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Key Points:
- Global economic wealth is tied to rates of primary energy consumption
- Rates of economic growth depend on past growth, resource availability, and decay
- Human and climate systems can be coupled using essentially the same physics

Supporting Information:
- SupportingInformation_TableS1.pdf
- SupportingInformation_TableS2.pdf
- SupportingInformation_TableS3.txt
- SupportingInformation_text.pdf
- Readme.txt

Corresponding author:
T. J. Garrett, tim.garrett@utah.edu

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Long-run evolution of the global economy: 1. Physical basis

Timothy J. Garrett[1]

[1]Department of Atmospheric Sciences, University of Utah, Salt Lake City, Utah, USA

Abstract Climate change is a two-way street during the Anthropocene: civilization depends upon a favorable climate at the same time that it modifies it. Yet studies that forecast economic growth employ fundamentally different equations and assumptions than those used to model Earth's physical, chemical, and biological processes. In the interest of establishing a common theoretical framework, this article treats humanity like any other physical process; that is, as an open, nonequilibrium thermodynamic system that sustains existing circulations and furthers its material growth through the consumption and dissipation of energy. The link of physical to economic quantities comes from a prior result that establishes a fixed relationship between rates of global energy consumption and a historical accumulation of global economic wealth. What follows are nonequilibrium prognostic expressions for how wealth, energy consumption, and the Gross World Product (GWP) grow with time. This paper shows that the key components that determine whether civilization "innovates" itself toward faster economic growth include energy reserve discovery, improvements to human and infrastructure longevity, and reductions in the amount of energy required to extract raw materials. Growth slows due to a combination of prior growth, energy reserve depletion, and a "fraying" of civilization networks due to natural disasters. Theoretical and numerical arguments suggest that when growth rates approach zero, civilization becomes fragile to such externalities as natural disasters, and the risk is for an accelerating collapse.

1. Introduction

As with any other natural system, civilization is composed of matter. Internal circulations are maintained by a dissipation of potential energy. Oil, coal, and other fuels "heat" civilization to raise the potential of its internal components. Dissipative frictional, resistive, radiative, and viscous forces return the potential of civilization to its initial state, ready for the next cycle of energy consumption.

Burning coal at a power station raises an electrical potential or voltage allowing for a down-voltage electrical flow. The potential energy is dissipated along the journey from the power station to the appliance. The appliance sustains people, who themselves dissipate heat. And, because what the appliance does is useful in their minds, the cycle is completed with the human desire for more coal to burn. Similarly, energy is dissipated as cars burn gasoline to propel vehicles to and from desirable destinations. Or, people consume food to maintain the circulations of their internal cardiovascular, respiratory, and nervous systems while dissipating heat and renewing their hunger.

Such cycles are fairly rapid; at least the longest might be the annual periodicities that are tied to agriculture. This paper provides a framework for the slower evolution of civilization, over timescales where rapid cyclical behavior tends to average out, where the material growth and decay of civilization networks is driven by a long-run imbalance between energy consumption and dissipation.

As sketched in Figure 1, the approach is to develop a general framework for describing the current state of systems and their spontaneous emergence, starting from physical first principles and using a simple theoretical framework outlined previously in *Garrett* [2012c]. From this point, the paper exploits a fixed link between rates of global primary energy consumption (or power production) and a general measure of global wealth that was described in *Garrett* [2011] (see also supporting information). This leads to prognostic formulae for economic innovation and growth that are expressible in units of currency. The equations are presented in a form that can be evaluated against available economic statistics for past behavior. Potentially they may be used to provide physically constrained scenarios for the future, linking human and natural systems where the two are increasingly becoming coupled.

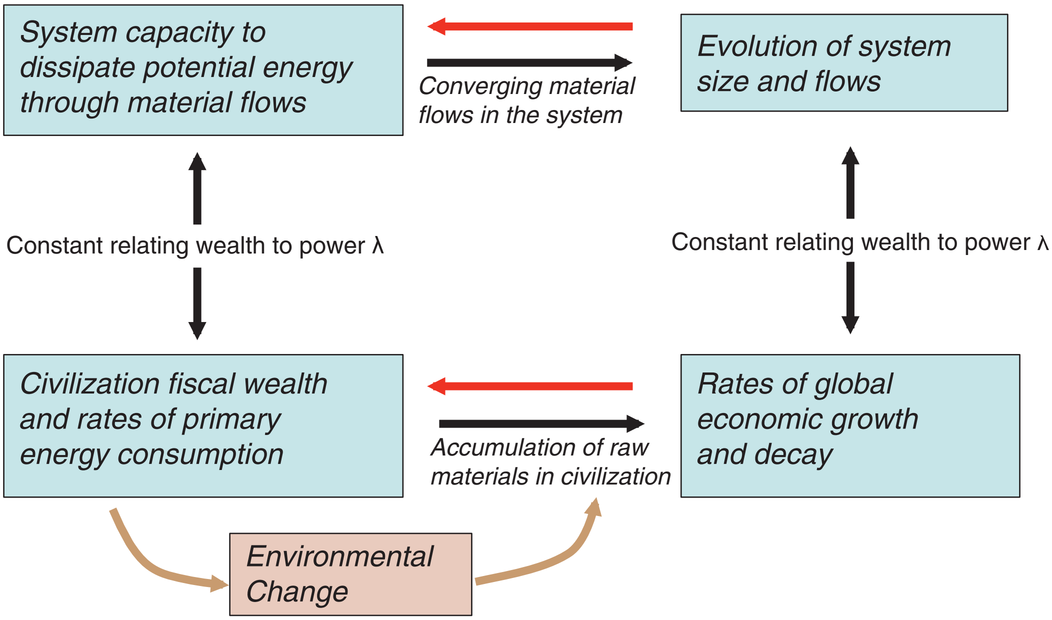

Figure 1. Diagram of the approach taken in this paper where physical first principles are used to derive analytical expressions for the long-run evolution of the global economy during the Anthropocene. Black arrows indicate a differential process. Red arrows indicate an additive or integral process.

There have been many prior efforts to link economic models to climate models [e.g., *Yohe et al*., 2004; *Stern*, 2007; *Tol*, 2009; *Nordhaus*, 2010. This paper differs by describing the human system in terms of the same thermodynamic laws that underpin parameterizations of gradients and flows in model representations of the earth's physical processes [e.g., *Bitz et al*., 2012]. Of course, many might argue that we should not subject human systems to physical laws due to the complexities of human behavior [*Scher and Koomey*, 2011]. Others might note that even physical systems at their most simple can easily become so sensitive to initial conditions as to become inherently unpredictable. But while we would not dream of predicting local weather beyond a week or so [*Lorenz*, 1963], forecasting the global mean surface temperature a century out is an accepted challenge. The primary requirement for maintaining predictability is that we degrade temporal and spatial resolution. In this case, little or nothing might be said about the short-term, finer-scale details of the system; yet, broader, constrained forecasts can be made for more slowly evolving behaviors [*Bretherton et al*., 2010; *Temam and Wirosoetisno*, 2011].

From the standpoint of forecasting the human role in climate change, a broad brush may be all that is necessary given that carbon dioxide is both well-mixed and long-lived in the atmosphere. What the article presents is long-range prognostic equations for global economic quantities by stepping back and viewing civilization as a whole, as it evolves slowly over "long" timescales and subject to such global externalities as resource availability and increasing natural disasters from a changing global climate.

The hope is to help resolve an apparent disconnect between how we forecast Earth's future during the Anthropocene, by moving away from traditional macroeconomic models and more toward treating civilization as a dissipative physical system like any other on our planet. Section 2 of this paper describes an underlying thermodynamic framework for emergent systems. Section 3 connects this framework to basic economic quantities. Section 4 discusses prognostic solutions for economic innovation and growth. Section 5 identifies formulations for distinct modes of growth in economic systems, and Section 6 summarizes the conclusions of this study.

2. Energetic and Material Flows to Systems

Before proceeding to a description of the growth of civilization, the starting point is to define what is meant by "short" and "long" timescales, and then to build from first principles a general thermodynamics for the emergence and evolution of dynamic systems over long timescales.

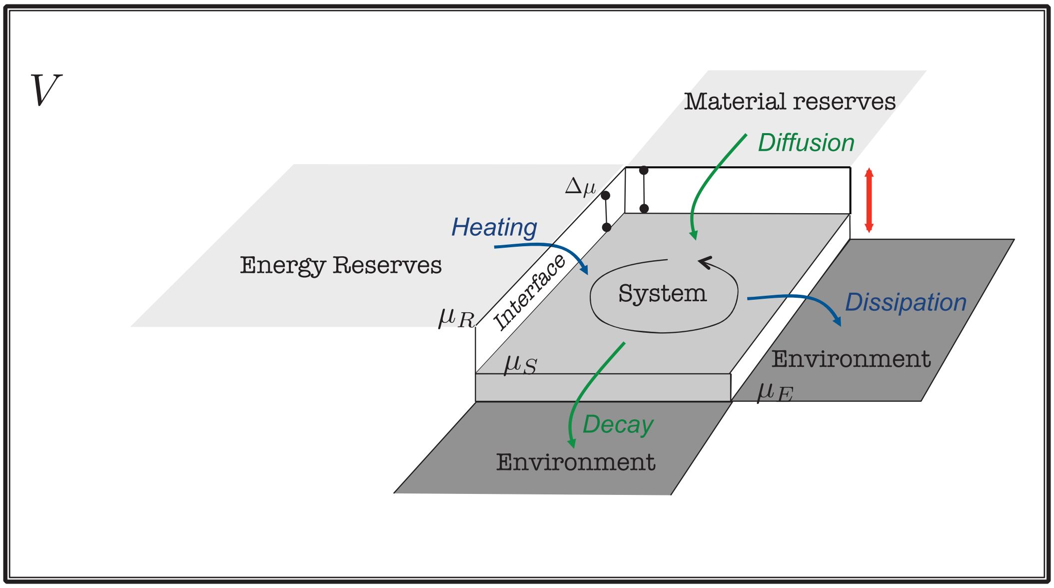

Figure 2. Schematic for the thermodynamics of an open system within a fixed volume V. Energy reserves, the system, and the environment lie along distinct constant potential surfaces μ_R, μ_S, and μ_E. Internal material circulations within the system are sustained by heating and dissipation of energy that is coupled to a material flow of diffusion and decay. The level μ_S is a time-averaged potential. Over shorter time-scales, the legs of a heat-engine cycle would show the system rising up and down between μ_E and μ_R in response to heating and dissipation, as shown by the red arrow, allowing for material diffusion to the system and decay from the system. If flows are in balance then the system is at equilibrium and it does not grow.

In the most abstract sense, the universe is a continuum of matter and potential energy in space. Local gradients drive thermodynamic flows that redistribute matter and energy over time. In the sciences, we invoke the existence of some "system" or "particle" from within this continuum, requiring as a first step that we define some discrete contrast between the system and its surroundings as shown in Figure 2. This discrete contrast can be approximated by an interfacial jump in potential energy $\Delta\mu$ between the system potential μ_S and some higher level μ_R; or, $\Delta\mu = \mu_S - \mu_E$ with respect to a lower level μ_E. Matter that lies along the higher potential μ_R has a higher temperature and/or pressure, so it can be viewed as a "reserve" for downhill flows that "pour" into the system potential level μ_S. Flows also "drain" from μ_S to the lower potential environment lying along the potential surface μ_E.

