Smolarkiewicz 1984; Smolarkiewicz and Margolin 1998). Note we must also advect $\zeta$ in $m$ space because of the change in crystal mass during vapor deposition/sublimation, although this has no impact on the total mass mixing ratio of rime ice (i.e., $\zeta$ integrated over mass spectrum). Sublimation is assumed to decrease total rime ice mass mixing ratio only when all of the remaining ice mass in a bin consists of rime ice [i.e., when $q_r(i) = m(i)N_i(i)$]. This assumption is uncertain given that rime ice tends to accumulate on the crystal edges and leading face (e.g., Pruppacher and Klett 1997), resulting in eventual encasement of the vapor diffusionally grown ice. However, it is meant to represent the development of more rounded crystal forms that are produced by sublimation (e.g., Korolev et al. 1999) (i.e., more akin to crystals with larger $F_r$). A sensitivity test (not shown) of the case described in section 3 in which rime ice is depleted before diffusionally grown ice during sublimation suggests little sensitivity to this assumption.

The results of a test to demonstrate the ability of the scheme to produce physically consistent evolution of growth by vapor deposition is shown in Fig. 1. This test assumes an initial rime mass mixing ratio of 0.7, constant temperature of $-10^\circ C$, and ice supersaturation corresponding to liquid water saturation. Depositional growth results in a shift of $\phi$ to larger masses. The transport of particles with a given rime mass mixing ratio into larger mass bins due to vapor deposition (i.e., transport of $\zeta$ in $m$ space) results in a decrease in $F_r$, as expected. This test also shows limited sensitivity of diffusional growth to doubling of the number of bins over the same mass range.

4) AGGREGATION

Particle growth due to hydrometeor collection is calculated using the gravitation collection kernel and linear flux method (Bott 1998). Aggregation efficiencies for ice–ice collisions $E_{agg}$ are set to 0.1 following Field et al. (2006). Mitchell et al. (2006) found that $E_{agg}$ of 0.07 produced the most realistic evolution snow-size spectra in a steady-state snow growth model. While in reality values of $E_{agg}$ are likely to be a more complex function of size, temperature, habit, etc. (e.g., Pruppacher and Klett 1997), it is uncertain and assumed to be constant here for simplicity.

Ice–ice collection (aggregation) impacts $\phi$ and $\zeta$ but has no impact on the total rime ice mass (i.e., $\zeta$ integrated...
over the mass spectrum). Figure 2 shows results of a simple test to demonstrate the ability of the scheme to predict a physically consistent solution for growth by aggregation. In this test, the initial rime mass fraction is assumed to be constant and equal to 0.7. All other growth processes are turned off except aggregation. As expected, aggregation results in a decrease of $f$ for small mass bins and an increase for the large bins, and also exhibits little sensitivity to increasing bin resolution. The value of $F_r$ of the newly formed aggregates is represented by the mass-weighted sum of the $F_r$ of the individual aggregating particles. Thus, in this test $F_r$ does not change from its initial specified value of 0.7 (for mass bins that actually contain ice). Ice particle breakup via collision is neglected for simplicity owing to its significant uncertainty.

5) RIMING

Efficiencies for collisions between liquid drops and crystals $E_{ci}$ have been studied by several researchers (e.g., Pitter and Pruppacher 1974; Martin et al. 1981; Lew and Pruppacher 1983; Lew et al. 1985; Mitchell 1995; Wang and Ji 1992; Khain et al. 2001). Values of $E_{ci}$ obtained from various studies are often inconsistent and furthermore they are found to be strongly dependent on crystal habit; thus, uncertainty remains in the representation of $E_{ci}$ for various habits. Here, prior to complete filling in of the crystal with rime (see section 2b), for crystals colliding with droplets of $d < 40 \mu m$, we use the relationship of Hall (1980) that was fitted to the data of Pitter and Pruppacher (1974) and Pitter (1977) for plate crystals. For crystals that are filled in with rime we use the results of Beard and Grover (1974) for spheres, similar to the approach of Hall (1980). For collisions of crystals with large drops of $d > 100 \mu m$, we follow the results for numerical calculations of plate–drop interaction from Lew et al. (1985). A close fit to their data is given by

$$E_{ci} = \tan^{-1}\left(\frac{D}{D_0}\right) - 0.5,$$

where $D$ is maximum crystal size (m) and $D_0 = 1.4 \times 10^{-3}$ m. Note that in all instances $E_{ci}$ is restricted to be within the range from 0 to 1. For collisions of crystals with drops of $40 < d < 100 \mu m$ we linearly interpolate $E_{ci}$ between the values for $d < 40 \mu m$ and $d > 100 \mu m$. For simplicity it is assumed that in subfreezing conditions, drop–crystal collisions result in instantaneous freezing of the collected liquid (i.e., “dry growth” regime).

