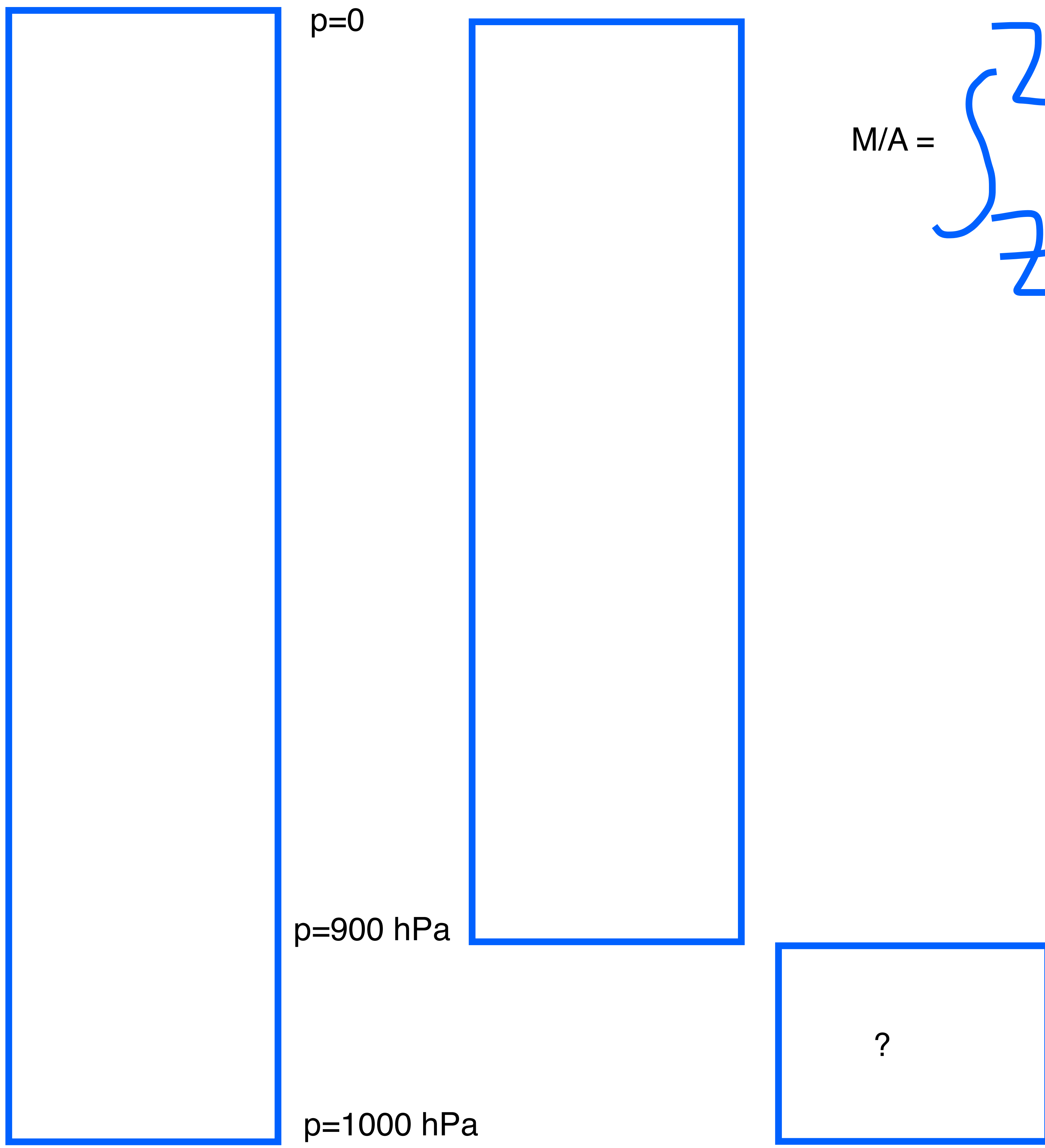
1. What is the mass per unit area of a column of the atmosphere extending from the surface, where $p = 1000$ hPa, to where $p = 900$ hPa?



$$M/A = \int_{z_1}^{z_2} \rho \, dz$$

1. What is the mass per unit area of a column of the atmosphere extending from the surface, where $p = 1000$ hPa, to where $p = 900$ hPa?

Solution: The mass M of a column of air above a level exerts a downwards force equal to Mg at that level, where g is the acceleration of gravity. Because pressure is force per unit area, the pressure p at a given level is equal to Mg/A, where M/A is the mass per unit area.