Viewed from a strictly thermodynamic perspective, any system that is defined by a constant potential must implicitly lie along a surface within which there is no *resolved* internal contrast, i.e., one where the assumption is that there is a fixed potential energy per unit matter μ_S and no internal gradients. This specific potential represents the time-integrated quantity of work that has been required to displace each unit of matter within the surface through an arbitrary set of force-fields that point in the opposite direction of the potential vector μ: e.g., the gravitational potential per block in a pyramid is determined by the product of the downward gravitational force on each block and its height.

Although internal gradients and circulations are not resolved within a constant potential surface, the existence of the continuum requires that they exist nonetheless. When a bathtub is filled, internal gradients force the water to slosh from side to side. While the short timescale of these small waves might be of interest to a child, a typical adult cares only about the time-averaged water level of the bathtub as a whole, and that it gradually rises as the water pours in. The definition of what counts as a "system" is only a matter of perspective. It depends on what timescale is of most interest to the observer looking at the system's variability. As a guiding principle, however, coarse spatial resolution corresponds with coarse time resolution [e.g., *Blois et al*., 2013].

The total energy of a system, or its enthalpy H_S, can be expressed as a product of the amount of matter in the system N_S and the specific enthalpy given by

$$e_S^{\mathrm{tot}} = \left(\frac{\partial H_S}{\partial N_S}\right)_{\mu_S} \quad (1)$$

The specific enthalpy can be decomposed into the product of the total number of independent degrees of freedom ν in the system and the oscillatory energy per independent degree of freedom e_S

$$e_S^{\mathrm{tot}} = \nu e_S \tag{2}$$

The quantity e_S represents the circulatory energy per degree of freedom per unit matter. For example, nitrogen gas has a specific enthalpy that is the product of the specific heat at constant pressure c_p and the system temperature T_S, or $e_S^{\mathrm{tot}} = c_p T_S$. The specific enthalpy can be decomposed into a total $\nu = 7$ degrees of freedom at atmospheric temperatures and pressures each with a time-averaged kinetic energy of $kT_S/2$ where k is the Boltzmann constant. Thus,

$$H_S\left(\mu_S\right) = N_S e_S^{\mathrm{tot}} = \nu N_S e_S \tag{3}$$

Conservation of energy considerations dictate that enthalpy is the energetic quantity that rises when there is net heating of the system at a constant pressure [*Zemanksy and Dittman*, 1997], i.e.

$$\left(\frac{\partial H_S}{\partial t}\right)_p = \left(\frac{\partial Q^{\mathrm{net}}}{\partial t}\right)_p \tag{4}$$

and that net heating of the system is a balance between a supply of energy to the system at rate a and a dissipation at rate d

$$\left(\frac{\partial Q^{\mathrm{net}}}{\partial t}\right)_p = a - d \tag{5}$$

The Second Law requires that dissipation redistributes enthalpy to some lower potential, draining some higher potential reserve. Not all enthalpy in the reserve H_R is necessarily *available* to the system. For example, unless the temperature of the system is raised to extremely high levels, the nuclear enthalpy of a reserve $H_R = mc^2$ might normally be inaccessible. Thus, available enthalpy is distinguished here by the symbol ΔH_R.

Heating is coupled to material flows through an idealized four step cycle or "heat engine", whose circulation is shown by the red arrow in Figure 2. A system that is initially in equilibrium with the environment at level μ_E is heated, which raises the potential level of the system μ_S an amount $2\Delta\mu$ to level μ_R with a timescale of $\tau_{\mathrm{heat}} \sim 2\Delta\mu/a$. It is at this point that, according to the Gibbs-Duhem equation, the surface μ_S comes into diffusive equilibrium with respect to external sources of raw materials, allowing for a material flow to the system [*Kittel and Kroemer*, 1980]. There is then cooling through dissipation of heat to the environment with timescale $\tau_{\mathrm{diss}} \sim 2\Delta\mu/d$, which brings the system back into diffusive equilibrium with surface μ_E, allowing for material decay.

How the thermodynamics should be treated depends on the question at hand, and whether the timescale of interest is short or long compared to τ_{heat}.

2.1. Systems in Material Equilibrium Over Short Timescales

Over timescales much shorter than τ_{heat}, the legs of the heat engine are resolved, so that the amount of matter in a system N_S would appear to change sufficiently slowly that it could be considered to be fixed. In this case, the response to net heating would be that the specific enthalpy per unit matter rises at rate

$$\left(\frac{\partial e_S^{\mathrm{tot}}}{\partial t}\right)_{p,N_S} = \frac{1}{N_S}\left(\frac{\partial Q^{\mathrm{net}}}{\partial t}\right)_{p,N_S} \tag{6}$$

For the example that heating is a response to radiative flux convergence, then it may be that the temperature rises according to

$$c_p\left(\frac{\partial T}{\partial t}\right)_{p,N_S} = \frac{1}{N_S}\left(\frac{\partial Q^{\mathrm{net}}}{\partial t}\right)_{p,N_S} \tag{7}$$

where c_p is the specific heat of the substance at constant pressure and $\partial Q^{\mathrm{net}}/\partial t$ is the radiative heating. In a materially closed system, the response to net heating is for the temperature to rise.

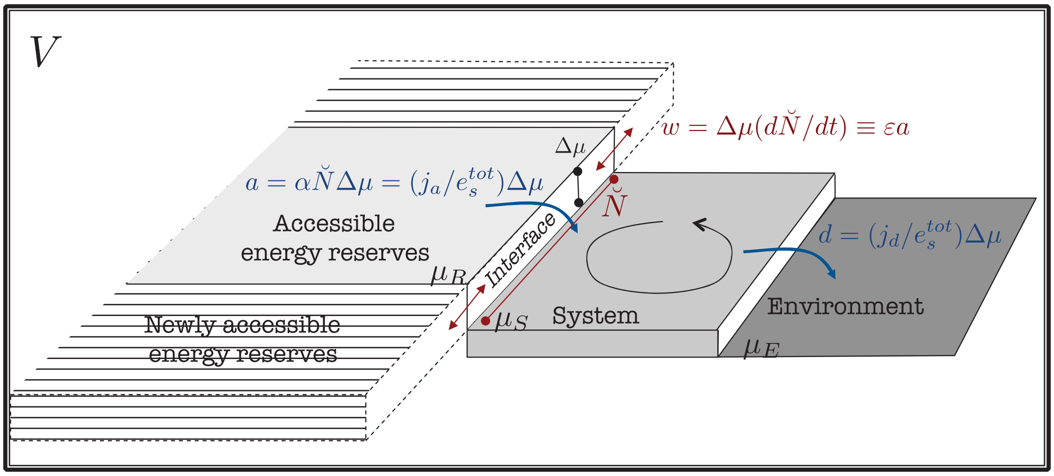

Figure 3. Schematic for the thermodynamic evolution of a system within a constant volume V. Energy reserves, the system, and the environment lie along distinct constant potential surfaces μ_R, μ_S, and μ_E. The size of an interface $\breve{N}\Delta\mu$ between surfaces determines the rate of heating a and the speed of downhill material flow j_a. The system grows or shrinks according to a net material flux convergence $j_a - j_d$ along μ_S. System growth is related to expansion work w that is done to grow the interface, extending the system's access to previously inaccessible energy reserves. The efficiency of work is determined by $\epsilon = w/a$.

In the atmospheric sciences, equation (7) expresses the short-term temperature response to radiative heating [*Liou*, 2002]. At timescales longer than τ_{heat}, however, the establishment of a temperature gradient ultimately leads to a diffusive, material flow that restores equilibrium and that we call the wind.

2.2. Systems in Material Disequilibrium Over Long Timescales

Over timescales much longer than τ_{heat}, the legs of the heat engine are not resolved. Instead, because the heat engine cycles are much faster than the timescales that are of interest to the observer, what is seen is only some average level of μ_S that lies in between the points of maximum and minimum potential energy, μ_R and μ_E (Figure 2).

In this case, energetic and material flows have the appearance of being instantaneously coupled. An illustration of this coupling is shown in Figure 3, which recasts Figure 2 in terms of a single co-ordinate. Where there is a disequilibrium, material convergence along a surface of constant potential μ_S corresponds with growth of the system enthalpy at rate

$$\left(\frac{\partial H_S}{\partial t}\right)_{\mu_S} = \left(\frac{\partial Q^{\text{net}}}{\partial t}\right)_{\mu_S} = e_S^{\text{tot}}\left(\frac{\partial N_S}{\partial t}\right)_{\mu_S} \tag{8}$$

so that from equation (5), the bulk grows at rate

$$\begin{aligned}\left(\frac{\partial N_S}{\partial t}\right)_{\mu_S} &= \frac{\left(\partial Q^{\text{net}}/\partial t\right)_{\mu_S}}{e_S^{\text{tot}}} \\ &= \frac{a-d}{e_S^{\text{tot}}}\end{aligned} \tag{9}$$

If there is zero time-averaged net heating, then $\left\langle \left(\partial Q^{\text{net}}/\partial t\right)_{\mu_S} \right\rangle = 0$ because $\langle a \rangle = \langle d \rangle$, in which case the size of the system N_S does not change. Like water pouring into and draining from a bathtub at equal rates, circulations within the system maintain a steady-state. Although local entropy production $(\partial Q^{\text{net}}/\partial t)_\mu/\mu$ is zero, global entropy $\sum_\mu (\partial Q^{\text{net}}/\partial t)_\mu/\mu$ grows from a continuous redistribution of matter through a flow from high to low values of μ.

Material growth occurs when there is the nonequilibrium condition that energy consumption exceeds dissipation, in which case $\left\langle \left(\partial Q^{\text{net}}/\partial t\right)_{\mu_S} \right\rangle > 0$. There is a net convergence of matter along the potential surface μ_S at rate j^{net}. Material flows into civilization at rate j_a, and out of civilization at the decay rate j_d, to form a balance defined by

$$j^{\text{net}} = \left(\frac{\partial N_S}{\partial t}\right)_{\mu_S} = j_a - j_d \tag{10}$$

so that the timescale for growth of the system is $\tau_{\text{growth}} \sim N_S/j^{\text{net}}$. Combined with equation (9), this implies that

$$j_a = a/e_S^{\text{tot}} \tag{11}$$

$$j_d = d/e_S^{\text{tot}} \tag{12}$$

$$j^{\text{net}} = \frac{a - d}{e_S^{\text{tot}}} \tag{13}$$

A straightforward and familiar example of this physics is what happens when we boil a pot of water. Once the water reaches the boiling point, the temperature of the water is maintained at a constant 100°C. Any energy input from the stove goes into turning liquid water into bubbles. Setting aside the energetics of forming the bubble surface, and assuming the pot is well insulated, the energy input that is required to vaporize a single liquid water molecule is $e_S^{\text{tot}} = l_v$ where l_v is the latent heat of evaporation at boiling. Thus, vapor molecules contained in the bubbles are created at a rate that is proportional to the rate of energetic input: $j_a = a/e_S^{\text{tot}} = a/l_v$.