Values of $E_{ci}$ as a function of droplet diameter $d$ for various crystal sizes ($D = 75, 200, 700,$ and $3000 \mu m$) are shown in Fig. 3. For plates, there is a sharp cutoff for $d < 20 \mu m$; for spheres the cutoff is less steep and extends to smaller droplet sizes. For both spheres and plates, values of $E_{ci}$ increase with increasing crystal size, except in the case of large plates with $D >$ about 1 mm. Note that experimental results using natural snow crystals suggest that droplets with $d < 10 \mu m$ have small but finite $E_{ci}$ ($\sim 0.04–0.2$), which may be important given that much of the liquid water content may be contained in droplets of this size (Mitchell 1995). We also note that turbulence may have important impacts on crystal–droplet collision.
kernels (Pinsky and Khain 1998; Pinsky et al. 1998). Comprehensive testing of the various formulations for collision efficiencies and kernels is beyond the scope of this paper but should be examined in future work.

Although total ice particle concentration (i.e., $\phi$ integrated over the mass spectrum) is not affected by riming, total ice mass and total rime ice mass increase. As described previously, this requires both advection of rime mass mixing ratio $\zeta$ in mass space and an additional source term corresponding to the increase in rime mass. Here this is calculated in a two-step approach. First, the increase of particle mass due to riming results in transport of $\phi$ and $\zeta$ to larger mass bins, which is calculated using a modified version of the Bott (1998) flux method (it also results in a decrease of $\psi$ accounting for the loss of liquid drops). Second, the rime mass mixing ratio is added to the mass bins into which particles were transported by the first step; the increase of rime mass mixing ratio is equal to the increase of total ice mass calculated from the first step and the decrease of liquid mass. Results of a test demonstrating the ability of the scheme to produce a physically consistent solution for riming growth is shown in Fig. 4. In this test all other growth processes are turned off, the droplets have a radius of $10 \mu m$ with a concentration of $30 \text{ cm}^{-3}$, and the collection efficiency is assumed to be a constant of 0.75. The growth of particles to larger sizes is seen by the shift of $\phi$ to larger masses. Correspondingly, $F_i$ increases across all sizes from its initial constant value of 0.7 because of the increase in rime mass mixing ratio (i.e., increase of $\zeta$) as expected. These results illustrate the efficacy of this approach and also show limited sensitivity of riming to increasing the bin resolution for ice.

3. Application in kinematic flow model

a. Description of the kinematic framework and case study

The bin mixed-phase microphysics scheme was implemented in a 2D kinematic model following MG08, which is similar to that presented by Szumowski et al. (1998), Grabowski (1998, 1999), and MG07. The kinematic model employs a specified flow field, which allows for testing of the microphysics in a framework that includes advective transport and particle sedimentation but avoids complications due to feedbacks between the dynamics and microphysics. In addition to the spectral equations describing conservation of the microphysical quantities, the kinematic model solves conservation equations for the potential temperature and water vapor mixing ratio. The equations include advective transport and sources/sinks due to the microphysical processes. Transport in the physical space is calculated using a 2D version of MPDATA (Smolarkiewicz 1984; Smolarkiewicz and Margolin 1998).

The specified flow field varies in time, representing the evolution of an idealized shallow convective plume. The flow pattern consists of low-level convergence, upper-level divergence, and a narrow updraft at the center of the domain. The horizontal flow includes weak vertical shear. Equations describing the streamfunction and flow velocities are detailed in MG07, with modifications described by MG08. The updraft speed is held constant at $1 \text{ m s}^{-1}$ for the first 15 min, intensifies to a peak of $8 \text{ m s}^{-1}$ at 25 min, and decays to zero after 40 min (see Fig. 4 in MG08). The simulated time period is from $t = 0$ to 120 min.
This case, detailed by Szumowski et al. (1998), was originally developed for warm conditions. MG08 subsequently applied the sounding to cold conditions (see their Fig. 5); here, we reduce the temperature profile of MG08 by 7 K so that the entire domain is below freezing. The initial water vapor mixing ratio is also reduced so that the initial relative humidity is the same as MG08 and Szumowski et al. (1998). The vertical and horizontal grid spacing is 50 m over a domain 9 km wide and 3 km deep. The model time step is 1 s, with substepping as required for condensation/evaporation and deposition/sublimation calculations to maintain numerical stability. Following MG08, entrainment and subgrid turbulent mixing are neglected. Droplet activation for the liquid microphysics component is calculated from water supersaturation and specified aerosol characteristics using Kohler theory (see MG07 for details). A single lognormal aerosol size distribution with a total concentration of 300 cm\(^{-3}\) is assumed here.

