Collision-coalescence

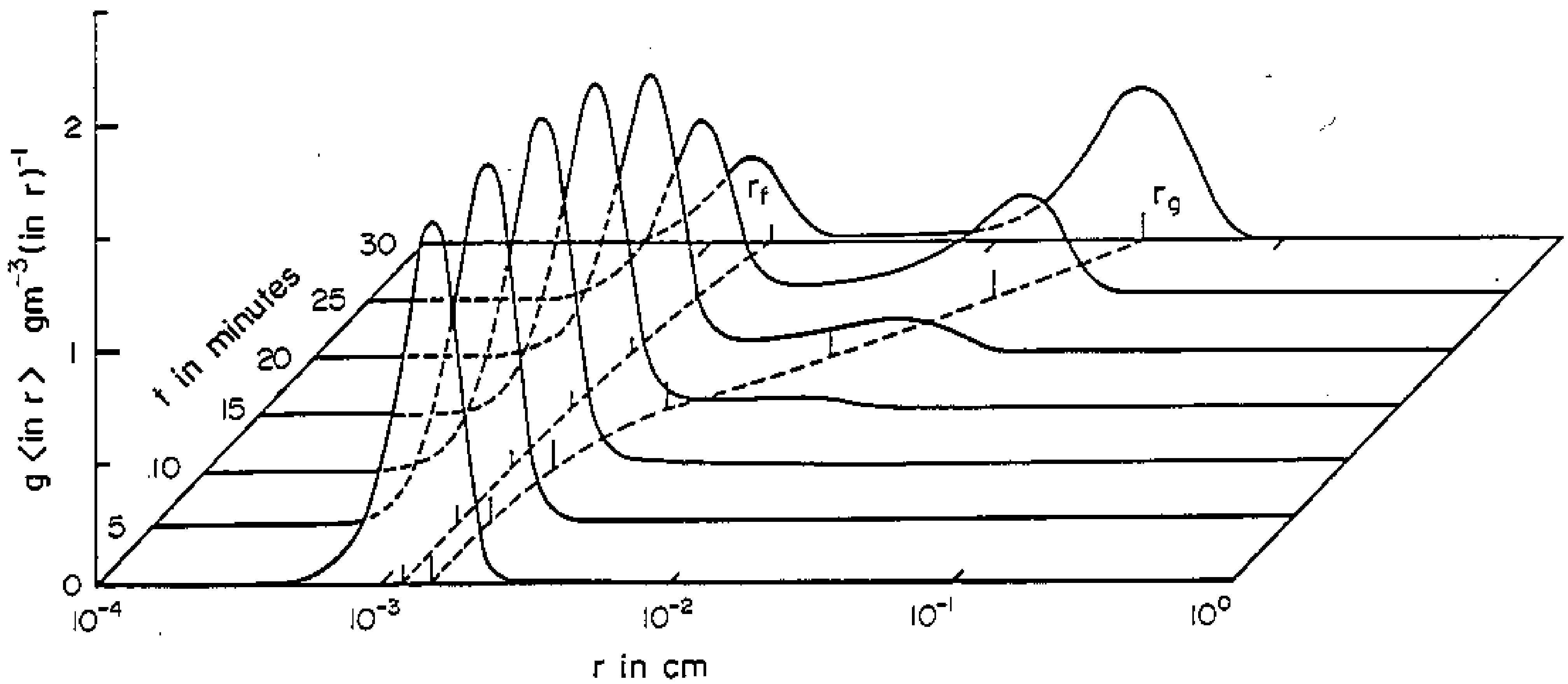
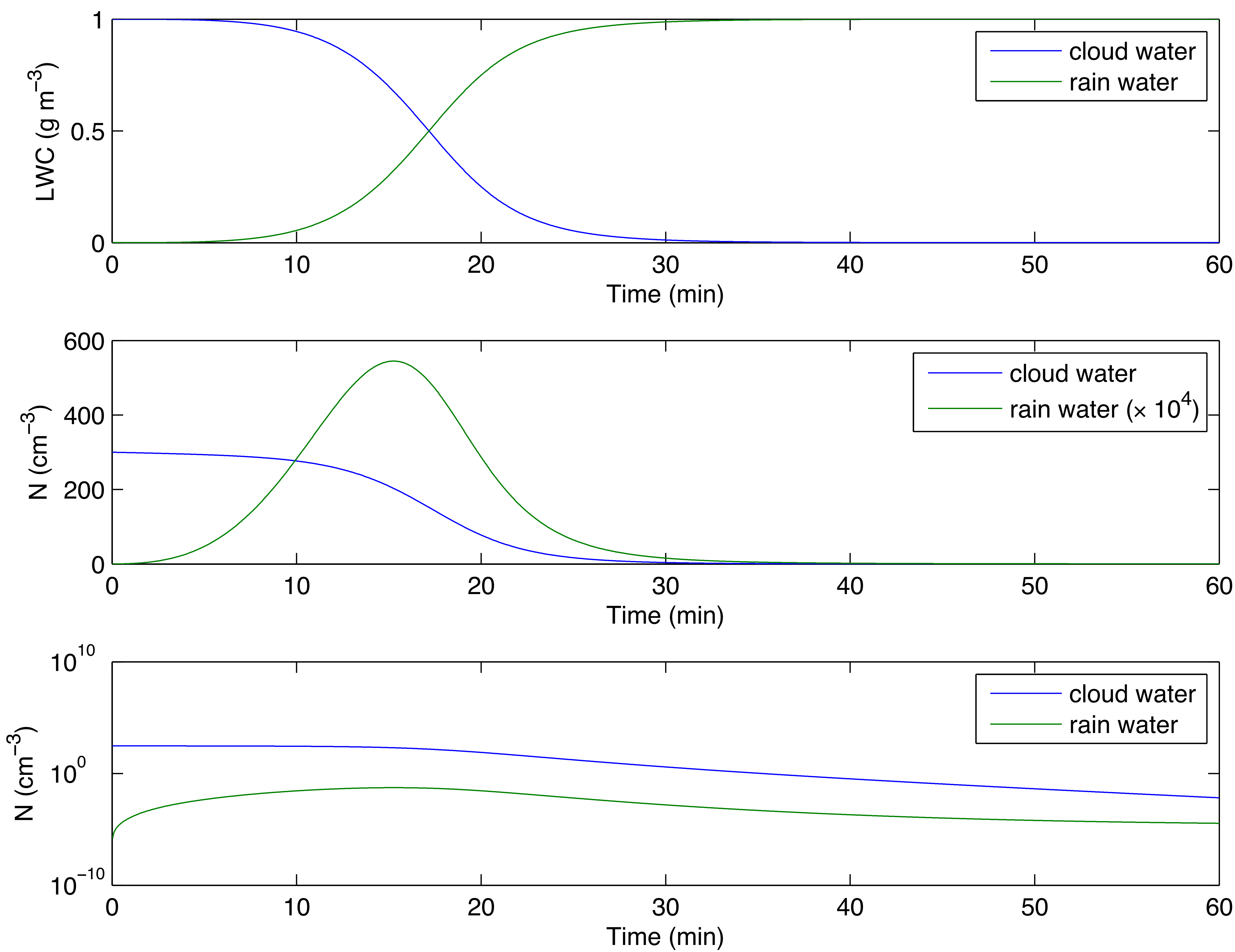


FIG. 8.10. Example of the development of a droplet spectrum by stochastic coalescence. (From Berry and Reinhardt, 1974b.)