Heating creates an internal circulation of bubbles that we call a boil. When bubbles rise to the surface, molecules escape the fluid at rate j_d, and there is an associated evaporative cooling of the water at rate $d = j_d e_S^{\text{tot}} = j_d l_v$. With a steady simmer, a constant vapor concentration N_S is maintained within the pot because heating equals cooling. In this case, from equation (13), $j_a \simeq j_d$ and $j^{\text{net}} = 0$.

If the output from the heating element is suddenly raised to high, then there is a nonequilibrium adjustment period of $\tau_{\text{growth}} \sim N_S/(j_a - j_d)$ during which heating temporarily exceeds dissipation and bubble production at the bottom of the pot j_a exceeds bubble popping at its top j_d. The size and number of vapor bubbles in the water increases, and a new stasis is attained only when evaporative cooling d rises to come into equilibrium with the element heating a. At this point, the pot has gone from a simmer to a rolling boil.

2.3. Gradients and Flows in Material Disequilibrium

As shown in Figure 3, a material flow at rate j can be regarded as the diffusion of matter downhill, across a material interface toward the system. The interface between the system and its higher potential reservoirs can be defined by a potential step with a rise $\Delta\mu = \mu_R - \mu_S$ and an orthogonal quantity of material that lies along the interface $\breve{N}$. The total energy required to grow the interface is the product of these two quantities: i.e., $\Delta G = \breve{N}\Delta\mu$. Because the presence of the gradient is required to enable flows, there is a proportional dissipation of available potential energy ΔH_R at rate

$$a = \alpha\Delta G = \alpha\breve{N}\Delta\mu \tag{14}$$

where α is a rate coefficient with units of inverse time. The quantity $\Delta G = \breve{N}\Delta\mu$ in equation (14) is a related but different quantity from the available enthalpy $\Delta H_R = N_R\Delta\mu$. The available enthalpy is a reserve of energy that is eventually available to be consumed. In contrast, ΔG represents the gradient that is instantly available to drive flows due to a material interface between the system and ΔH.

From equations (10) and (11), when a system is considered over long timescales, then energy consumption is coupled to a material flux $j_a = \left(\partial N_S/\partial t\right)_{\mu_S}$. Thus, from equation (14)

$$j_a = \alpha\breve{N}\Delta\mu/e_S^{\text{tot}} \tag{15}$$

The magnitude of the interface $\breve{N}$ reflects the respective sizes of the two components it separates. In general, when there is a diffusive flow to a system, $\breve{N}$ is proportional to a product of the available enthalpy within a high potential energy "reservoir" $\Delta H_R = N_R\Delta\mu$ and the size of the system N_S taken to a one third power [*Garrett*, 2012c], or that

$$\breve{N} = kN_S^{1/3}N_R \tag{16}$$

where the dimensionless coefficient k is related to the object shape. For a sphere, $k = (48\pi^2)^{1/3}$ [*Garrett*, 2012c].

At first glance, one might guess that the system interface should be proportional to $N_S N_R$ instead of $N_S^{1/3} N_R$: both the size of the system and the size of the reserve are what drive flows between the two. In general, a system's size is proportional to its volume $V_S = N_S/n_S$, where N_S is the number of elements in the system and n_S is the internal density; V_S and N_S are proportional to a dimension of length cubed, or volume. What is important to recognize is that flows to a system are not determined by a volume. Rather, flows are down a linear gradient that lies normal to a surface. The surface area has dimensions of length squared or $N_S^{2/3}$, and the linear gradient has dimensions of inverse length or $N_S^{-1/3}$. Both factors control the flow rate across the interface, and their product yields a one third power, or a length dimension: $N_S^{2/3} \times N_S^{-1/3} = N_S^{1/3}$.

The significance is that, if it were assumed that $\breve{N}$ is proportional to the product $N_S N_R$, then the implication would be that wholes interact with wholes, implying a perfect mixture of the system and its reserve. The objection to this formulation is that any supposed existence of a perfect mixture would mean that it would be impossible to resolve flows between N_S and N_R: the two components of the mixture would be indistinguishable. A second consequence is that assuming a unity exponent for N_S removes any element of persistence or memory from rates of system growth, as will be shown below. Unphysically it would isolate what happens in the present from and what has happened in the past. As a general principle, the Second Law allows for neither perfection nor isolation in our universe.

Since $\Delta H_R = N_R \Delta\mu$, equations (14) and (15) for energy dissipation and material flows can now be expressed as

$$j = \alpha k N_S^{1/3} \Delta H_R / e_S^{\text{tot}} \tag{17}$$

$$a = \alpha k N_S^{1/3} \Delta H_R \tag{18}$$

In *Garrett* [2012c] it was shown that the quantity $\alpha k N_S^{1/3}$ can be expressed in an equivalent fashion in terms of a length density times a diffusivity $\Lambda\mathcal{D}$, where the length density is analogous to the electrostatic capacitance within a volume and the diffusivity has dimensions of area per time. For the diffusional growth of a spherical cloud droplet of radius r, vapor condenses at rate $j = 4\pi r \mathcal{D} N_R/V$, where N_R/V is equivalent to the excess vapor density relative to saturation. In this case $\alpha k N_S^{1/3} \mathcal{D} = \Lambda\mathcal{D} = 4\pi r \mathcal{D}/V$. For more dendritic structures like snowflakes, there is no clearly definable "radius", yet it is still a length dimension within a volume Λ or "capacitance density" that drives diffusive growth [*Pruppacher and Klett*, 1997].

Thus, the flow and dissipation equations can be alternatively expressed as

$$j = \mathcal{D} \Lambda \Delta H_R / e_S^{\text{tot}} \tag{19}$$

$$a = \mathcal{D} \Lambda \Delta H_R \tag{20}$$

The rate of material flows is proportional to a rate of energy dissipation a, which in turn is proportional to some measure of the length density within the system Λ or its accumulated size N_S to a one third power, and the number of potential energy units in the reserve $N_R = \Delta H_R/\Delta\mu$. The final component is e_S^{tot}, which expresses the amount of energy that must be dissipated to enable each unit of material flow toward the system.

2.4. Efficiency and Growth

As described above, a system grows if there is an imbalance so that net heating drives an accumulation of matter in the system through diffusive material flows (equations (10) and (13)). Growth of the size of the system N_S increases an interface with the energy reserves $\Delta G = \breve{N}\Delta\mu$ that enable diffusive flows.

Taking the approach that the resolved "rise" of the interface $\Delta\mu$ is fixed, then future flows evolve because the magnitude of the "step" $\breve{N}\Delta\mu$ grows laterally in response to a convergence (or divergence) of current

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flows (Figure 3). Here, this material expansion or "stretching" of the interface $\check{N}$ and the potential difference ΔG is termed "work" w, where

$$w = \left(\frac{\partial \Delta G}{\partial t}\right)_{\mu_R,\mu_S} = \left(\frac{\partial \check{N}}{\partial t}\right)_{\mu_R,\mu_S} \Delta\mu \tag{21}$$

The efficiency of converting heating to a rate of doing work is normally defined by the ratio

$$\epsilon = \frac{w}{a} \tag{22}$$

Here efficiency can be either positive or negative depending on whether the interface is shrinking or growing in response to heating, and therefore on the sign of w (equation (21)).

From equation (21), the relative growth rate of the interface can be defined by

$$\eta = \frac{w}{\Delta G} = \frac{\mathrm{d}\ \ln \Delta G}{\mathrm{d}t} = \frac{\mathrm{d} \ln \check{N}}{\mathrm{d}t} \tag{23}$$

where η has units of inverse time. In other words, $1/\eta$ is the characteristic time for exponential growth of ΔG and $\check{N}$.

Since from equations (21) and (22), $w = \mathrm{d}\Delta G/\mathrm{d}t = \epsilon a$ and from equation (14), $a = \alpha \Delta G$, it follows that the relationship between the growth rate η and the efficiency ϵ and the heating rate a is given by

$$\eta = \alpha\epsilon \tag{24}$$

$$= \frac{\mathrm{d} \ln a}{\mathrm{d}t} \tag{25}$$

This formulation has the advantage of expressing η in terms of a measurable flux a. The implication is that systems that are efficient are able to incorporate matter more quickly; such efficient incorporation causes the system to accelerate growth of an interface with respect to energy reserves. Ultimately, higher efficiency allows the system to consume energy more rather than less.

For the special case of pure exponential growth where η is a constant, then $a = a_0 \exp(\eta t)$, but, more generally, nothing is ever fixed in time: η constantly changes as the interface evolves, and it can even change sign if it shrinks. The growth rate η is positive if the efficiency ϵ is greater than zero meaning that the system is able to do net work on its surroundings in response to heating (i.e., $\mathrm{d} \ln \check{N}/\mathrm{d}t > 0$). Otherwise, the growth rate is negative and the system collapses (i.e., $\epsilon < 0$ and $\mathrm{d} \ln \check{N}/\mathrm{d}t < 0$).

2.5. Emergence, Diminishing Returns, and Decay

In the case of the pot of boiling water that was discussed previously, there was an external agency that had its hand on the energetic flow. "Emergent systems" differ from this situation by displaying a spontaneous development of structure. One way to illustrate how emergence works is shown in Figure 3. Here, heating and dissipation sustain internal circulations. If heating exceeds dissipation then, over longer timescales, a net incorporation of matter into the system allows it to expand into newly accessible energy reserves. The thermodynamic recipe for emergence is only that sufficient energy reserves exist to be "discovered" in order to sustain the disequilibrium between heating and dissipation that drives growth.

Emergent phenomena are ubiquitous in nature. After all, something must emerge at some point for us to observe it. Where emergence is commonly discussed is with regards to living organisms, since they survive by eating, drinking, and inhaling a matrix of matter and potential energy, which is then diffused through a linear of network of vascular structures. Consumption of the potential energy in carbohydrates, proteins, and fats sustains the organism and facilitates an incorporation of water, chemicals, vitamins, and minerals. Meanwhile, heat is dissipated, and matter is lost, through a combination of radiation, perspiration, exhalation, and excretion.

The flow of raw materials and the dissipation of potential energy are coupled within cardiovascular, respiratory, gastro-intestinal, and nervous networks. Over short timescales, dissipation is tied to the internal circulations that allow for further consumption. In the long-run though, where consumption is in excess of dissipation, flows are out of equilibrium, and the organism networks grow. The demand for energy by the organism goes toward sustaining internal circulations within previously grown networks and toward furthering greater network growth.

For a given availability of energy supplies $\Delta H = N_R \Delta\mu$, then from equations (16) and (23) the instantaneous growth rate is related to the system size N_S or its network length density Λ through

$$\eta = \left(\frac{\partial \ln \breve{N}}{\partial t}\right)_{N_R} \tag{26}$$

$$= \left(\frac{\partial \ln N_S^{1/3}}{\partial t}\right)_{N_R} \tag{27}$$

$$= \left(\frac{\partial \ln \Lambda}{\partial t}\right)_{N_R} \tag{28}$$

If the rate of emergent growth η is positive then a positive feedback loop dominates and this length dimension grows exponentially (i.e., $\Lambda = \Lambda_0 \exp(\eta t)$). Negative values of η correspond with decay.