### b. Baseline simulation

In this section we focus on the baseline shallow precipitating cumulus simulation using the new bin microphysics schemes (referred to hereafter as BIN). The time–height plots of maximum values of cloud water, rain, and ice/snow mixing ratios are shown in Fig. 5. These plots, similar to those used in MG08, are created by combining, at a given time, the horizontal maximum of a given field at each model vertical level into a single column and subsequently displaying the time evolution of these columns. Hence, the plots trace vertical movement of the horizontal field maxima but provide no information about their horizontal location. Cloud and rainwater are distinguished using a threshold radius of 40 \(\mu\text{m}\) following MG07. The bulk rime mass fraction (total rime mass mixing ratio integrated over the mass spectrum divided by total ice mass integrated over the spectrum) at the location of the maximum ice mixing ratio is shown in Fig. 6. Time evolution of the average cloud liquid water path (LWP), ice water path (IWP), total cloud optical depth \(t_c\), total ice optical depth \(t_i\), and the surface precipitation rate (PREC) are shown in Fig. 7, together with the same quantities for some of the sensitivity simulations described later. Time-averaged values of these quantities are given in Table 2. The cloud water optical depth is calculated assuming spherical particles and the geometric optics limit. Ice optical depth is calculated following Fu (1996). For additional detail on the cloud water and ice optical depth calculations, see MG08.

As illustrated in Fig. 5, cloud water is produced as the updraft increases in strength between \(t = 0\) and 25 min, with maximum droplet mixing ratios exceeding 1.4 g kg\(^{-1}\). Ice is initiated via deposition and condensation freezing nucleation as well as freezing of cloud droplets. The ice mixing ratio is primarily grown by vapor deposition initially (owing to the low riming collection efficiencies associated with small crystals), but the rime mass fraction increases quickly once riming begins (Fig. 6). As the updraft weakens after \(t = 25\) min, a shaft of ice precipitation with high rime mass fraction \((F_r > 0.7)\) develops and falls toward the surface. A second shaft with somewhat lower rime mass fraction \((0.3 < F_r < 0.7)\) develops about 10 min later, although there is not much
distinction between the two shafts. Ice mixing ratios reach about 1.6 g kg\(^{-1}\) at about \(t = 45\) min. PREC reaches a maximum around \(t = 50\) min with a broad secondary peak at \(t = 83\) min associated with the second shaft of precipitation with lower \(F_r\). Liquid water is rapidly depleted as the ice falls through the cloud layer. After about \(t = 60\) min the rime mass fraction steady decreases because of the reduction in droplet mixing ratio and hence the decrease in the riming rate.

To further investigate the performance of the new scheme, we also examine the simulated particle size spectra. Examples of the liquid and ice size spectra as well as \(F_r\) and ice particle fall speed as a function of particle size for a few representative times and locations are shown in Figs. 8–11. Here we have chosen points corresponding to the center of the horizontal domain near the core of the updraft. These examples illustrate the particle spectra both in highly rimed conditions within the main precipitation shaft and in lightly rimed conditions near the end of the simulation.

Within the first precipitation shaft at \(t = 40\) min and a height of 2000 m (see Fig. 8) there is significant liquid, with a maximum spectral density occurring at about 20–25 \(\mu\)m, corresponding to the cloud droplet mode, and a second broader mode corresponding with drizzle drops (diameter from about 80 to several hundred micrometers). The ice spectral density has a broad peak near 70 \(\mu\)m with a nearly exponential decrease at sizes up to approximately 1000 \(\mu\)m. At larger sizes the distribution is superexponential (i.e., enhanced concentration in the tail of the distribution relative to exponential). The \(F_r\) increases sharply with ice particle size between 100 and 1000 \(\mu\)m and is nearly constant at about 0.9 for larger sizes. This sharp increase of \(F_r\) likely reflects the steep increase in efficiency of collisions between droplets with \(d < 20\) \(\mu\)m and ice with \(D\) increasing from about 100 to 700 \(\mu\)m (see Fig. 3). Fall speed increases with ice particle size and exceeds 2 m s\(^{-1}\) for \(D \geq 2\) mm. These values are in good agreement with observed graupel particle fall speeds (Mitchell 1996).