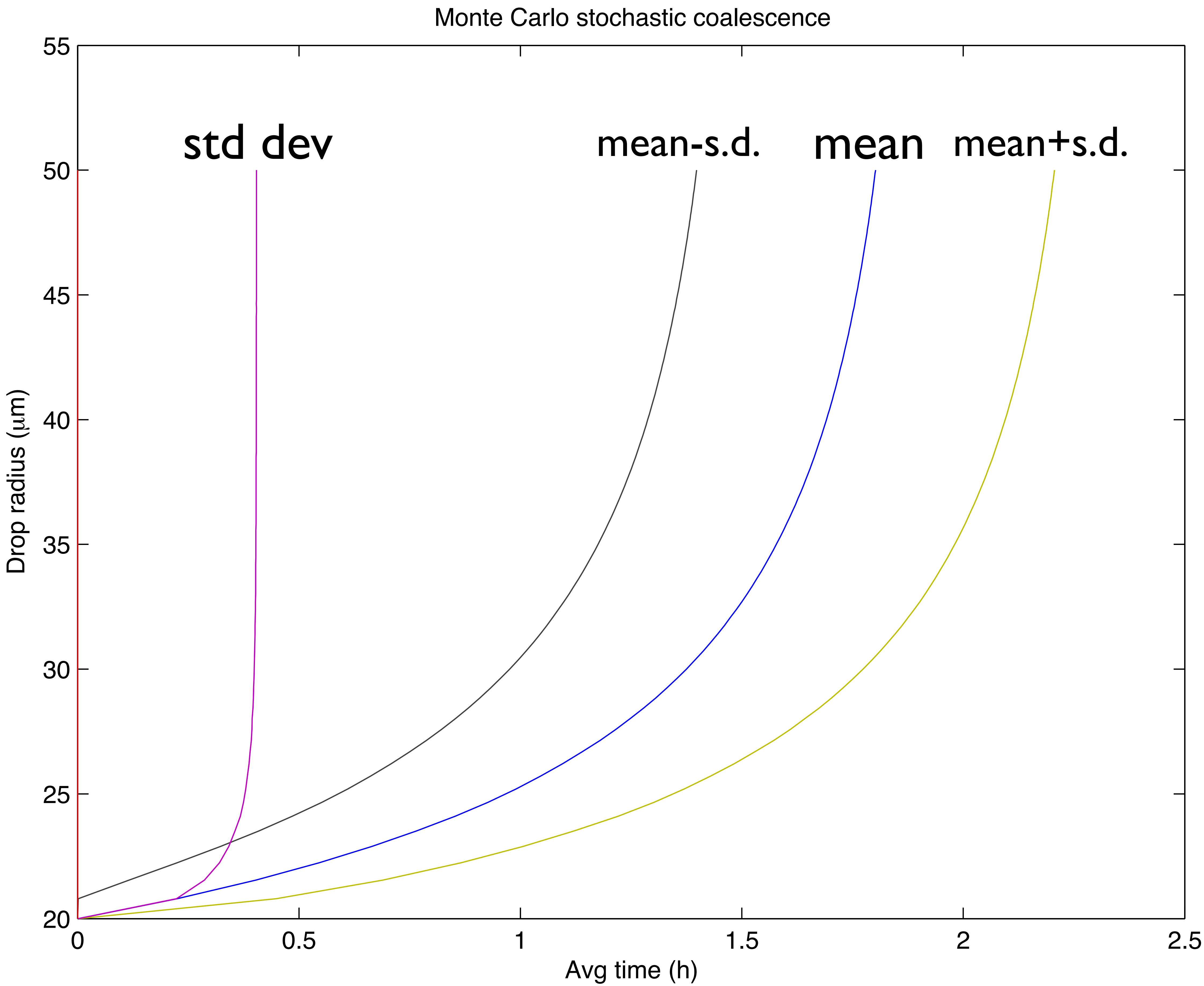

LWC (g m^−3)
cloud water
rain water
Time (min)
N (cm^−3)
cloud water
rain water (× 10^4)
Time (min)
N (cm^−3)
cloud water
rain water
Time (min)

5. (a) What is the mathematical expression that describes the probability that a droplet will capture a smaller droplet during a unit time interval? Label each factor.
(b) Why does this probability increase rapidly as the collector drop radius increases?

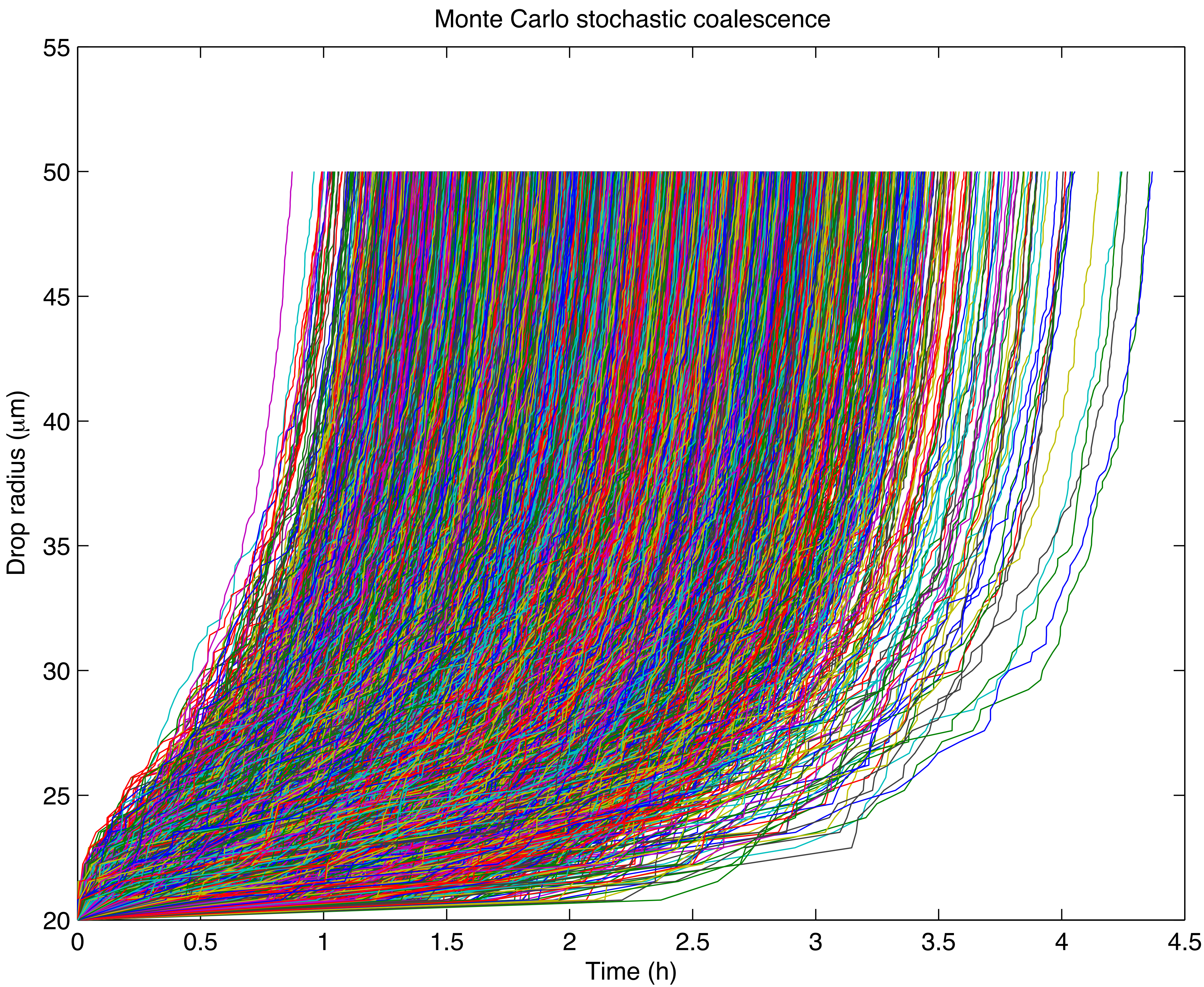
Answer:
(a) $P(R, r)dr = \pi(R + r)^2[u(R) - u(r)]E(R, r)n(r)\,dr$ where $\pi(R + r)^2[u(R) - u(r)]$ is the volume swept out by the larger droplet per unit time, $E(R, r)$ is the collection efficiency, and $n(r)$ is the number concentration of smaller droplets of with radius between r and $r + dr$.
(b) For droplets with radius up to about 30 μm, $u(R) \sim R^2$, so $P(R, r) \sim R^4$.