From equations (10) and (27), the rate of emergent growth can be related to rates of material consumption j_a and decay j_d through

$$\eta = \frac{1}{3N_S}\left(\frac{\partial N_S}{\partial t}\right)_{N_R} \tag{29}$$

$$= \frac{1}{3}\frac{j_a - j_d}{\int_0^t \left(j_a - j_d\right) dt'} \tag{30}$$

$$= \frac{1}{3}\frac{j^{\text{net}}}{\int_0^t j^{\text{net}} dt'} \tag{31}$$

Note that the timescale for growth of the system discussed earlier τ_{growth} is related to the growth rate of flows through $\eta = 3/\tau_{\text{growth}}$.

A "decay parameter" δ can be defined as the rate of material decay relative to the rate of material consumption

$$\delta = \frac{j_d}{j_a} \tag{32}$$

and, since the current system size is the time integral of past net material flows, $N_S = \int_0^t j^{\text{net}} dt'$, it follows that the rate of emergent growth is given by

$$\eta = \frac{1}{3}(1-\delta)\frac{j_a}{N_S} \tag{33}$$

$$= \frac{1}{3}\frac{(1-\delta) j_a}{\int_0^t (1-\delta) j_a dt'} \tag{34}$$

The final step is to account for the motive force for current flows to the system, which is obtained by substituting equation (17) into equation (33) to yield

$$\eta = \alpha k (1-\delta)\frac{N_S^{1/3} N_R \Delta\mu}{N_S e_S^{\text{tot}}} \tag{35}$$

$$= \alpha k (1-\delta)\frac{\Delta H_R}{N_S^{2/3} e_S^{\text{tot}}} \tag{36}$$

Equation (35) for emergent growth has seven parameters. Three—α, k, and $\Delta\mu$—have been considered to be constants in this treatment (i.e., η represents a partial derivative). With this condition, current growth rates η are determined by the quantity of energy $\Delta H_R = N_R \Delta\mu$ that is available to drive material flows to the system; the amount of energy e_S^{tot} that must be dissipated to incorporate each unit of matter into the system; the fraction $1-\delta$ of this new matter whose addition is not offset by decay; and, crucially, past flows leading to the current system size N_S: as past flows grow a system, there is a natural propensity for the growth rate to slow with time.

The memory of past growth introduces a "law of diminishing returns". As was mentioned above, had it been assumed that flows were proportional to $N_S N_R \Delta\mu$ rather than $N_S^{1/3} N_R \Delta\mu$ in equation (17), then this dependence of current growth rates on past flows $\int_0^t j^{net} dt'$ would not be present—the N_S terms would have canceled in equation (35). Clearly, this would be inconsistent with our observations of emergent systems. Expressed logarithmically, large objects tend to grow more slowly than small objects. And, the growth of all emergent systems is somehow tied to the past through existing matter that has been accumulated from prior growth. "Great oaks from little acorns grow".

3. Thermodynamics of the Growth of Wealth

The above discussion is intended to be quite general for system evolution. Here civilization can be considered as a special case. Taken as a whole, civilization might be viewed as an example of a living emergent system that consumes a matrix of matter and energy. For civilization, "food" includes raw materials such as water, wood, cement, copper, and steel. The potential energy is contained in fossil fuels, nuclear fuels, and renewables. The linear networks are our roads, shipping lanes, communication links, and interpersonal relationships.

Over short timescales, civilization can be characterized by the internal circulations that govern our daily lives, including our bodily functions, commuting to work, and communications. But, as shown by comparing Figures 3 and 4, if civilization is examined with an eye to more slowly evolving behaviors, where the internal circulations are not explicitly resolved, then energy consumption at a rate a enables civilization to raise raw materials across a potential energy barrier. This then enables an incorporation through diffusion of matter into civilization's bulk at rate j_a. The amount of energy that is required to turn raw materials into the stuff of civilization is the enthalpy of rearranging matter into a new form. Section 2.2 included a discussion of how heating transforms liquid into vapor within a pot of boiling water. A similar "phase transition" can be seen when we burn oil to extract such things as iron ore and trees from the ground, and then reconfigure raw materials from their low potential, natural state into carefully arranged steel girders and houses.

In what we might call the economy, energy consumption sustains all of civilization's existing internal circulations against a continuous dissipation of heat at rate d and a material decay at rate j_d. Civilization radiates heat to space while we and our physical infrastructure fall apart.

If civilization consumes energy at rate a through the exothermic reaction of primary energy reserves (e.g., through combustion and nuclear reactions), and it dissipates energy at an equivalent rate d, then the size of civilization stays fixed. But if there is a positive disequilibrium between consumption and dissipation, then a remnant of consumed power is able to go toward incorporating new raw materials at rate $j_a - j_d$. The new materials grow a length density of civilization networks Λ.

So civilization might be considered to fall under a class of "emergent systems" because this disequilibrium allows civilization to expand into new reserves of raw materials and energy; the expansion leads to a positive feedback that accelerates emergent growth. From equation (26), growth rates are equivalent to an expansion of a length density Λ that is tied to the system's accumulated bulk to a one third power $N_S^{1/3}$. The growth rate can be thought of as a lengthening and concentration of the networks that form civilization's fabric.

From equation (35), we can infer that civilization growth is promoted when a combination of the following three conditions are satisfied: civilization has access to large reserves of available energy $\Delta H_R = N_R \Delta\mu$; the amount of energy e_S^{tot} that is required to incorporate raw materials into civilization's structure is low;

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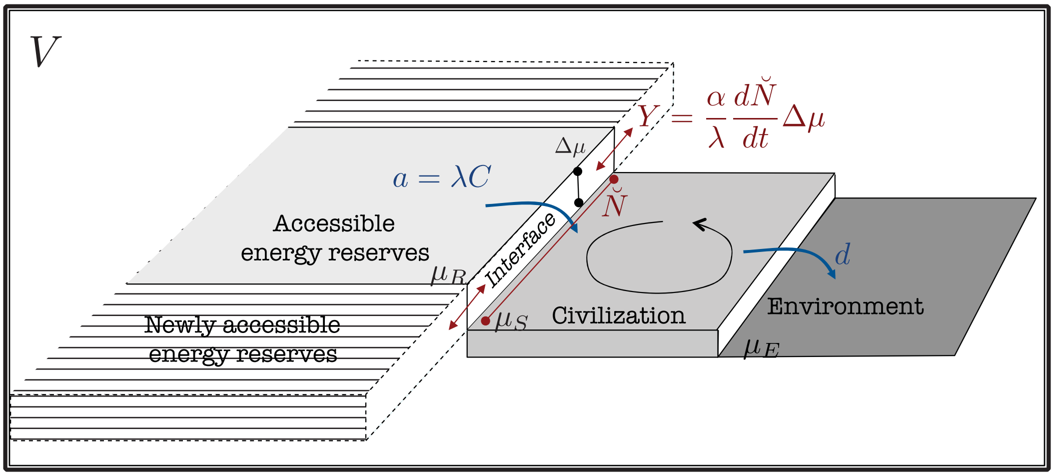

Figure 4. Representation of Figure 2 in terms of global economic wealth *C* and economic production *Y*, as linked to rates of primary energy consumption *a* and the size of an interface with respect to energy reserves $\breve{N}$. Economic production *Y* is tied to interface growth, representing the material expansion of civilization through a linear growth of its networks. Growing networks increase the capacity to draw from newly accessible energy reserves. Energy consumption sustains civilization circulations against dissipation to the environment at rate *d*.

and civilization does not fray too quickly, or that the decay parameter $\delta = j_d/j_a$ expressing relative rates of decay is small.

In what follows, these concepts are extended to provide specific formulations for the long-term evolution of civilization, expressible in such purely economic terms as rates of return on wealth, economic production, innovation, and technological change.

3.1. Expression of Economic Quantities in Thermodynamic Terms

In *Garrett* [2011], it was hypothesized that global rates of energy consumption *a* can be linked to a very general metric of global economic wealth *C* through a constant λ:

$$a = \lambda C \tag{37}$$

where current wealth is the time integral of past inflation-adjusted economic production

$$C = \int_0^t Y\left(t'\right) dt' \tag{38}$$

The motivation for these expressions is that global energy consumption at rate *a* sustains the short timescale internal circulations of civilization against an associated power dissipation *d*. If the capacity to sustain the global economy's circulations is what we implicitly value, then primary energy consumption should be fundamentally tied to a general representation of economic wealth (Figure 4). Such complicating factors as debt and trade do not need to appear in equation (38) because wealth is inclusive of all civilization elements.

The hypothesis that λ is a constant is falsifiable. Since the Gross World Product (GWP) is the total productivity within a period of one year, equation (38) can be calculated from

$$C_i = \sum_i GWP_i \tag{39}$$

where *i* is a time index starting from the beginnings of civilization.

Equation (37) can be tested using historical estimates of world GWP *Maddison* [2003] and available statistics for global primary energy consumption as outlined in *Garrett* [2011], *Garrett* [2012a], and in the supporting information for the article here. What was found is that, when expressed in inflation-adjusted US year 2005 dollars, λ has maintained a steady value for each of forty-one years of observations. Effectively, what sustains the purchasing power embodied in each one thousand dollar bill, and distinguishes it from a mere piece of paper, is a continuous 7.1 ± 0.1 W of primary energy consumption.

For example, in the year 2010, a global wealth of US $2352 trillion was supported by 16.9 terawatts of primary energy consumption; and, in 1980, $1300 trillion 2005 was sustained by 9.6 terawatts. The ratio of these two quantities remained essentially unchanged in each year between 1970 and 2010, with a standard deviation of just 3% over a time period when wealth increased by 111% and GWP increased by 238%.

The implication is that economic wealth can be considered to be a human representation of the magnitude of the associated circulations that power consumption can support. The identity $a = \lambda C$ is important because it presents the possibility of using the tools of physics to evaluate and forecast economic growth.

There are not only similarities but also some important differences between this approach and standard macroeconomic models. Wealth C, as defined by equation (37), has units of currency it might appear to be much like the term "capital" K that is used in traditional economic treatments [e.g., *Solow*, 1956],

There is a key difference, however. The term capital is normally reserved for the additive value of fixed "physical" structures such as bank accounts, buildings and roads. Economic output Y is not considered to be directly additive to physical capital unless it is a "savings" or "investment" that is not immediately "consumed" by people. The motivation for subtracting "consumption" from output is that it seems logical to separate people from nonliving structures. The argument is that the economy is human. Human labor L uses physical capital K to enable future consumption by humans. It is certainly not perceived to be the reverse where, e.g., cars use people as a means to access gasoline which they then consume while enabling the creation of more people. Rather, the presumption is that human consumption of food in the distant past should be subtracted from production because it has no obvious relationship to physical capital in the present.

Certainly the traditional approach offers a self-consistent way to track financial accounts. However, it would appear to violate physical laws because it does not appeal foremost to a coupling of energy dissipation and material flows, something that has been pointed out by a few economists. The suggested remedy has been to include energy among labor and capital as a factor of production [e.g., *Costanza*, 1980; *Georgescu-Roegen*, 1993; *Warr and Ayres*, 2006; *Kümmel*, 2011].