By \(t = 56\) min the main precipitation shaft has reached the surface. At a height of 50 m the ice particle size distribution is rather narrow with a peak near 2.5 mm (see Fig. 9). The total particle number concentration is smaller than at higher altitudes in the cloud and there are substantially fewer particles smaller than 1 mm, which is likely due to size sorting. The extension of the PSD to smaller sizes occurs through sublimation, which also results in \(F_r\) near unity for particles smaller than 1 mm since we have assumed that the rime mass mixing
ratio does not decrease via sublimation until complete sublimation of the depositionally grown mass [see section 2c(4)]. In other words, sublimation increases \( F_r \) when \( F_r < 1 \). However, it should be noted that \( F_r \) has little impact on particle properties for particles smaller than about 100 \( \mu m \) because the \( m-D \) and \( A-D \) relationships correspond with those of solid ice spheres regardless of \( F_r \) (see section 2b). For particles larger than about 2 mm, the fall speed is about 2 m s\(^{-1}\). At larger sizes (>4 mm), the decrease in \( F_r \) results in a smaller ratio of particle mass to projected area and hence small decrease in particle fall speed. There are no drizzle drops at this altitude because of the rapid collection of drizzle by ice at higher altitudes.

Near the end of the simulation at \( t = 112 \) min the cloud is completely glaciated and the ice spectral densities are shifted to smaller sizes than earlier in the simulation, corresponding to the reduced mass of ice. At a height of 2000 m the spectral density has a peak near 160 \( \mu m \) and is rather narrow with almost no particles larger than 1 mm (see Fig. 10). Values of \( F_r \) increase from zero to about 0.16 between \( D \) of approximately 200 and 700 \( \mu m \). These values of \( F_r \) are much smaller than earlier in the simulation because of the lack of liquid water and hence riming growth. The particle fall speeds are correspondingly lower for a given particle size relative to points earlier in the simulation because of the smaller \( F_r \) and hence lower ratio of particle mass to projected area.

At \( t = 112 \) min and a height of 50 m, the spectra are bimodal, with a broad peak near 50 \( \mu m \) and a more significant peak at 1 mm (see Fig. 11). Interestingly, values of \( F_r \) are fairly constant within each mode but differ widely between the modes, with \( F_r \) of unity for the smaller mode and \( F_r \sim 0.4 \) for the larger mode. The large values of \( F_r \) at small sizes appear to result from sublimation of larger partially rimed crystals produced earlier in the simulation as explained above. Fall speeds for the larger particle mode are relatively small for a given particle size compared to earlier in the simulation because of the smaller \( F_r \).

**c. Sensitivity tests**

In addition to the baseline bin simulation using the new approach for ice microphysics, we have run several sensitivity tests. The first test uses the bulk microphysics scheme described by MG08 (referred to as BULK) to assess differences between the bulk and bin approaches. Additional tests use the bin scheme but with a single component approach whereby the \( m-D \) and \( A-D \) relationships follow those of either unrimed snow (SNOW) or graupel (GRAUPEL); more specifically, the \( m-D \) relationship is given by (10) and (12) for SNOW and GRAUPEL, respectively. We also test the sensitivity to the specified \( m-D \) relationship of the unrimed crystals (HABH07 and HABSP). HABH07 assumes the \( m-D \) formulation derived by Heymsfield et al. (2007, hereafter H07); HABSP uses the \( m-D \) relationship for side planes following Mitchell et al. (1990).