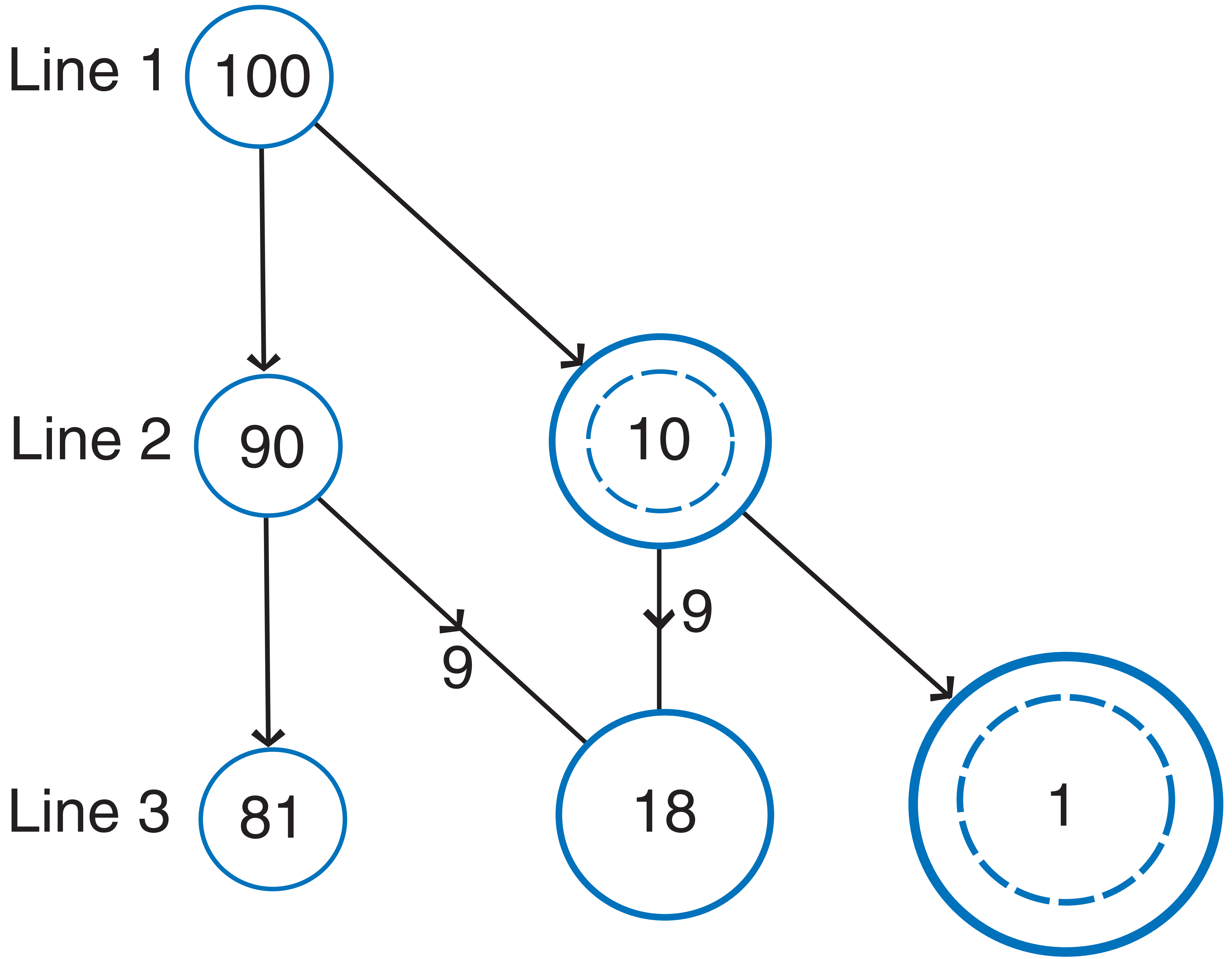
(c) The attached **plot** shows the coalescence growth of several individual droplets, all originally 20 μm in radius. They are collecting smaller droplets of 10 μm radius. What does this plot tell you about:
(i) The average time to grow from 20 to 30 μm radius versus the average time to grow from 30 to 100 μm radius?
(ii) The range of times to grow from 20 to 30 μm radius versus the range of times to grow from 30 to 100 μm radius?

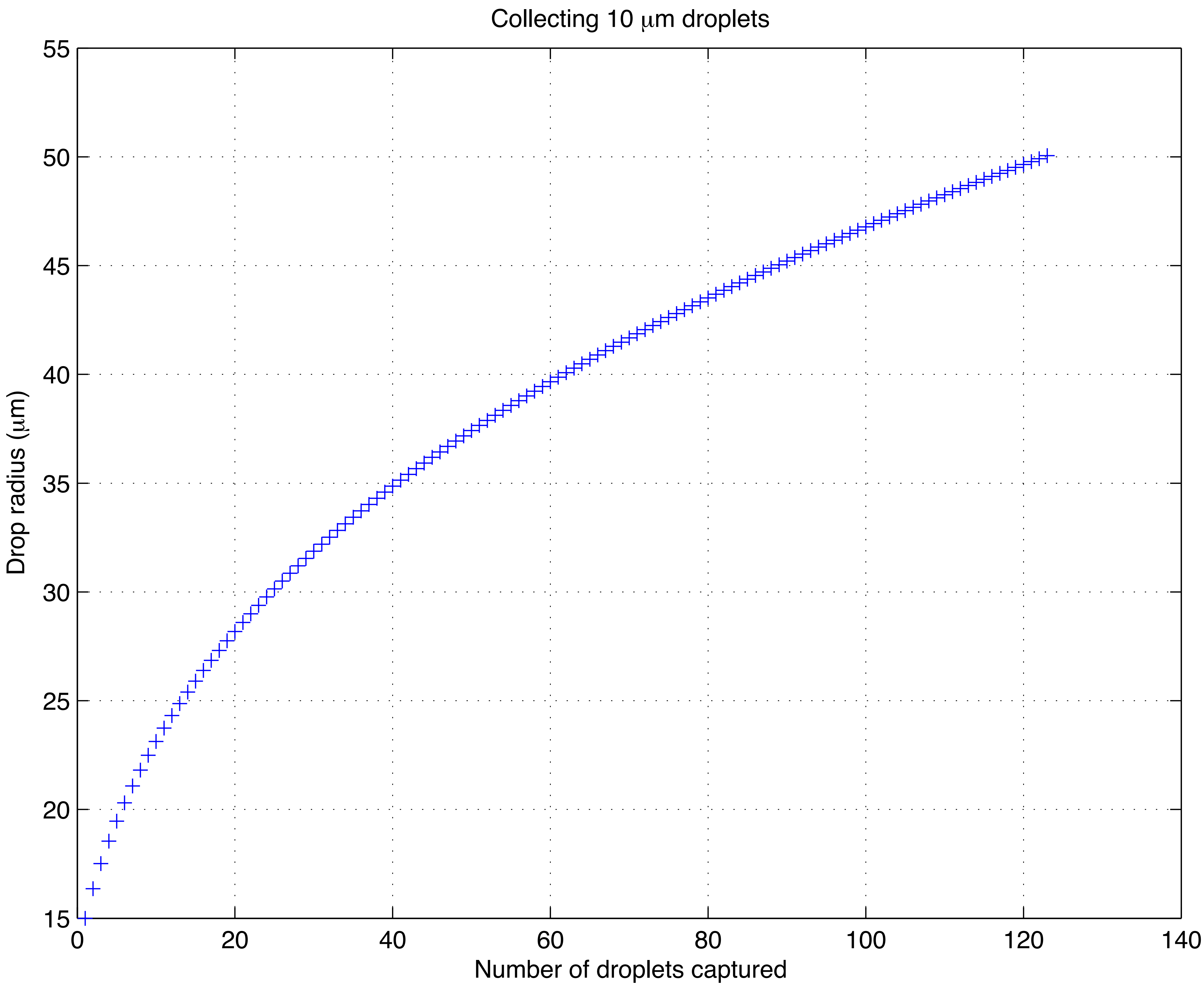
(c) (i) On average it takes about 1.25 h to grow from 20 to 30 μm, and about 0.5 h to grow from 30 to 100 μm.
(ii) The range of times is the same (about 2 h). This range is determined by the variability in the time to grow from 20 to 30 μm. There is relativel little variability in the time to grow from 30 to 100 μm.