But even if energy is included as one factor of production, there remain two important drawbacks to this approach. One is that, where some combination of labor, capital, and energy are included as distinct elements of production, the production functions lack dimensional consistency: the most commonly used form is $Y = AL^{\alpha}K^{\beta}$ where A is a variable and α and β are noninteger exponents. Further, the models are neither falsifiable nor inherently prognostic since A, α, and β are tuned to historical economic statistics rather than derived from first principles.

More importantly, the traditional treatment of additive capital and economic consumption appears to violate the Second Law of Thermodynamics. Although sometimes overlooked, perhaps the most profound implication of the Second Law is that it forbids the existence of isolated systems, either in space or time. By necessity, everything is connected through dissipative flows, even if the connection is very remote.

Assuming the Second Law applies equally to human systems, it would seem problematic to treat something like physical capital as being purely mathematically additive, as is presumed in traditional treatments. A better perspective might be that the magnitude of civilization wealth lies in its connections or a network, insofar as network elements allow for the dissipative flows that sustain it.

People need houses as much as houses need people in order to maintain their respective worth; removing one affects the utility of the other. Even our perceptions of worth cannot be meaningfully separated from our cardiovascular system and stomachs; each has no independent economic worth as each needs each other to work. Value lies not in any element individually but rather in its connections.

The Second Law would also require that human consumption cannot simply disappear to the past, as is presumed in traditional models (and expressed in past criticisms of the approach here [*Cullenward et al.*, 2011]. All past actions unavoidably have some connection to present actions. Or, in the language of time series analysis, all natural systems are "reddened" such that they exhibit a bias toward low frequency variability. Even if someone is only "consuming" a hamburger, a hamburger is nourishing and satisfying in a way that does more than to simply sustain current short timescale human interactions with the rest of civilization. It also carries some lingering, long timescale memory of the pleasures of hamburger consumption into the future. Another example could be that figs and barley eaten by the ancient Greeks facilitated in some small way the construction of the monuments, culture, and population growth that sustain modern Greece today.

Thus, there is no embodied value within any object by itself, as is sometimes considered [*Costanza*, 1980]. There is only value insofar as something currently has ties with other elements of civilization, as they have been built up over time. A brick of solid gold is worthless—if it is forgotten and lost in the middle of the desert. The same brick is worth much more if it facilitates financial flows as part of a previously built economic network. Wealth includes people, their knowledge, their buildings, and their roads, but only to the extent that they are interconnected through networks to the rest of the accumulated whole.

So the alternative approach that is proposed here is to treat civilization as a system with constant specific potential μ_S as shown in Figure 4, one whose collective wealth is an economic expression of how its elements are intertwined through networks that mutually support global scale diffusive and dissipative flows. From equations (14) and (37)

$$C = \frac{\alpha}{\lambda}\breve{N}\Delta\mu \qquad (40)$$

where through equation (16), $\breve{N}$ is related to the system size through $N_S^{1/3}$ and a quantity of potential energy $N_R\Delta\mu$ Or, from equation (20)

$$C = \frac{\mathscr{D}}{\lambda}\Lambda\Delta H_R \qquad (41)$$

The financial value of civilization lies in the accumulated length density of a global network $\Lambda = \int_0^t d\Lambda/dt'\,dt'$, with the caveat that the total network must be coupled to reserves of potential energy ΔH_R through an interface so that there can be diffusive flows with diffusivity $\mathscr{D}$. As will be discussed in more detail below, and is illustrated in Figure 4, the consumption of potential energy is tied to wealth C, but consumption is not part of production Y. Instead, consumption is orthogonal to production. Production of value occurs only when heating exceeds dissipation in the consumptive flow. In this case, there is material growth of the length of the interface between civilization and its energy reserves. Economic production how we value civilization growth.

Value or wealth can have many forms. One may be the linear networks that diffuse energy from a coal fired power plant to a toaster—insofar as there is coal and an unbroken electrical grid (and the homeowner likes toast). Or, there is value in cars and roads that transport people and goods—insofar as there is a supply of petroleum and the roads and vehicles are maintained. The diffusion of knowledge and goods within human systems might be expressed in terms of a network length density and a proximity to intellectual and material resources. In fact, this is an approach that has previously been employed by a few others, albeit in less strictly thermodynamic terms [e.g., *Barabási and Albert*, 1999; *Jackson*, 2010; *Bahar et al.*, 2012; *Bettencourt*, 2013].

Of course, the complexity of civilization is extraordinary, so treating it as a physical system might seem overly simplistic. Certainly, it would be extremely challenging if not impossible to model all possible network interactions. But, as a first step, thermodynamic principles offer the simplification of lowering resolution so that human and physical capital are regarded at global scales, with the trade-off that nothing can be said about the internal details, except perhaps in a statistical sense [e.g., *Ferrero*, 2004]. A global scale economic model can say nothing about unemployment in the European Union or the trade imbalance between the United States and China. But, a straight-forward link between physical and economic quantities allows for stepping back to view civilization as a whole.

3.2. Thermodynamics of Nominal and Inflation-Adjusted Economic Production

Where economic wealth is defined holistically by $C = \int_0^t Y(t')\,dt'$ (equation (38)), there appears to be a fixed relationship to thermodynamic flows through $a = \lambda C$ (equation (37)). Thus, the physical principles described in Section 2 can provide the basis for the derivation of prognostic expressions for economic quantities. The first that is described here is the growth of wealth through inflation-adjusted economic production.

The simplest and most physical expression of the production function is that it is orthogonal to consumption so that it adds directly to economic wealth as it has been defined above. Taking the derivative of equation (38)

$$Y = \frac{dC}{dt} \tag{42}$$

where Y is inflation-adjusted (or real) economic output or productivity, with units of currency per time. However, since $a = \lambda C$, $w = \Delta\mu d\check{N}/dt$, and from equations (40) and (41), any of the following expressions also apply, where α, $\mathscr{D}$, and λ are constants:

$$Y = \frac{1}{\lambda}\frac{da}{dt} \tag{43}$$

$$= \frac{\alpha}{\lambda} w \tag{44}$$

$$= \frac{\mathscr{D}}{\lambda}\frac{d}{dt}\left(\Lambda \Delta H_R\right) \tag{45}$$

Thus, economic production is an expression of the amount of physical work w that is done to increase the network density Λ and available energy reserves ΔH_R. Real production is valuable only to the extent that it accelerates the energetic flows a that sustain the circulations in civilization. (Figure 4).

From equations (24) and (37), a more purely economic expression of the production function is one that is related to wealth and rates of energy consumption through

$$Y = \frac{dC}{dt} = \eta C \tag{46}$$

where, η is a variable rate of emergent growth for thermodynamic systems. For economic systems, the rate of emergent growth η can be termed the "rate of return" since, like money in the bank, it expresses the growth rate of global wealth through

$$\eta = \frac{d \ln C}{dt} \tag{47}$$

From equations (13) and (28), the rate of return and economic production are positive when energy consumption exceeds dissipation, or when the incorporation of new raw materials exceeds physical decay. In contrast to traditional economic treatments, consumption is not part of economic production. Rather, economic production equates to the net material growth of civilization, insofar as it grows an interface with available energy supplies (Figure 4). Real production is a consequence of a convergence of materials flows that arises from an imbalance between consumption and dissipation. It is the historical accumulation of this imbalance that leads to the current wealth and capacity to consume.

Long-term statistics for the evolution of η are described in *Garrett* [2011] and *Garrett* [2012a]. In recent years, the inflation-adjusted rate of return has reached an all-time high of slightly more than 2.2% per year, although it has largely ceased to rise any further.

The balance of forces that determines the evolution of η was explored in *Garrett* [2012b] which presented a model that couples civilization growth to climate change. It was argued that the rate of return η can be expressed in terms of two components $\eta = \beta - \gamma$, expressing a source and a sink, in which case production is related to wealth through

$$\begin{aligned} Y &= (\beta - \gamma) C \\ &= \hat{Y} - \gamma C \end{aligned} \tag{48}$$

where β is a coefficient of nominal production, $\hat{Y} = \beta C$ is the nominal economic output, and γC is the magnitude of any correction to nominal production that is required to yield inflation-adjusted real production. From equation (29), the source is related to material consumption through

$$\beta = \frac{j_a}{3N_S} \tag{49}$$

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and, the sink is related to material decay through,

$$\gamma = \frac{j_d}{3N_S} \tag{50}$$

or, from equation (32)

$$\gamma = \delta \frac{j_a}{3N_S} \tag{51}$$

Expressed thermodynamically, β can be viewed as a rate coefficient for interface growth and γ as a rate coefficient for interface decay, each with units of inverse time.

What is interesting is that there is a simple link here to rates of inflation. Normally, the GWP deflator is what is used to represent the degree of any revisions to calculations of nominal output, i.e., the nominal GWP is revised downward by a factor $\hat{Y}/Y$. The GWP deflator is linked to inflation insofar that it is estimated from price changes in a very broad, moving basket of goods. For inter-annual calculations, the factor by which the nominal GWP must be adjusted to be compared to the nominal GWP in a prior year is:

$$\text{GWP Deflator} = \frac{\hat{Y}}{Y} \simeq 1 + \langle i \rangle \tag{52}$$

where $\langle i \rangle$ is the calculated average inflation rate for the year. Assuming the inflation rate is much less than 100% per year, it follows that

$$\langle i \rangle = \frac{\widehat{\text{GWP}} - \text{GWP}}{\widehat{\text{GWP}}} \simeq \frac{\hat{Y} - Y}{\hat{Y}} = \frac{\langle \gamma \rangle}{\langle \beta \rangle} \tag{53}$$

From equations (49) and (51), this leads to the very simple result that global-scale inflation rates can be viewed as a economic expression of the decay parameter $\delta = j_d/j_a$:

$$\begin{aligned} \langle i \rangle &= \frac{\langle \gamma \rangle}{\langle \beta \rangle} \\ &\simeq \langle \delta \rangle = \frac{\langle j_d \rangle}{\langle j_a \rangle} \end{aligned} \tag{54}$$

What this analysis suggests is that civilization decay and global inflation are two sides of the same coin. Decay corresponds to an inflationary pressure because it "devalues" the productive capacity of existing assets. It takes away that which has previously been built, learned, or born. A "fraying" of networks occurs because people die or forget, buildings crumble, and machines oxidize. For example, it has been estimated that 10% of our twentieth century accumulation of steel has been lost to rust and war [*Smil*, 2006]. Where human and physical networks fall apart, there is a diminished capacity to enable the thermodynamic flows that sustain civilization wealth. Any monetary assets that were previously created to support human and physical wealth no longer possess the same real purchasing power. To see the sources of inflationary trends, since $j_a = a/e_S^{\text{tot}}$ (equation (11)) and $a \propto \Delta H_R$ (equation (17)), then assuming that e_S^{tot} changes slowly

$$\begin{aligned} \frac{d \ln \langle i \rangle}{dt} &= \frac{d \ln \langle j_d \rangle}{dt} - \frac{d \ln \langle j_a \rangle}{dt} \\ &\simeq \frac{d \ln \langle j_d \rangle}{dt} - \frac{d \ln \langle \Delta H_R \rangle}{dt} \end{aligned} \tag{55}$$

so, rising inflation might occur if material decay j_d accelerates, perhaps from the types of global scale natural disasters that might be associated with climate change [*Zhang et al.*, 2007; *Lobell et al.*, 2011]. Alternatively, inflation might be driven by a declining availability of energy reserves ΔH_R [*Bernanke et al.*, 1997].