The time–height plots of maximum values of cloud water and ice/snow mixing ratios for BULK are shown in Fig. 12 (rain mixing ratios in this simulation are \(<1 \times 10^{-3} \) g kg\(^{-1}\) and therefore not shown). A time–height plot of \( F_r \) at the location of the maximum ice/snow mixing ratio in the horizontal is shown in Fig. 13. The bulk scheme produces results that are generally similar to BIN, including the liquid water path and optical depth as well as the timing of the onset of precipitation (see Fig. 7). However, there are notable differences. The most significant differences are the larger maximum ice mixing ratios in the main precipitation shaft and the much greater peak precipitation intensity in BULK, despite the smaller domain-average IWP. This simulation also exhibits a much sharper peak in PREC when compared with BIN, with values decreasing very rapidly after \( t = 60 \) min. Previous studies have suggested that excessive size sorting in two-moment bulk schemes with exponential distributions can produce biases in surface precipitation relative to bin simulations (Wacker and Seifert 2001); furthermore, the bin results discussed in the previous section showed narrower size distributions in the main precipitation shaft. However, an additional test using
FIG. 7. Time evolution of domain-average cloud LWP (g m$^{-2}$), IWP (g m$^{-2}$), droplet optical depth $\tau_d$, ice optical depth $\tau_i$, total cloud optical depth $\tau_t$, and surface precipitation rate (PREC) (mm h$^{-1}$). BIN and BULK are the baseline bin and bulk model simulations, respectively. SNOW and GRAUPEL refer to sensitivity tests using the bin model with larger ice treated as unrimed snow or graupel, respectively.
the bulk scheme but with the same fall speed (average of the mass- and number-weighted fall speed) applied to both the number concentration and mixing ratio, thereby eliminating size sorting, still exhibits the same overall features (not shown). Furthermore, an additional test with the bin scheme but using the same riming collection efficiency as the bulk scheme (i.e., a constant of 0.7 for collisions of cloud droplets and ice and a constant of 1 for collisions of rain and ice) produced results similar to the baseline bin scheme. Hence, differences between BIN and BULK cannot be attributed to the simple treatment of riming collection efficiency in the bulk scheme. It is possible that these differences originate from the assumption of constant $F_r$ with particle size in the bulk scheme. As shown in section 3.2, values of $F_r$ predicted by the bin scheme tend to be constant with particle size only for sizes larger than about 800 $\mu$m, with smaller values of $F_r$ for smaller sizes (except in the case when sublimation dominates). Larger values of $F_r$ in the bulk relative to the bin scheme for crystals with $D < 800 \mu$m will result in a larger fall speed for a given particle size, which may explain the rapid fallout and relatively sharp peak of the surface precipitation rate. However, since the assumption of constant $F_r$ as a function of particle size was an integral part of the design of the bulk scheme (see MG08), modifying this assumption, while possible, would require a major reworking of the scheme and hence further testing of this hypothesis is beyond the scope of this study. Given that the focus of this paper is on the design and testing of the bin microphysics scheme, we note general similarity between BIN and BULK and leave more detailed testing and evaluation of the bulk scheme for future work.

Results for the SNOW and GRAUPEL sensitivity runs are shown in Fig. 7 and Table 2. LWP and IWP are generally similar to BIN in these runs. However, $\tau_i$ is much larger (smaller) in SNOW (GRAUPEL) compared with BIN. This is mostly a direct result of the larger projected area for a given particle mass using the $m-D$ and $A-D$ relationships for unrimed ice in SNOW, which results in a decrease in the effective radius and hence increase in $\tau_i$ relative to BIN. The opposite situation occurs in GRAUPEL. The relatively small ratio of particle projected area to mass in GRAUPEL also increases the particle fall speed relative to SNOW (Fig. 14).

Also shown in Fig. 14 are the fall speeds assuming $F_r$, a constant of 0.5; these values match well with fall speeds for rimed plates derived by Mitchell (1995, see his Fig. 4.4).

Despite the increase in fall speed, the decrease in projected area for a given particle mass (as well as $E_{ci}$ for relatively small ice particles) in GRAUPEL relative to BIN and SNOW results in a large decrease in riming rate (Fig. 15, left panel). Faster riming growth combined with slower fall speeds is consistent with the somewhat larger IWP in SNOW relative to BIN and GRAUPEL before $t = 90$ min. For vapor depositional growth, the situation is more complicated because there are several factors involved, including maximum particle dimension $D$ for a given mass (determined by the $m-D$ relationship), capacitance factor for a given $D$ [about 2 times as large for graupel when compared with unrimed snow; see section 2c(3)], and particle fall speed through ventilation. Mitchell et al. (2006) explored the role of the $m-D$ relationship on vapor deposition and found that the small $D$ for a given $m$ for isometric crystals (an $m-D$ relationship similar to that of to graupel) led to reduced depositional growth when compared with other ice habits, suggesting that the $m-D$ coefficients themselves are important in controlling growth. Here, these three factors tend to be offsetting so that there are not large differences in bulk vapor deposition among the BIN, SNOW, and GRAUPEL simulations (Fig. 15, right panel). For example, even though $D$ for GRAUPEL is small relative to the value for SNOW for a given $m$, the larger fall speed and hence increased ventilation as well as larger capacitance factor lead to similar depositional growth rates.