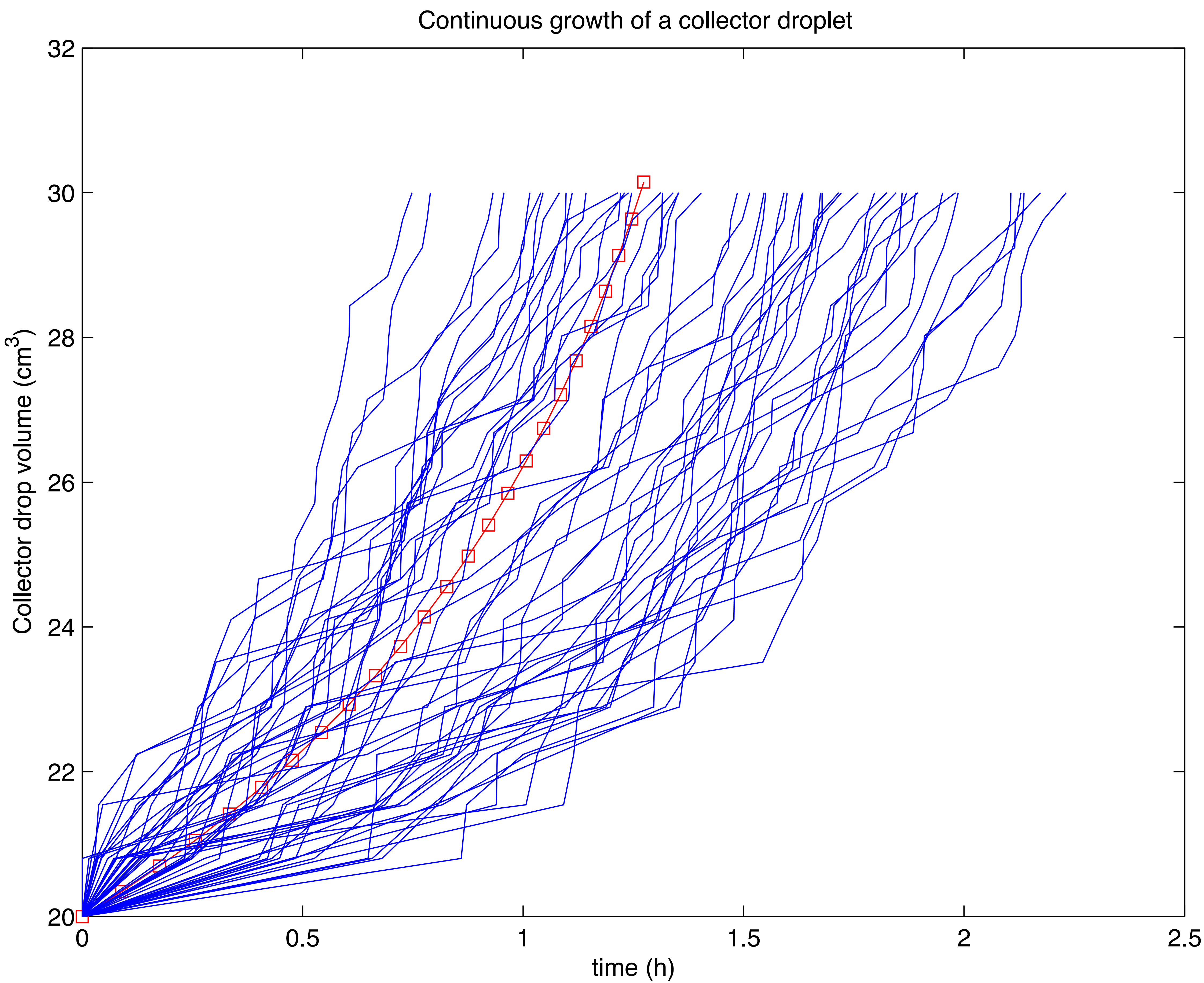


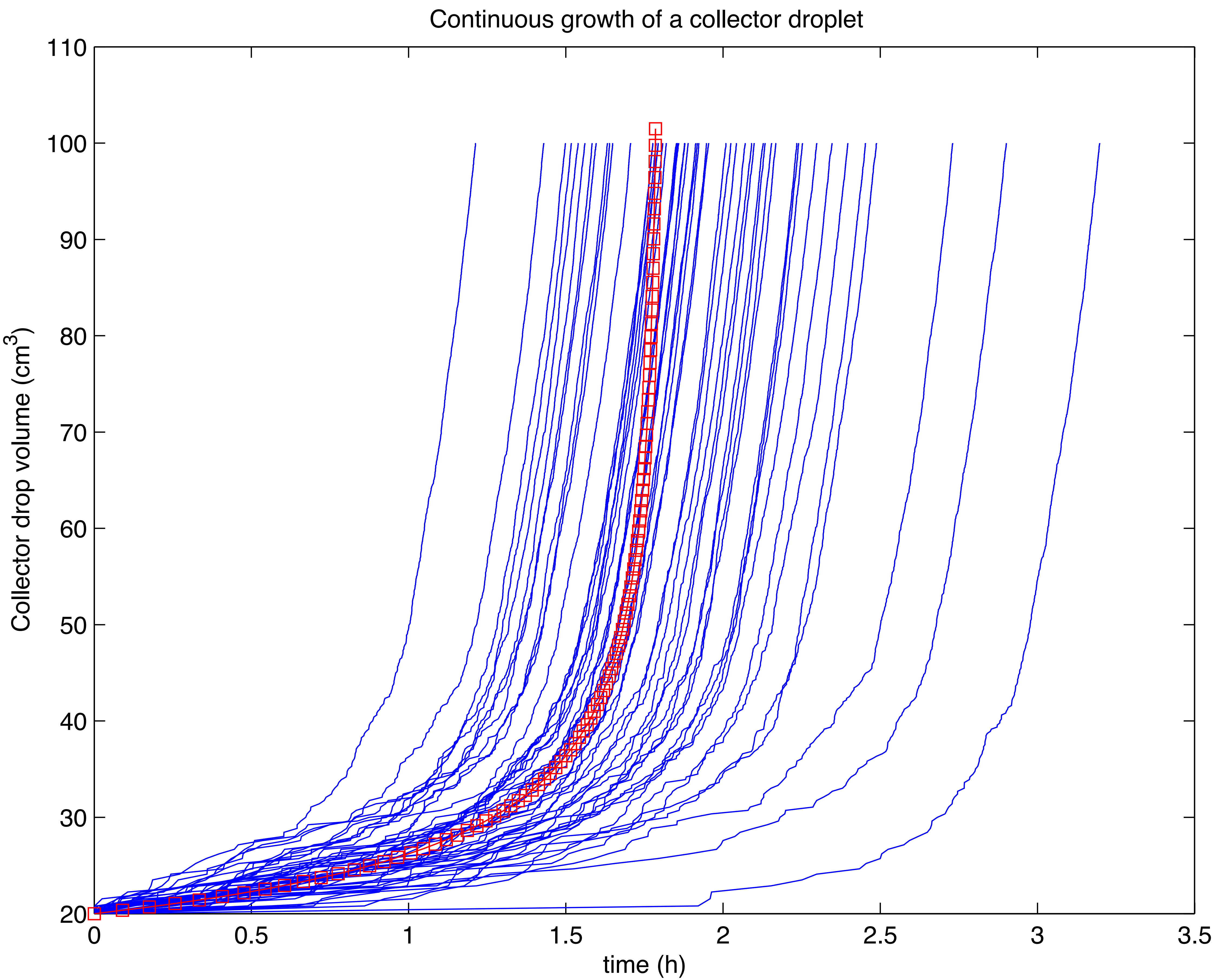
Monte Carlo stochastic coalescence
std dev
mean-s.d.
mean
mean+s.d.
Drop radius (μm)
Avg time (h)
55
50
45
40
35
30
25
20
0
0.5
1
1.5
2
2.5


Monte Carlo stochastic coalescence
Drop radius (μm)
Time (h)
55
50
45
40
35
30
25
20
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5