As a caution, traditional interpretations of price inflation [e.g., *Parkin*, 2008] may not be a perfect match for the treatment described here. Pure price inflation is a form of devaluation that arises because the existing money supply has a lower purchasing power with respect to some predefined basket of goods. Inflation is often viewed as being simply a matter for control of the money supply by central banks.

However, banks are not resolved in the very general expression of global wealth C that has been discussed here, which extends beyond money and physical assets to comprise a “basket of goods” that includes all our physical and human relationships and networks. The implication is that inflation through devaluation does not necessarily apply to monetary assets alone. Devaluation might also arise from such factors as depopulation through wars, or because previously acquired skills are no longer needed by others because our capacity for work goes idle for lack of an energetic impetus. For instance, car production might fall if oil becomes scarce and expensive. The workers and their factories remain but the external demand for petroleum driven transportation declines and this leads to car manufacturer layoffs and unemployment [*Lee and Ni*, 2002]. In fact, an apparent short-term trade-off between unemployment and price inflation is well known in the field of Economics and has been termed the “Phillips Curve” [*Phillips*, 1958].

4. Thermodynamics of Global Technological Change, Innovation, and Growth

Thus far, it has been shown that an economic growth model can be defined by the coupled equations for the production function for real output Y, and the growth of real wealth C given by $dC/dt = Y$ and $Y = \eta C$, where η is a variable rate of return on wealth. As described here and in *Garrett* [2011], these equations can be viewed as being a more thermodynamically justified (and dimensionally self-consistent) form of the Solow-Swan neo-classical economic growth model [*Solow*, 1956] that is typically used to estimate the social costs of preventing and adapting to climate change [*Yohe et al*., 2004; *Stern*, 2007; *Tol*, 2009; *Nordhaus*, 2010]. Here C is a generalized form of physical capital (K) that encompasses labor (L), and η is analogous to the total factor productivity (A), whose changes relate to technological change ($d\ln A/dt$).

Technological change is often seen as a primary driver of long-run economic growth [*Solow*, 1957], and a potential tool for limiting future energy consumption and carbon dioxide emissions [*Nakicenovic*, 2004; *Pacala and Socolow*, 2004; *Raupach et al*., 2007; *Raupach et al*., 2007; *Pielke et al*., 2008]. The source of technological change remains somewhat of a puzzle; however, there has been a shift toward regarding it as having endogenous origins, perhaps due to government investments in research and development [*Romer*, 1994].

What follows shows how the forces behind technological change can also be seen in light of a more strictly thermodynamic context. Viewed globally, the rate of return η evolves according to a deterministic expression obtained by taking the partial derivative of the logarithm of equation (35), holding fixed the rate, shape, and specific potential α, k, and $\Delta\mu$:

$$\frac{d\ln\eta}{dt} = -2\frac{dN_S/dt}{3N_S} + \frac{d\ln(1-\delta)}{dt} + \frac{d\ln \Delta H_R}{dt} - \frac{d\ln e_S^{\mathrm{tot}}}{dt} \tag{56}$$

$$\begin{aligned} &= -2\eta + \eta_\delta + \eta_R^{\mathrm{net}} - \eta_e \\ &= -2\eta + \eta_{\mathrm{tech}} \end{aligned} \tag{57}$$

Here the term $d\ln\eta/dt$ is referred to as *economic innovation* because positive values of equation (56) represent an acceleration of existing rates of return η. Innovations are what are required for rates of return on wealth to rise. Defining $\tau_\eta = 1/(d\ln\eta/dt)$ as the characteristic time for innovation, then wealth grows from an initial value C_0 as

$$C = C_0 e^{\eta\tau_\eta\left(e^{t/\tau_\eta}-1\right)} \tag{58}$$

If innovation is positive, then wealth grows explosively or super-exponentially. In the limit of no innovation and $\tau_\eta \to \infty$, the growth of wealth reduces to the simple exponential form $C = C_0\exp(\eta t)$.

Since $a = \lambda C$, the form of equation (58) applies equally to rates of energy consumption, and if the carbonization of the energy supply is nearly fixed, then to carbon dioxide emission rates as well [*Garrett*, 2011]. In what is sometimes termed Jevons' Paradox, technological gains accelerate global energy consumption and carbon dioxide emissions by increasing the efficiency of civilization growth into the reserves of energy that sustain it. For a further discussion, see *Garrett* [2012b].

The sum $\eta_{\text{tech}} = \eta_\delta + \eta_R^{\text{net}} - \eta_e$ is termed here as the *rate of technological change* η_{tech} because it is the driving force behind innovation $d \ln \eta/dt$. It represents the sum of reductions to net decay (η_δ), rates of net energy reserve expansion (η_R^{net}), and reductions in the amount of energy required to access raw materials ($-\eta_e$).

It should be mentioned that equation (56) could be expanded to include variability in α, k and $\Delta\mu$. However, in this treatment these quantities are held fixed, partly in the interests of simplicity, and partly because a quantity like civilization shape is difficult to quantify with available statistics. By taking the partial derivative, the emphasis shifts instead to how best to define variability in the energy reserve $\Delta H_R = N_R \Delta\mu$. If the "rise" $\Delta\mu$ is treated as being fixed, N_R might be expressed by civilization in units of millions of barrels of oil equivalent (mmboe), where the fixed potential energy of combustion contained in one barrel is equivalent to $\Delta\mu$. With respect to the rate and shape coefficients α and k, energy economists distinguish between energy "resources" and energy "reserves" [*Höök et al*., 2010]. Resources represent what is potentially available to be exploited. Reserves represent what is currently accessible given existing technological, physical, and political considerations. The presumption that is made here is that ΔH_R represents reserves not resources. Variability in α and k would represent more efficient access of resources. If α and k are fixed, the variability is manifested instead in changes in the statistics for what civilization considers to be reserves.

The following examines each component of technological change in equation (56) in more detail.

4.1. Innovation Through Increased Longevity

The first component of technological change is η_δ, which relates to reductions in the decay parameter δ (equation (32)). From equation (54), and assuming that the global inflation rate is much less than 100%, the decay parameter is approximately equal to the inflation rate through $\langle i \rangle \simeq \langle \delta \rangle$. In this case, the first order expansion in η_δ yields

$$\eta_\delta = \frac{\mathrm{d} \ln (1-\delta)}{\mathrm{d}t} \simeq -\frac{\mathrm{d} \langle \delta \rangle}{\mathrm{d}t} \tag{59}$$

Since $\delta = j_d/j_a$ (equation (32)), one way of interpreting η_δ is through

$$\eta_\delta = -\frac{1}{j_a} \left(\frac{\partial j_d}{\partial t} \right)_{j_a} \tag{60}$$

or, for a given rate of material consumption j_a, innovation is favored by decreasing decay rates j_d. If people are enabled to live longer through advancements in health [*Casasnovas et al*., 2005], or their structures are built so that they last longer [*Kalaitzidakis and Kalyvitis*, 2004], then this is a form of positive technological change that contributes to faster growth. From equation (54), it follows that this innovative force would show up in global scale economic statistics as declining inflation and/or unemployment. In other words:

$$\eta_\delta \simeq -\frac{d \langle \delta \rangle}{dt} \simeq -\frac{d \langle i \rangle}{dt} \tag{61}$$

4.2. Innovation Through Discovery of Energy Reserves

The expression η_R^{net} refers to the net rate of expansion of available energy reserves such as fossil fuels ΔH_R. Having a newly plentiful supply of energy accelerates economic innovation and growth [*Smil*, 2006; *Ayres and Warr*, 2009].

There are two forces here. One is for energy reserves to decline due to potential energy consumption at rate a. The second is that civilization discovers new reserves of energy at rate D. The balance is given by

$$\begin{aligned} \frac{\mathrm{d} \ln \Delta H_R}{\mathrm{d}t} &= \frac{\text{Discovery} - \text{Depletion}}{\text{Existing}} \\ &= \frac{D - a}{\Delta H_R} \\ &= \eta_D - \eta_R \end{aligned} \tag{62}$$

Net reserve expansion occurs when rates of reserve discovery η_D exceed rates of reserve depletion η_R, requiring that $\eta_D/\eta_R > 1$.

As illustrated in Figure 4, civilization consumes energy as it grows, and it grows into surroundings that may or may not contain new reserves of fuel [*Murphy and Hall*, 2010]. If $\eta_D/\eta_R > 1$, then civilization discovers new reserves faster than it depletes previously discovered reserves. In some global sense, faster growth follows because current energy consumption becomes "cheaper" relative to the existing quantity of wealth C that was built from past consumption.

4.3. Innovation Through Increased Efficiency of Raw Material Extraction

The expression $-\eta_e$ in equation (56) refers to changes in the specific enthalpy of civilization e_S^{tot}. Since $e_S^{\text{tot}} = a/j_a$ (equation (11)), a decline in e_S^{tot} would appear as a decrease in the amount of power a that is required for civilization to extract raw materials and incorporate them into civilization at rate j_a.

Comparing equations (28) and (29), civilization networks grow through raw material consumption. If growing civilization requires less energy per unit matter, then civilization can grow faster for any given rate of global energy consumption a. This is an innovative force because the material growth of civilization increases access to the resources that sustain it. Effectively, since economic wealth and energy consumption are linked, raw materials become cheaper.

For example, roads improve our ability to access and consume new reserves of energy and raw materials. But it takes energy and raw materials to build roads. While a given length of road requires a more or less fixed amount of bitumen, efficiency gains can be made if less energy is required in order to mine bitumen and lay asphalt. Lowering the specific enthalpy of road production $e_S^{\text{tot}} = a/j_a$ means that more roads can be built for a given amount of current energy consumption. Thereby, civilization is able to accelerate its growth into existing and new reserves of energy and raw materials.

4.4. Diminishing Returns as a Drag on Innovation

The final term in equation (56), -2η, expresses a drag on how fast rates of return can grow. Innovation slows naturally due to a *law of diminishing returns*. In the absence of technological change, wealth converges on a steady-state where rates of return approach zero. This is a result that also appears in more traditional frameworks [*Romer*, 1986]), but where the economy is viewed as a physical system, diminishing returns is a consequence of the dilution of current growth within the accumulated bulk built from past growth (equation (35)). Each incremental addition of raw materials into civilization j^{net} has a decreasing impact relative to the summation of previously incorporated matter $\int_0^t j^{\text{net}}\left(t'\right) \text{dot}'$. The consequence is that the rate of return slows unless technological change is sufficiently rapid. Mathematically, from equation (56), what is required for net innovation with increasing growth rates is that the technological change rate exceed the current rate of return by a factor of 2, and $\eta_{\text{tech}} > 2\eta$.