Interestingly, there is a significant delay in the onset and peak surface precipitation rate in both SNOW and GRAUPEL relative to BIN; the maximum PREC occurs 20 and 32 min later in GRAUPEL and SNOW, respectively. This result is somewhat unexpected for GRAUPEL, given the relatively large particle fall speeds in this run, and is due to the reduced riming growth rate in GRAUPEL, which compensates for the larger particle fall speeds (for a given particle mass). Thus, both the particle growth rates and the fall speeds are important in determining the onset and peak precipitation.

### Table 2: Time- and domain-averaged cloud LWP (g m$^{-2}$), IWP (g m$^{-2}$), water optical depth $\tau_w$ (unitless), ice optical depth $\tau_i$ (unitless), total cloud optical depth $\tau_{tot}$ (unitless), and surface precipitation rate PREC (mm h$^{-1}$).

<table>
<thead>
<tr>
<th>Run</th>
<th>LWP</th>
<th>IWP</th>
<th>$\tau_w$</th>
<th>$\tau_i$</th>
<th>$\tau_{tot}$</th>
<th>PREC</th>
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<tbody>
<tr>
<td>BIN</td>
<td>57.2</td>
<td>428.7</td>
<td>6.7</td>
<td>20.7</td>
<td>27.5</td>
<td>0.30</td>
</tr>
<tr>
<td>BULK</td>
<td>66.5</td>
<td>220.2</td>
<td>9.7</td>
<td>13.0</td>
<td>22.8</td>
<td>0.33</td>
</tr>
<tr>
<td>SNOW</td>
<td>53.4</td>
<td>488.8</td>
<td>6.6</td>
<td>37.3</td>
<td>43.9</td>
<td>0.33</td>
</tr>
<tr>
<td>GRAUPEL</td>
<td>70.3</td>
<td>437.5</td>
<td>8.3</td>
<td>13.3</td>
<td>21.6</td>
<td>0.28</td>
</tr>
<tr>
<td>HAB07</td>
<td>72.4</td>
<td>387.1</td>
<td>8.6</td>
<td>7.4</td>
<td>16.0</td>
<td>0.30</td>
</tr>
<tr>
<td>HABSP</td>
<td>63.6</td>
<td>430.9</td>
<td>7.5</td>
<td>17.0</td>
<td>24.5</td>
<td>0.30</td>
</tr>
</tbody>
</table>
rate. Results suggest that the physically based treatment of riming and conversion to graupel in the new approach, including representation of crystals with varying degrees of riming, is important for simulating the onset and peak surface precipitation in this case. These differences in the onset and evolution of precipitation may also have important consequences for the dynamics, which is an effect that is disabled in the kinematic framework employed here.

Finally, we test the sensitivity to the $m-D$ relationship for unrimed crystals in the same manner as MG08 (note that the $m-D$ relationship for unrimed crystals also impacts the derived $m-D$ relationship for partially rimed crystals, as described in section 2b). In the first
test (HABH07), we use the recent $m-D$ formulation derived by H07. This formulation is based on crystal ensembles observed in convectively generated ice cloud layers during the 2002 Cirrus Regional Study of Tropical Anvil and Cirrus Layers–Florida-Area Cirrus Experiment (CRYSTAL-FACE). The relationship is given by

$$m = (0.004 015 7 + 0.000 060 6T)D^{1.75}$$

(cgs units), where $T$ is the air temperature in degrees Celsius. Here we use $T = -15^\circ C$, which is near the middle range of temperatures in the domain. The second test (HABSP) uses the $m-D$ relationship for side planes following Mitchell et al. (1990), where $m = 0.004 19D^{2.3}$ (cgs units; the baseline