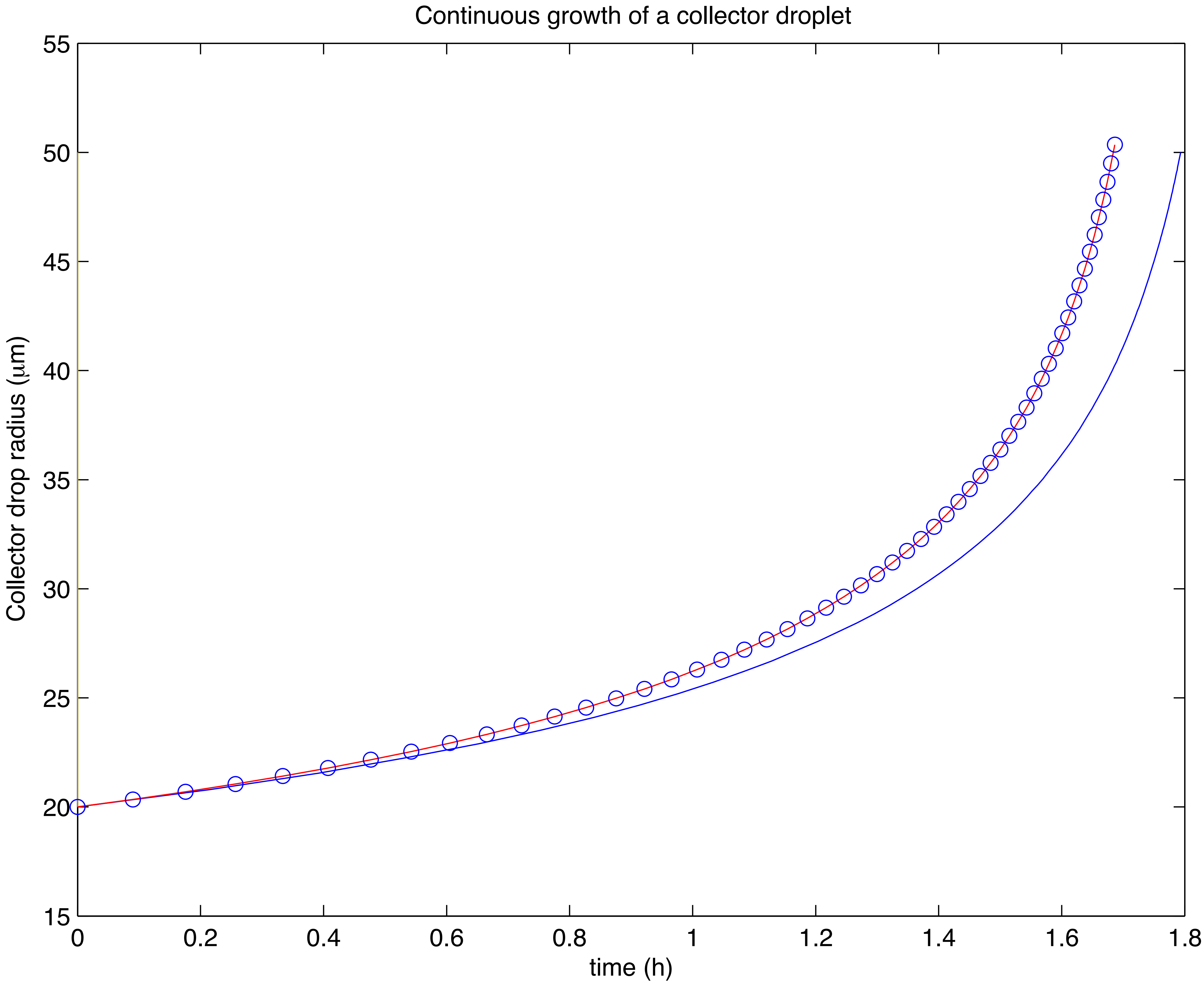

Line 1
100
Line 2
90
10
9
9
Line 3
81
18
1

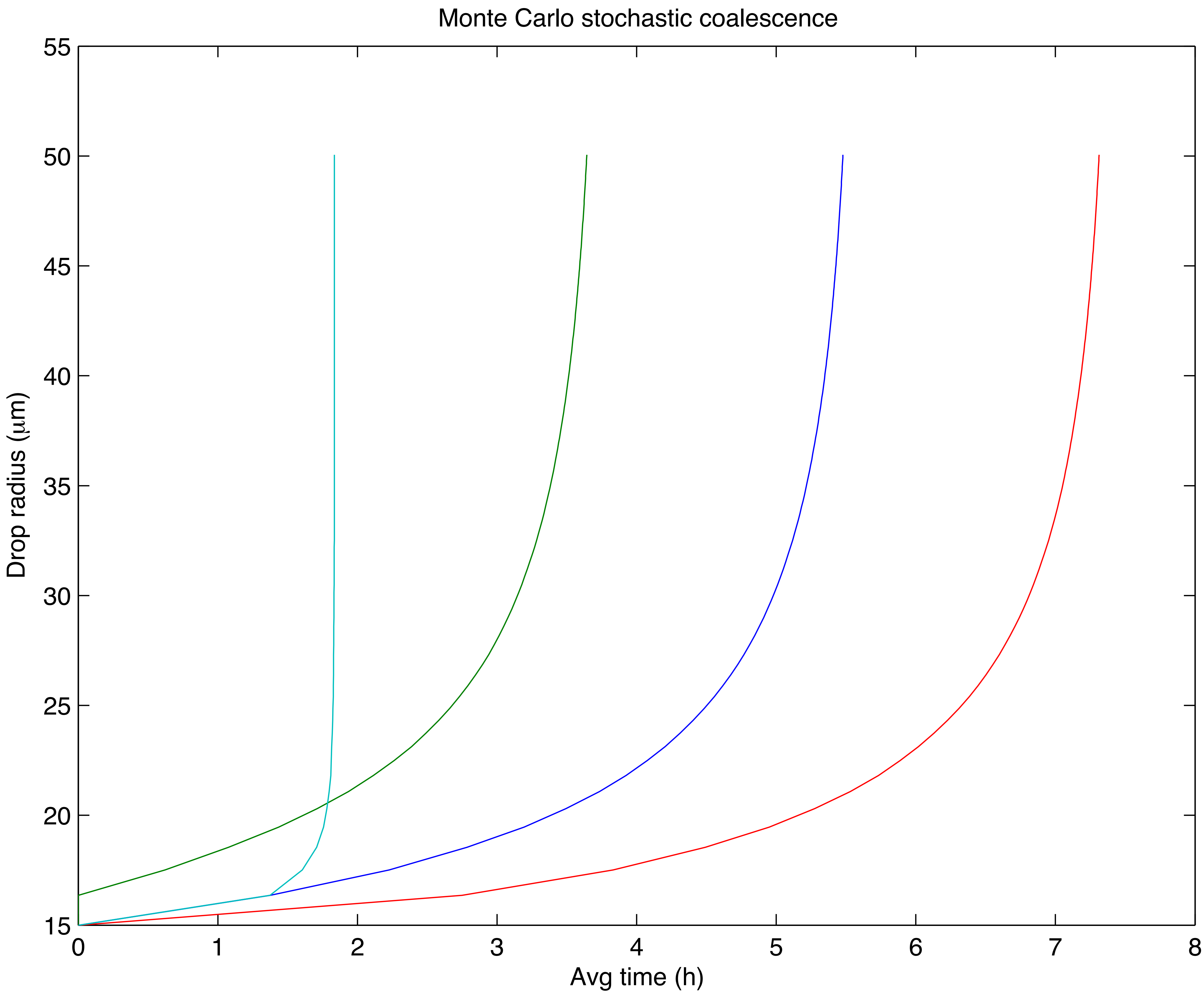

Collecting 10 μm droplets
Drop radius (μm)
Number of droplets captured
55
50
45
40
35
30
25
20
15
0
20
40
60
80
100
120
140


Continuous growth of a collector droplet
Collector drop volume (cm^3)
time (h)
32
30
28
26
24
22
20
0
0.5
1
1.5
2
2.5

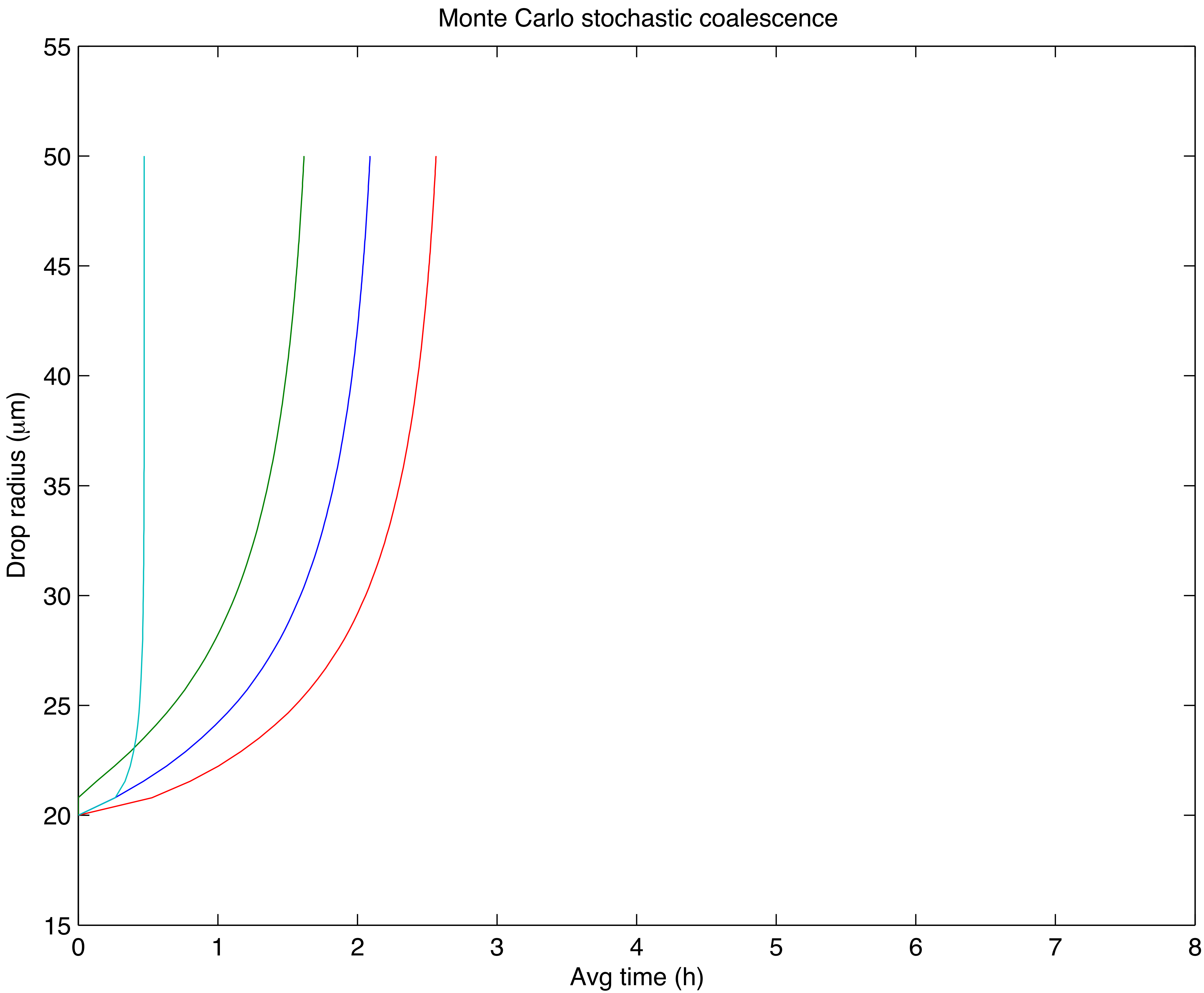


Continuous growth of a collector droplet
Collector drop volume (cm3)
time (h)
110
100
90
80
70
60
50
40
30
20
0
0.5
1
1.5
2
2.5
3
3.5