5. Modes of Growth and Decay in Economic Systems

5.1. Technological Change and Rates of Return

The above expressions open the way to deterministic solutions for global wealth and energy consumption. Previous studies have identified characteristic sigmoidal or logistic behavior in the effects of technological change on economic growth. They show that, after overcoming a period of initial resistance, technological change appears to initially accelerate growth, but it is then followed by saturation [*Landes*, 2003; *Smil*, 2006; *Marchetti and Ausubel*, 2012]. In *Garrett* [2011] similar behavior was shown in long-term historical statistics for η, where rates of return accelerated exceptionally rapidly between 1950 and 1970 but have leveled off since (see also the supporting information).

Equation (56) can also be expressed in the form of the logistic equation

$$\frac{\mathrm{d}\eta}{\mathrm{d}t} = \eta_{\text{tech}}\eta - 2\eta^2 \tag{63}$$

If rates of technological change η_{tech} are constant, then the solution has the sigmoidal or "S-curve" form

$$\eta\left(t\right) = \frac{G\eta_0}{1 + (G-1)\exp\left(-\eta_{\text{tech}}t\right)} \tag{64}$$

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Table 1. Modes of Growth in Economic Systems[a]

	Innovation	DR and TC	DR and TD	Decay	Collapse
Growth number $\eta_{tech}/2\eta_0$	$G > 1$	$0 < G < 1$	$G < 0$	$G > 1$	$G < 1$
Initial rate of return	$\eta_0 > 1$	$\eta_0 > 0$	$\eta_0 > 0$	$\eta_0 < 0$	$\eta_0 < 0$
Limiting rate of return	$\eta_{tech}/2$	$\eta_{tech}/2$	0	0	$-\infty$

[a]DR, diminishing returns; TC, technological change; TD, technological decline.

where η_0 is the initial value for the rate of return, and

$$G = \frac{\eta_{tech}}{2\eta_0} \tag{65}$$

represents a "growth number" [*Garrett*, 2012c] that partitions solutions for $\eta(t)$ into varying modes of growth summarized in Table 1.

In this framework, the four modes of growth that are available to civilization are innovation, diminishing returns, decay, and collapse. Each is partitioned by the growth number G and the initial rate of return η. Innovation is characterized by growing rates of return; diminishing returns is associated with declining rates of return, either to a limiting value $\eta_{tech}/2$ or to zero. Where rates of return are initially negative, decay rates either slow with time to approach zero, or they accelerate in which case civilization enters a mode of collapse.

Figure 5 carves these modes within a space of η_{tech} and η, along with associated trajectories for any given value of η_{tech}. For example, for values of $G > 1$, civilization is in a mode of innovation because technological innovation is sufficiently rapid to overcome diminishing returns. At first, rates of return increase exponentially. They then saturate to approach a value of $G\eta_0 = \eta_{tech}/2$. If η is initially 1% per year and rates of technological change η_{tech} are sustained at a nominal 4% per year, then one would expect rates of return η to grow sigmoidally toward 2% per year. The exponential phase of the sigmoidal growth would have a characteristic time of $1/\eta_{tech}$, or 25 years.

5.2. Technological Change and GWP Growth

Changes in GWP growth rates are the most commonly cited quantity in discussions of mitigation and adaptation to climate change [*Tol*, 2009]. Since $Y = \eta C$ (equation (71)), and the rate of return is given by $\eta = d\ln C/dt$ (equation (47)), it follows that:

$$\frac{d\ln Y}{dt} = \eta + \frac{d\ln \eta}{dt} \tag{66}$$

Thus, GWP growth rates are a simple sum of the current rate of return η and the innovation rate $d\ \ln \eta/dt$. GWP growth increases when there is innovation, and it declines otherwise.

From equation (56), the rate of return itself evolves at rate $d\ln \eta/dt = -2\eta + \eta_{tech}$, where rates of technological change $\eta_{tech} = \eta_\delta + \eta_R^{net} - \eta_e$ are a summation of reductions to net decay, net energy reserve expansion, and improvements to the efficiency of raw material extraction and incorporation into civilization. Since GWP growth is related to the sum of the innovation rate and the rate of return in equation 66, it follows that:

$$\frac{d\ln Y}{dt} = -\eta + \eta_{tech} \tag{67}$$

The GWP growth rate is buoyed when,

$$\eta_{tech} > \eta \tag{68}$$

which, since $\eta = d\ \ln a/dt$, means that, in the long run, GWP growth rates increase only when technological change is more rapid than the current rate at which energy consumption is growing.

One way to express equation (68) is to suppose that technological change is driven by the net discovery of new energy reserves as described by equation (62). In this case equation (68) can be rewritten as

$$\frac{da}{dt} > \Delta H_R (D - a)\, a \tag{69}$$

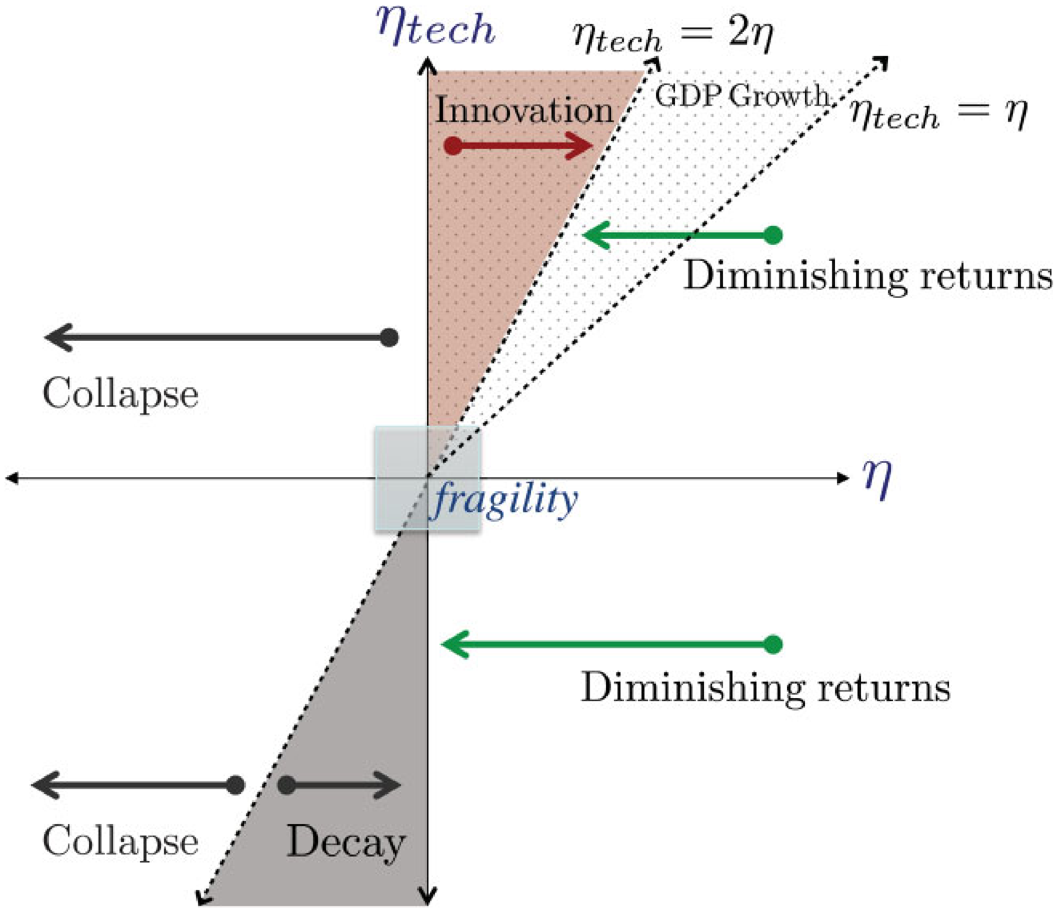

Figure 5. Modes of growth in economic systems, partitioned within a space of rates of technological change η_{tech} and rates of return on wealth η. Arrows represent trajectories for rates of return, assuming that η_{tech} is a constant. The dotted region shows the domain of parameter space associated with GWP growth. See text for details.

Equation (69) is a logistic equation for energy consumption [*Bardi and Lavacchi*, 2009; *Höök et al*., 2010]. The implication is that GWP growth requires ever increasing energy consumption, which itself is sustained only when discovery of new reserves at rate D exceeds their depletion at rate a. Meanwhile energy consumption grows at rate da/dt so discoveries must keep pace. In a fossil fuel economy, maintained GWP growth cannot be decoupled from accelerating carbon dioxide emissions.

5.3. Fragility and Growth

On the other side of growth is decay and collapse. How does this happen? While growth must initially be positive for civilization to emerge, positive growth cannot be sustained forever. Civilization networks are always falling apart, and presumably in a world with finite resources, we will eventually lose the capacity to keep fixing them. What is interesting is that there is no spontaneous mathematical transition between the various modes of growth that are implied by equation (64). In the limit of $t \rightarrow \infty$, rates of return η either asymptotically approach a constant value, or they tend toward collapse.

So it would seem that transitions between modes must be forced by some external impetus such as an increase in the sorts of natural disasters that might be anticipated in the face of climate change. For example, in equation (35), if the decay parameter $\delta = j_d/j_a$ is greater than unity, then η must be negative. If some disaster suddenly forced material decay to exceed material consumption, then civilization would transition from positive to negative rates of return. Of course, things can go both ways. Equation (35) also allows for conditions that might suddenly favor growth, including decreased decay or significant discoveries of new energy reserves that increase ΔH_R. As they have in our past, these conditions might permit