FIG. 9. Simulated (a) ice particle size spectrum, (b) ice particle fall speed, and (c) ice particle rime mass fraction $F_r$ using the baseline bin scheme at $t = 56$ min, a height of 50 m, and in the center of the domain in the horizontal direction.
Simulations assume plates with sectorlike branches. Side planes may be expected in water saturated conditions as occur here and at temperatures between approximately $-20^\circ$ and $-25^\circ$C (Magono and Lee 1966). In both tests, the $A-D$ relationship is the same as baseline. In general there is only a limited impact on the IWP, LWP, and $\tau_I$ (see Fig. 16 and Table 2). However, $\tau_I$ is decreased by more than a factor of 2 using the $m-D$ formulation of H07, despite similarity of the IWP. This occurs because for a given particle mass, the H07 formulation tends to produce a smaller particle dimension $D$ (higher bulk particle density) than the baseline $m-D$ formulation. Hence, the ratio of bulk ice mass to bulk projected area (proportional to effective radius) tends to be larger than in BIN, leading to a reduction of optical depth. Since particle fall speed is similarly proportional...
to particle mass divided by its projected area, the mean particle fall speed is larger in HABH07 than BIN for particles of a given size and rime mass fraction. However, the increased fall speed is compensated by the smaller projected area for particles of a given mass, leading to a reduction in the collection of liquid drops. The net effect is an increase in LWP and a delayed onset of precipitation at the surface and timing of maximum PREC in HABH07 relative to BIN. HABH07 also exhibits a greater peak intensity of PREC (exceeding 1 mm h⁻¹).

The test using the $m-D$ formulation for side planes is closer to BIN than HABH07. However, it exhibits similar features to HABH07 relative to BIN, namely somewhat increased LWP, increased ice effective radius (and hence decreased optical depth), and a delay in the
peak intensity of the domain-average precipitation. Similarly to HABH07, these differences are consistent with the fact that the $m-D$ relationship for side planes (as given by Mitchell et al. 1990) produces a somewhat smaller $D$ for a given particle mass than the baseline $m-D$ relationship.

### 4. Summary and conclusions

This paper extends the approach for representing ice microphysics as outlined in MG08 to a detailed, size-resolving multicomponent bin microphysics scheme. The new ice scheme was combined with the liquid bin microphysics scheme of MG07. In addition to prediction of the ice number concentration, this scheme predicts the rime mass mixing ratio in each mass bin. This allows for the local diagnosis of rime mass fraction over the distribution of different particle sizes/masses to provide a physical basis for the treatment of snow with varying degrees of riming as well as the conversion of snow to graupel. Existing mixed-phase bin microphysics schemes typically treat different types of ice by a priori classification of different species within the same size/mass distribution based on particle mass (e.g., Ovtchinnikov and Kogan 2000) or by using a single component approach that includes separate distributions for each species (e.g., Geresdi 1998; Khain et al. 2004). The difficulty in such schemes is determining how to parameterize the transfer of particles from one species to another (e.g., the conversion of snow to graupel via riming) and how to treat particles with varying degrees of riming. Based on the conceptual model of Heymsfield (1982) describing the formation of rimed snow and graupel from the filling in of crystal interstices with rime, the mass–dimension ($m-D$), projected area–dimension ($A-D$), and terminal velocity–dimension relationships in the new scheme are derived from the total particle mass and rime mass fraction. The key point is that this scheme allows for a smooth transition from unrimed crystals to rimed snow and graupel, without applying thresholds and arbitrary conversion rates as done in traditional approaches. This scheme can be used to investigate physical processes and interactions in cloud model simulations of ice- and mixed-phase clouds, as well as serving as a basis for testing simpler bulk microphysics schemes.

**FIG. 12.** Time evolution of horizontal maxima of cloud water mixing ratio and ice mixing ratio at each vertical level for the bulk model simulation.
The main shortcoming of the new scheme is that it assumes only a single rime mass fraction within each mass bin at a given time and location. Thus, it may be less applicable to situations such as deep, vigorous convection in which mixtures of frozen drops, hail, and rimed crystals occur in close proximity. The focus of this paper is on the riming of snow and conversion to graupel via riming; extension of this scheme to include a separate treatment of frozen drops and hail, including conversion of graupel to hail in the wet growth riming regime (e.g., Pruppacher and Klett 1997), is left for future work. One approach could be to include a separate size distribution of ice particles with characteristics of frozen drops/hail, similar to other multispecies bin schemes (e.g., Khain et al. 2004). Inclusion of frozen drops/hail as a separate category from snow/rimed snow/graupel makes more physical sense than attempting to represent the continuum of snow, rimed snow, and graupel as separate species, given the substantial differences in physical characteristics and different formation mechanisms of frozen drops/hail versus rimed snow/graupel. Another approach would be to include multiple values of rime mass fraction in 2D bins that extend over total mass and rime mass fraction spaces, which is conceptually similar to the 2D particle distributions for water drops/aerosol proposed by Bott (2000). However, this approach would greatly increase the computational cost.