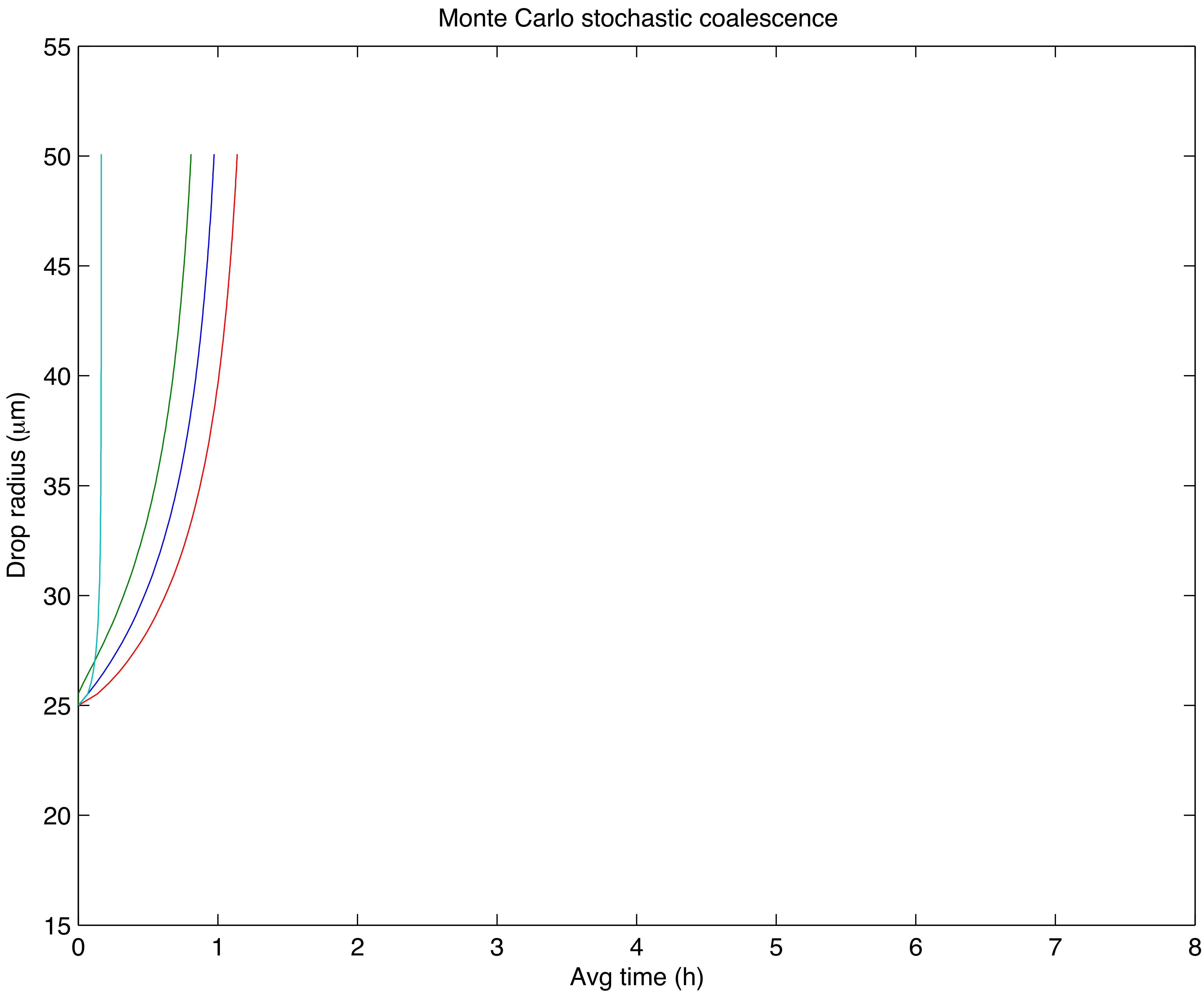

Continuous growth of a collector droplet
Collector drop radius (μm)
time (h)
55
50
45
40
35
30
25
20
15
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8