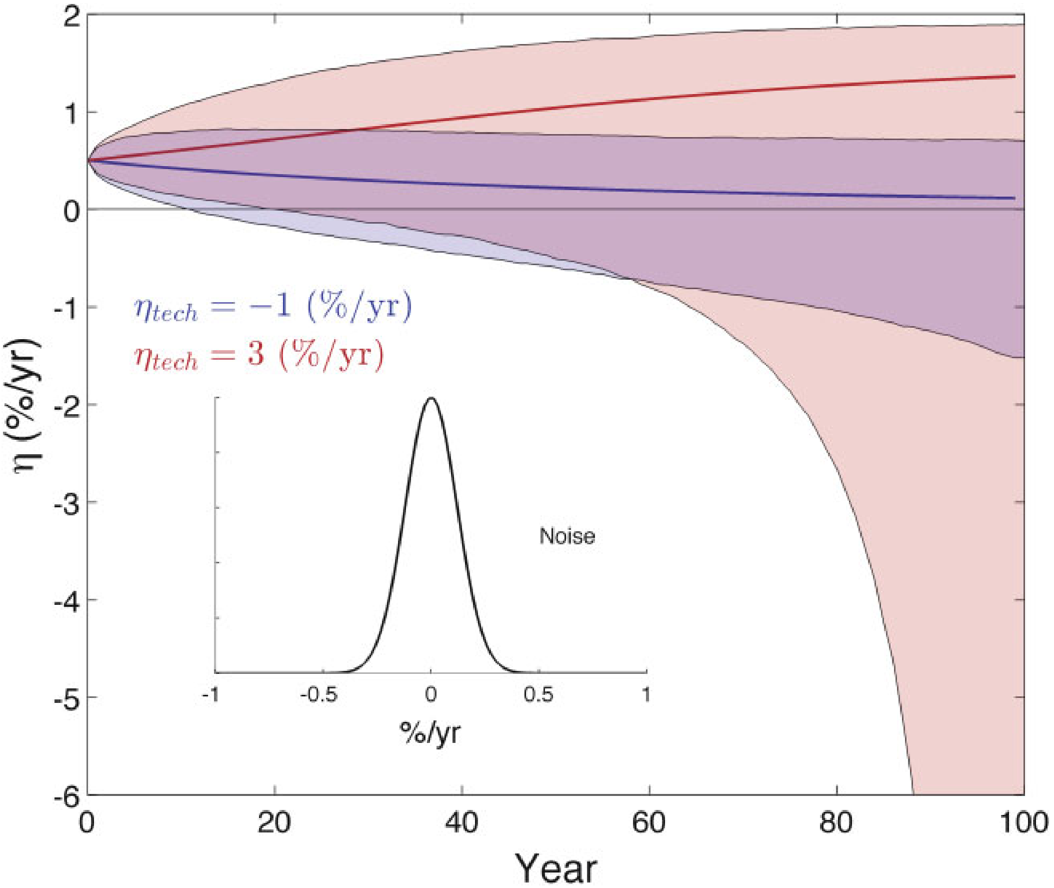

Figure 6. For an initial value for the rate of return η_0 of 0.5% per year, lines are trajectories of the evolution of $\eta(t)$ for scenarios with rates of technological change η_{tech} of 3% per year and -1% per year, as given by equation (64). The shaded region is derived from the upper and lower 5% bounds in an ensemble of 10,000 simulations where noise (inset) has been introduced that has a standard deviation of 0.1% per year for η.

civilization to transition from a mode of diminishing returns into one of innovation and super-exponential growth.

To account for both the good and the bad in the future, stochastic and largely unpredictable external events might be represented by introducing noise to equation (35). An example of how this might play out is illustrated in Figures 6 and 7. If there is no noise, then trajectories follow the logistic solutions provided by equation (64). But if random Gaussian noise is added to η, then the range of possible trajectories broadens. Notably, there are "unlucky" trajectories that could be associated with frequent and persistent global scale natural disasters. Disasters might push civilization into a transition toward a mode of irreversible decay or collapse. Most notably, a transition is particularly likely when rates of return approach zero.

The existence of "tipping points" where there has been a "slowing down" appears to be a feature of ecological and climate systems [*Dakos et al*., 2008, 2011]. What is interesting in the simulations above is that the most dramatic rates of collapse are associated with trajectories that were initially associated with innovation and super-exponential growth. It might seem that the same conditions that allow for the human system to respond especially quickly to favorable conditions are the same ones that allow the system to rapidly decay when conditions turn for the worse.

As illustrated in Figure 5, an innovative economy that enjoys relatively rapid technological change with a growth number $G > 1$ might alternatively be viewed as a "bubble economy" that lacks long-term resilience. Whether collapse comes sooner or later depends on the quantity of energy reserves available to support continued growth and the accumulated magnitude of externally imposed decay. By contrast,

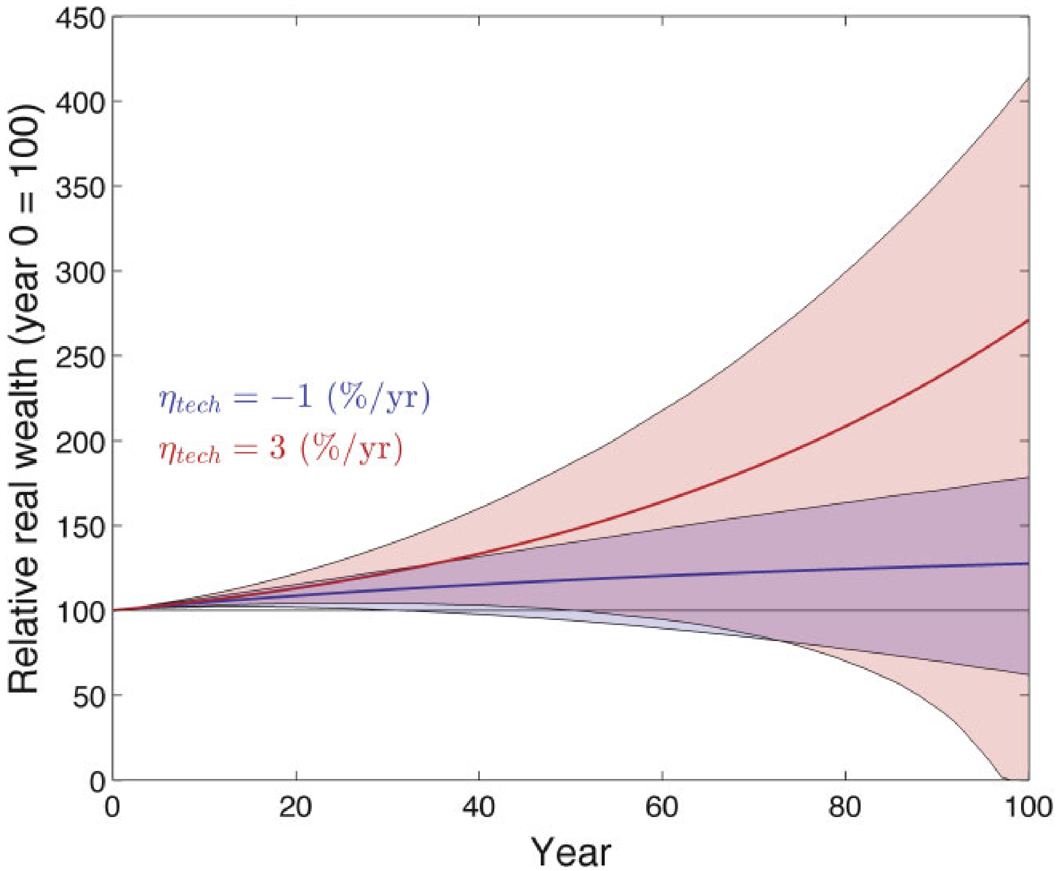

Figure 7. For the scenarios shown in Figure 5, corresponding values of global inflation-adjusted wealth, referenced to 100 in year 0. Wealth corresponds to energy consumption through $a = \lambda C$ and likewise to carbon dioxide emission rates, provided the carbonization of fuels holds steady.

an economy that is less innovative, with lower rates of return η, has a lower risk of rapid rates of decline. In the space shown in Figure 5, it lies "farther away" from modes of collapse.

6. Summary

This paper has presented a physical basis for interpreting and forecasting global civilization growth, with the intent that it might be used to develop a consistent theoretical basis for forecasting interactions between humanity and climate during the Anthropocene.

The perspective is that, like a living organism [*Vermeij*, 2009], energy consumption and dissipation drives material flows to civilization. If there is a net convergence of matter within civilization, then civilization grows. Growth increases the availability of new and existing reserves of matter and energy, and this leads to a positive feedback loop that allows growth to persist or even accelerate.

These rather general thermodynamic results can be expressed in purely economic terms because there appears to be a fixed link between global rates of primary energy consumption and a very general expression of human wealth: $\lambda = 7.1 \pm 0.1$ Watts of primary energy consumption is required to sustain each \$1000 of civilization value, adjusting for inflation to the year 2005 (see supporting information and *Garrett* [2012a]).

It was argued that wealth does not rest in inert "physical capital", as in traditional treatments. Rather, wealth can be interpreted to include all aspects of civilization, even the purely social. Value lies in the density of a network of connections between civilization elements, insofar as this network contributes to a global scale consumption and dissipation of energy (equation (41)). Global economic production Y is positive when consumption exceeds dissipation, and there is a net diffusion of matter to civilization that grows its size.

This leads to an economic growth model for wealth C and economic production Y that is more simple, physical, and dimensionally self-consistent than mainstream models:

$$\frac{dC}{dt} = Y \tag{70}$$

$$Y = \eta C \tag{71}$$

where Y is directly proportional to a lengthening of civilization's networks and growth of its energy reserves. The real rate of return on wealth η is somewhat analogous to the total factor productivity in traditional models. Prognostic expressions for η presented here show that its value is determined by a combination of rates of civilization decay, the quantity of available energy reserves, the amount of energy required to incorporate raw materials into civilization's structure, and the accumulated size of civilization due to past raw material flux convergence. Current values of the rate of return can be inferred from equation (71). For example, current global rates of return are about 2.2% per year [*Garrett*, 2012a]. Trends in η can be forecast based on estimates of future decay and rates of raw material and energy reserve discovery (equation (56)).

Thus, this paper offers a set of prognostic expressions for the growth of civilization, expressible in economic and energetic terms that can be linked to physically measurable quantities. The implications that have been described are summarized as follows:

- Civilization inflation-adjusted wealth is sustained by global energy consumption and grows only as fast.
- Some combination of price inflation and unemployment is related to rates of civilization decay.
- Rates of return on wealth decline in response to accelerated decay or increased resource scarcity.
- Rapid rates of current growth act as a drag on future rates of growth.
- Rates of return grow when there is "innovation" through technological change.
- The GWP grows when energy consumption grows super-exponentially (at an accelerating rate), or when global energy reserve discovery exceeds depletion.
- If growth rates of wealth approach zero, civilization becomes fragile with respect to externally forced decay. This appears to be particularly true if prior growth was super-exponential.

Many of these conclusions might seem intuitive, or as if they have been expressed already by others within more traditional economic perspectives. What is novel in this study is the expression of the economic system within a deterministic thermodynamic framework where a very wide variety of economic behaviors are derived from only a bare minimum of first principles.

More importantly, a sufficient set of statistics exists for global economic productivity, inflation, energy consumption, raw material extraction and energy reserve discovery that the nonequilibrium solutions presented here can be evaluated and falsified with no requirement for any *a priori* tuning or fitting to historical data. Such evaluation will be addressed in Part II. Specifically, it will be shown that the logistic equation given by equation (64) closely matches the evolution of global economic rates of return since 1950, allowing for observed rates of technological change defined by equation (56). Logistic behavior has been recognized in the evolution of human empires throughout history [*Marchetti and Ausubel*, 2012]. It will be shown to be evident in global rates of economic growth as well.

Global civilization has enjoyed explosive growth since the industrial revolution, but it is unclear how long this can be sustained when it is facing ongoing resource depletion, pollution, and climate change. Global economic wealth is tied to energy consumption, and energy consumption through combustion is tied to carbon dioxide emissions. Without a sufficiently rapid switch to noncarbon sources of energy, growing wealth is necessarily linked to growing emissions.

Yet accumulating carbon dioxide in the atmosphere is also likely to drive accelerating civilization decay through amplified hydrological extremes, storm intensification, sea level rise, and mammalian heat stress [*Hansen et al*., 2007; *Solomon et al*., 2009; *Vermeer and Rahmstorf*, 2009; *Sherwood and Huber*, 2010]. The prognostic expressions that have been derived here might be useful to help guide a physically plausible

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range of future timelines for civilization growth and decay, particularly in models that couple human and climate systems during the Anthropocene.

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