The new bin scheme was applied in a 2D kinematic model with a specified time-varying flow field mimicking the evolution of a single precipitating shallow cumulus cloud. The kinematic framework simplifies testing of the microphysics by neglecting microphysics–dynamics feedbacks while still allowing sedimentation and advective transport in a realistic flow field. The new scheme produces physically consistent results both in terms of bulk cloud properties (LWP, IWP, surface precipitation rate) and of the details of the predicted particle spectra. Together with idealized tests of the individual microphysical processes in isolation, these results demonstrate the efficacy of the scheme.

Additional simulations used the traditional single component approach that assumes all crystals within the distribution consist of either unrimed snow or graupel. Different $m$–$D$ relationships for unrimed crystals were also tested. These results illustrated compensating effects of changing particle characteristics. For example, reducing the particle size for a given mass (by assuming a different $m$–$D$ relationship or assuming that larger particles are graupel) without a compensating decrease in particle projected area resulted in an increase in particle fall speed but a decrease in riming and removal of droplets. The net effect was to increase the LWP and delay onset of precipitation at the surface. Conversely, increasing the particle size for a given mass (by again assuming a different $m$–$D$ relationship or assuming that larger particles are unrimed snow) resulted in a decrease in particle fall speed but increase in riming, and also led to...
to a delay in the onset of precipitation. Thus, assuming that larger particles consisted entirely of either unrimed snow or graupel led to a delay in the onset and timing of maximum surface precipitation compared to the simulation using the new scheme. These results suggest the importance of treating different ice types including partially rimed crystals (at least for this case), which is done in our approach in a self-consistent way (without applying thresholds or arbitrary conversion rates) based on the predicted rime mass fraction. We also note the potential sensitivity to other parameters in the scheme that were not explored here for brevity, such as aggregation efficiencies for ice–ice collisions and riming collection efficiencies. These parameters are not very well constrained by field or laboratory observations (e.g., Pruppacher and Klett 1997) and their impact on results should be tested in future work.

Results using the bin scheme were also compared with a simulation using the bulk microphysics scheme of MG08. Overall, the bulk and bin schemes produced fairly similar results in terms of the liquid and ice water paths, optical depths, and precipitation. Notably, timing of the onset and maximum rate of surface precipitation were similar, although the peak surface precipitation rate using the bulk scheme was about 4 times larger and showed a more rapid decrease toward the end of the simulation. Differences between the bulk and bin scheme may have resulted from the assumption of constant rime mass fraction with particle size in the bulk scheme. In the simulations with the bin scheme, rime mass fraction was generally only constant for particle sizes larger than about 700–1000 µm. However, since the focus of this paper is on the bin scheme, additional detailed testing of the bulk scheme is left for future work.

The multicomponent bin framework developed here could in principle be extended to any conserved spectral quantity. For example, prediction of the mass mixing ratio of aerosols embedded in the hydrometeors as a function of cloud/precipitation particle mass would allow for a more rigorous treatment of cloud–aerosol interaction without greatly increasing the computational cost and complexity. Combined with the new multicomponent ice scheme and a detailed representation of the liquid hydrometeors, such a scheme could provide a framework for simulating cloud–aerosol interactions over a wide range of cloud types containing liquid and ice, including deep convection. Multicomponent approaches that treat various aerosol species using 1D size/mass distributions that track average mass of the species contained in drops of a particular size have been previously used in several liquid bin microphysics schemes (e.g., Flossmann 1993; Ackerman et al. 1995; Feingold et al. 1996; Feingold and Kreidenweis 2002). More detailed approaches using two-dimensional particle distributions that allow for a distribution of aerosol masses within each drop mass/size bin have been proposed (Bott 2000), but they have significantly increased computational cost because of the large number of prognostic variables required.

Future work will focus on implementation of the new scheme in a fully 3D dynamical model allowing feedbacks between the microphysics and dynamics and on application of such a model to real-world cases to compare with observations. We will also continue to test sensitivity to key microphysical parameters in the scheme, as well as extending it to include treatment of the liquid water fraction on ice particles. This will allow detailed calculations of melting as well as hail growth in the “wet” regime.

![Figure 15](image-url)
FIG. 16. As in Fig. 7, but for the baseline bin model simulation (BIN) and sensitivity tests using the bin model with $m-D$ relationship from H07 (HABH07) and with $m-D$ relationship for side planes from Mitchell (1996) (HABSP).
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