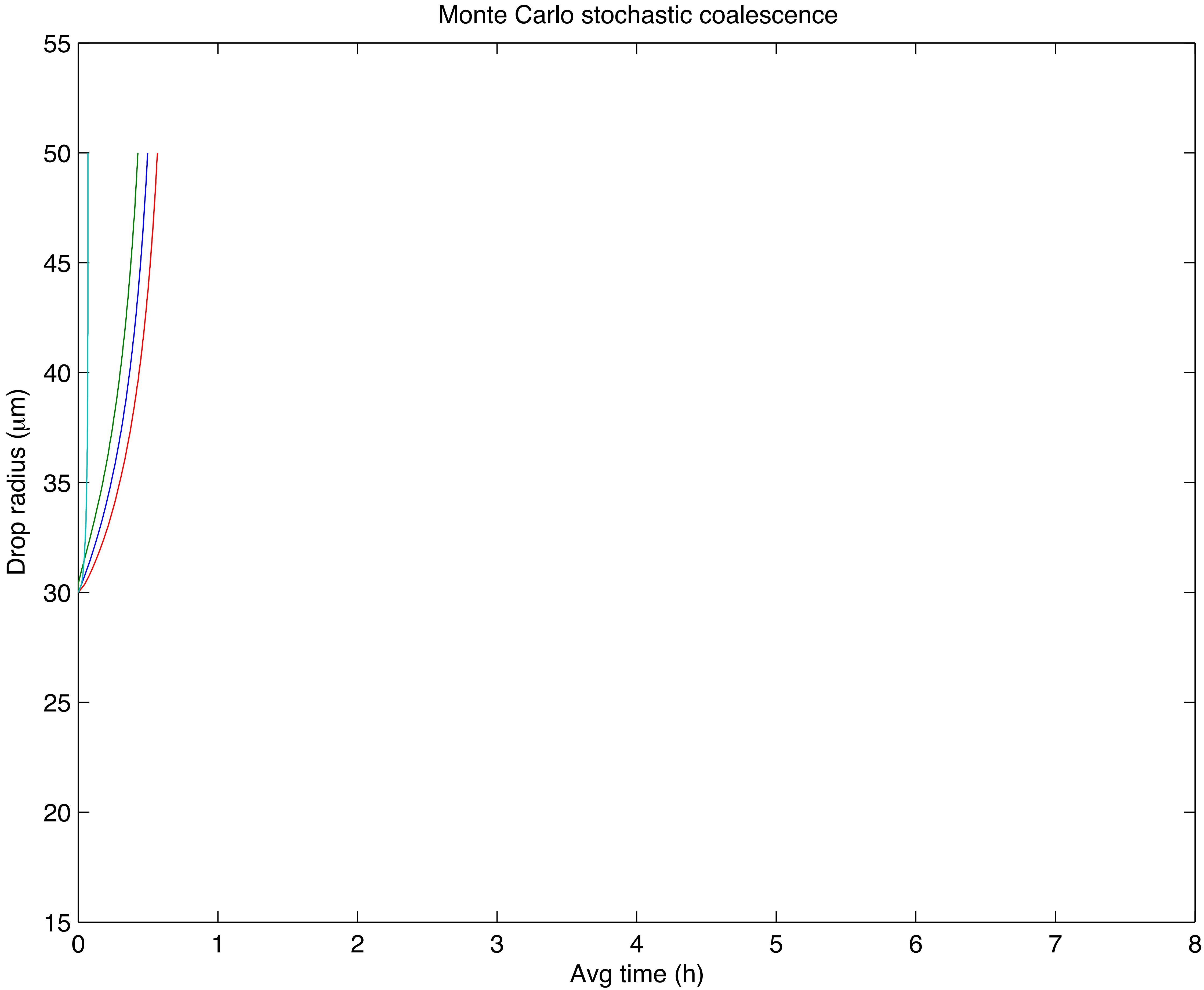


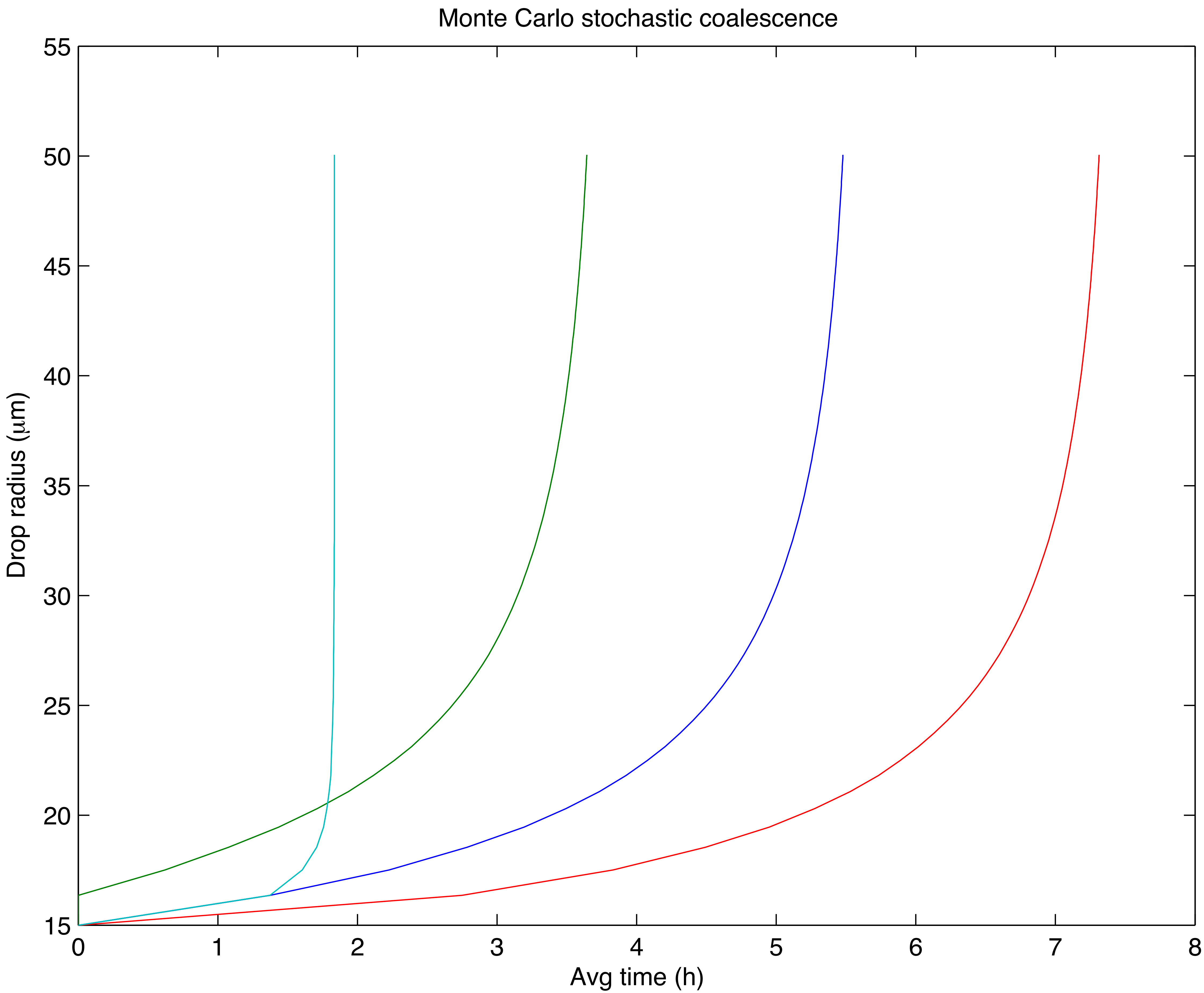
Monte Carlo stochastic coalescence
Drop radius (μm)
Avg time (h)
55
50
45
40
35
30
25
20
15
0
1
2
3
4
5
6
7
8

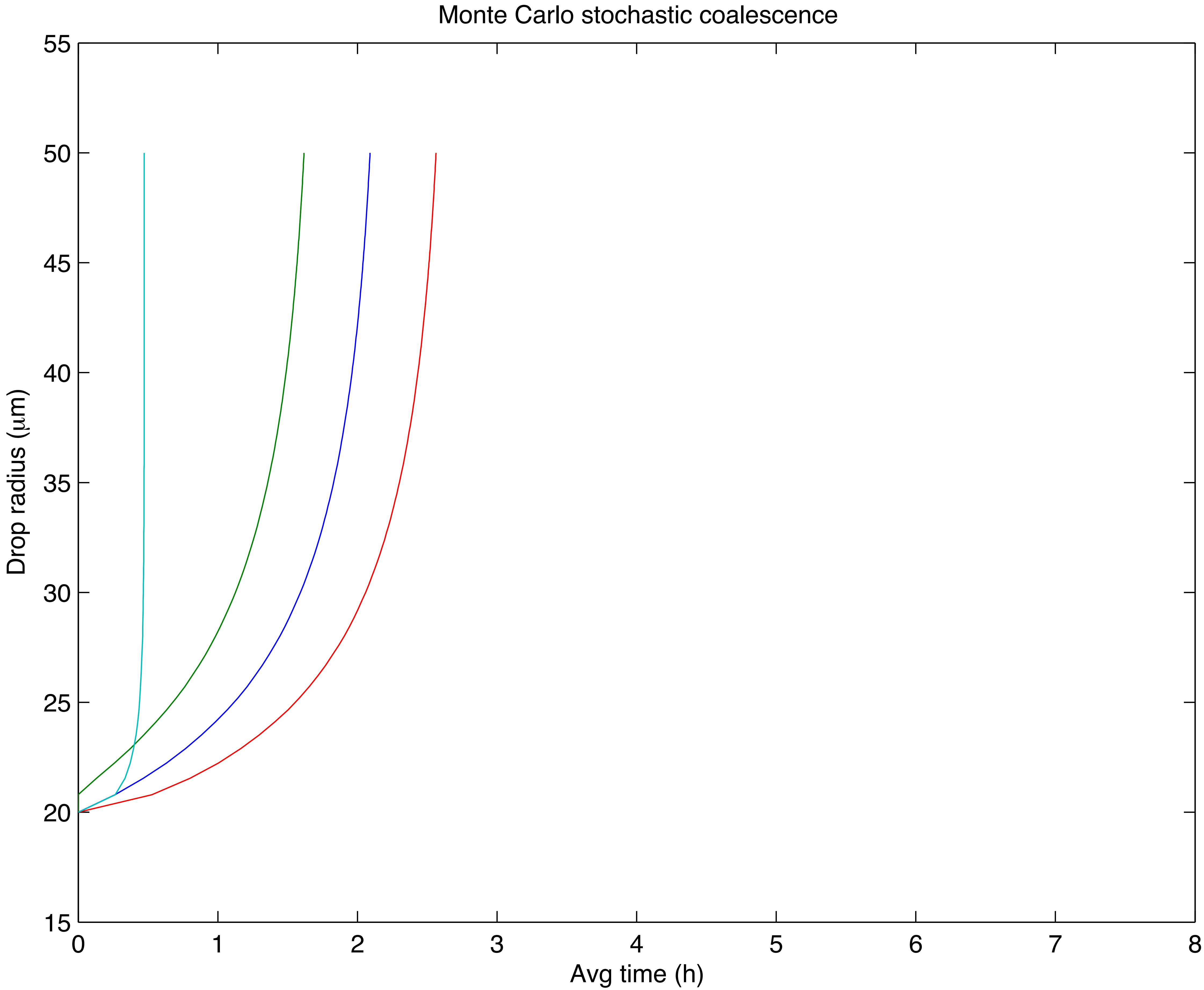


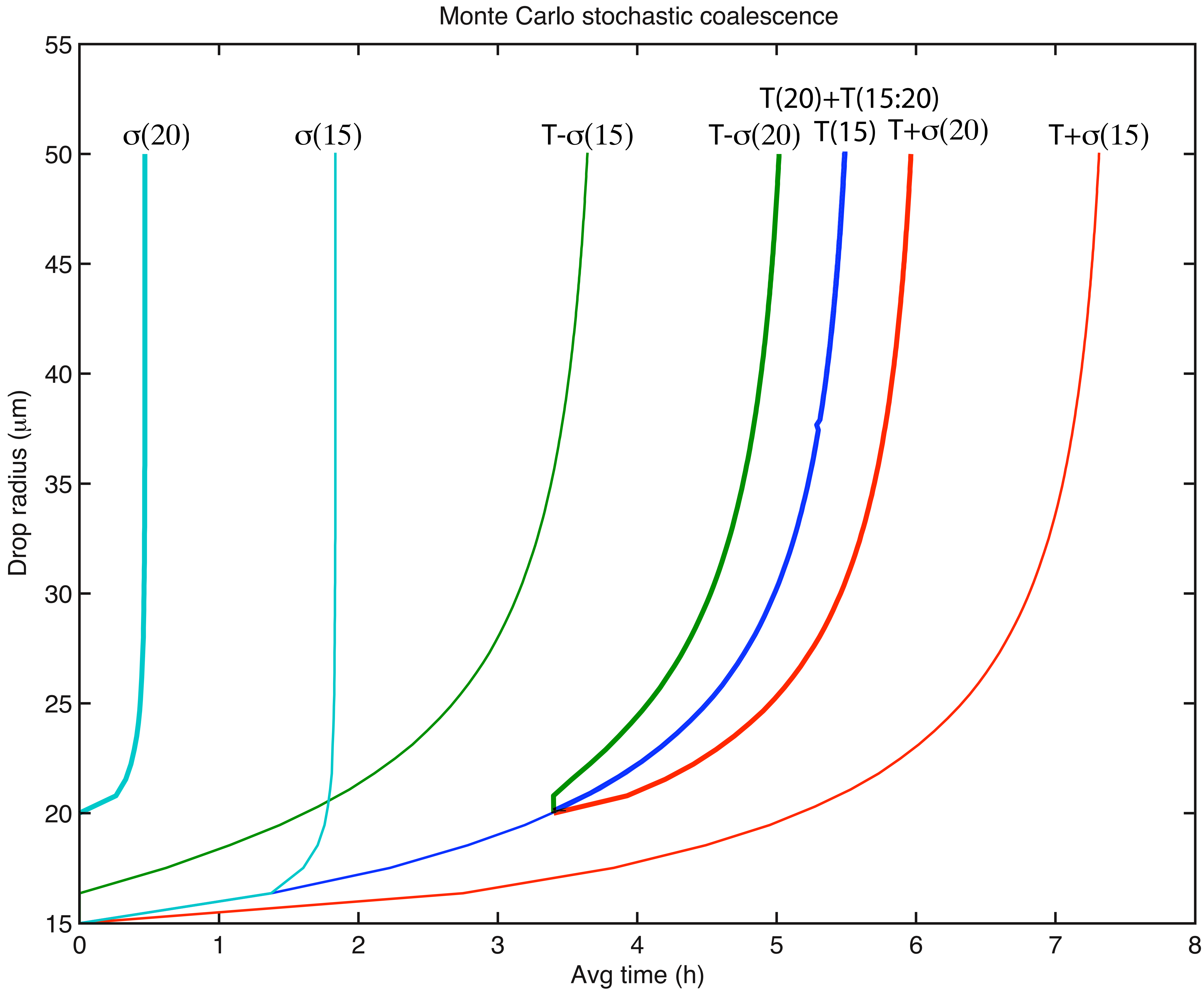
Monte Carlo stochastic coalescence
Drop radius (μm)
Avg time (h)
55
50
45
40
35
30
25
20
15
0
1
2
3
4
5
6
7
8


Monte Carlo stochastic coalescence
Drop radius (μm)
Avg time (h)
55
50
45
40
35
30
25
20
15
0
1
2
3
4
5
6
7
8


Monte Carlo stochastic coalescence
Drop radius (μm)
Avg time (h)
55
50
45
40
35
30
25
20
15
0
1
2
3
4
5
6
7
8


Monte Carlo stochastic coalescence
Drop radius (μm)
Avg time (h)
55
50
45
40
35
30
25
20
15
0
1
2
3
4
5
6
7
8


Monte Carlo stochastic coalescence
Drop radius (μm)
Avg time (h)
55
50
45
40
35
30
25
20
15
0
1
2
3
4
5
6
7
8



Monte Carlo stochastic coalescence
Drop radius (μm)
Avg time (h)
σ(20)
σ(15)
T-σ(15)
T-σ(20)
T(20)+T(15:20)
T(15)
T+σ(20)
T+σ(15)
55
50
45
40
35
30
25
20
15
0
1
2
3
4
5
6
7
8

6. For the conditions described, calculate the rates of growth (dR/dt) due to (a) condensation and (b) collection (of smaller droplets that are 10 μm in radius), for $R = 10$, 20, 30, and 40 μm, and plot dR/dt versus R for each process. Estimate the droplet radius for which the rates of growth are equal.

   The supersaturation is 0.2 percent, T is 10°C, p is 800 hPa, and the liquid water content of the smaller droplets is 1 g m^{-3}.

   For condensation growth, neglect curvature and solute effects. For the given T and p, the growth parameter $1/[F_k + F_d] = 100$ μm^2 s^{-1}.

   For coalescence growth, assume that $R \gg 10\mu$m, that the collection efficiencies are 0, 0.17, 0.37, and 0.55, for $R = 10$, 20, 30, and 40 μm, respectively, and that Stokes' Law is applicable so that the droplet fall speed is $u(R) = k_1 R^2$, with $k_1 = 1.2 \times 10^6$ cm^{-1}s^{-1}.

6. For the conditions described, calculate the rates of growth (dR/dt) due to (a) condensation and (b) collection (of smaller droplets that are 10 μm in radius), for $R = 10$, 20, 30, and 40 μm, and plot dR/dt versus R for each process. Estimate the droplet radius for which the rates of growth are equal.

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For coalescence growth, assume that $R \gg 10\mu$m, that the collection efficiencies are 0, 0.17, 0.37, and 0.55, for $R = 10$, 20, 30, and 40 μm, respectively, and that Stokes' Law is applicable so that the droplet fall speed is $u(R) = k_1 R^2$, with $k_1 = 1.2 \times 10^6$ cm^{-1}s^{-1}.

Please provide the two growth equations.

Answer:

For condensation growth:

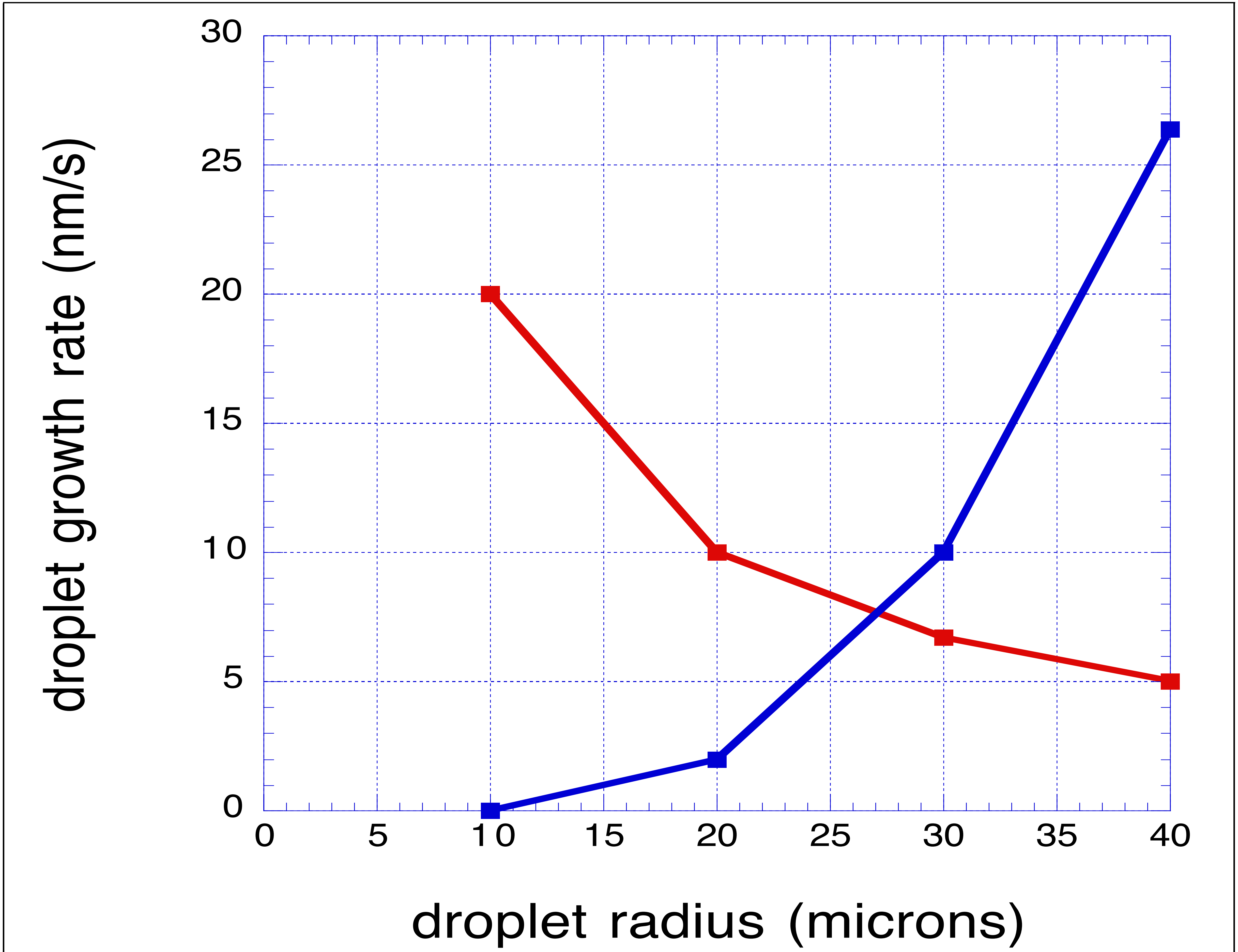
$$r\frac{dr}{dt} = \frac{S-1}{[F_k + F_d]}.$$

For coalescence growth:

$$\frac{dr}{dt} = \frac{EM}{4\rho_L}u(r),$$

where M is the LWC and ρ_L is the density of liquid water.

The table below lists dr/dt for condensation and collection in units of nm s^{-1}. The rates are equal at about 27 μm.



droplet growth rate (nm/s)
30
25
20
15
10
5
0
0
5
10
15
20
25
30
35
40
droplet radius (microns)