Therefore, at $p = 1000$ hPa

$$\frac{M(1000)}{A} = \frac{p(1000)}{g} = \frac{1000 \times 100 \text{ Pa}}{9.8 \text{ m s}^{-2}} = 10204 \text{ kg m}^{-2}.$$

Pa = F/A =nt/m^2 = kg m/s^2/m^2 =
kg/m/s^2
Pa/g = kg/m s^2 /(m/s^2) = kg/m^2

1. What is the mass per unit area of a column of the atmosphere extending from the surface, where $p = 1000$ hPa, to where $p = 900$ hPa?

Solution: The mass M of a column of air above a level exerts a downwards force equal to Mg at that level, where g is the acceleration of gravity. Because pressure is force per unit area, the pressure p at a given level is equal to Mg/A, where M/A is the mass per unit area.

Therefore, at $p = 1000$ hPa

$$\frac{M(1000)}{A} = \frac{p(1000)}{g} = \frac{1000 \times 100 \text{ Pa}}{9.8 \text{ m s}^{-2}} = 10204 \text{ kg m}^{-2}.$$

At $p = 900$ hPa

$$\frac{M(900)}{A} = \frac{p(900)}{g} = \frac{900 \times 100 \text{ Pa}}{9.8 \text{ m s}^{-2}} = 9184 \text{ kg m}^{-2}.$$

1. What is the mass per unit area of a column of the atmosphere extending from the surface, where $p = 1000$ hPa, to where $p = 900$ hPa?

Solution: The mass M of a column of air above a level exerts a downwards force equal to Mg at that level, where g is the acceleration of gravity. Because pressure is force per unit area, the pressure p at a given level is equal to Mg/A, where M/A is the mass per unit area.

Therefore, at $p = 1000$ hPa

$$\frac{M(1000)}{A} = \frac{p(1000)}{g} = \frac{1000 \times 100 \text{ Pa}}{9.8 \text{ m s}^{-2}} = 10204 \text{ kg m}^{-2}.$$

At $p = 900$ hPa

$$\frac{M(900)}{A} = \frac{p(900)}{g} = \frac{900 \times 100 \text{ Pa}}{9.8 \text{ m s}^{-2}} = 9184 \text{ kg m}^{-2}.$$

Therefore, the mass per unit area of the layer between $p = 1000$ hPa and $p = 900$ hPa is

$$\frac{M(1000)}{A} - \frac{M(900)}{A} = \frac{p(1000) - p(900)}{g} = \frac{100 \times 100 \text{ Pa}}{9.8 \text{ m s}^{-2}} = 1020 \text{ kg m}^{-2}.$$

Heat capacity

The ratio of the energy ΔH supplied to a body by heating to its corresponding temperature change ΔT is called the *heat capacity* C of the body:

$$C = \frac{\Delta H}{\Delta T}.$$

Heat capacity

The ratio of the energy ΔH supplied to a body by heating to its corresponding temperature change ΔT is called the *heat capacity* C of the body:

$$C = \frac{\Delta H}{\Delta T}.$$

The heat capacity C per unit mass M of a body, called the *specific heat capacity* c, is

$$c = \frac{C}{M} = \frac{\Delta H}{M \Delta T}. \tag{1}$$

Heat capacity

The ratio of the energy ΔH supplied to a body by heating to its corresponding temperature change ΔT is called the *heat capacity* C of the body:

$$C = \frac{\Delta H}{\Delta T}.$$

The heat capacity C per unit mass M of a body, called the *specific heat capacity* c, is

$$c = \frac{C}{M} = \frac{\Delta H}{M\Delta T}. \quad (1)$$

The specific heat capacity c is characteristic of the material of which the body is composed and the conditions under which the heating occurs.

Heat capacity

The ratio of the energy ΔH supplied to a body by heating to its corresponding temperature change ΔT is called the *heat capacity* C of the body:

$$C = \frac{\Delta H}{\Delta T}.$$

The heat capacity C per unit mass M of a body, called the *specific heat capacity* c, is

$$c = \frac{C}{M} = \frac{\Delta H}{M \Delta T}. \tag{1}$$

The specific heat capacity c is characteristic of the material of which the body is composed and the conditions under which the heating occurs.

In this case, the material is dry air and the heating is isobaric, so $c = c_p = 1004 \text{ J kg}^{-1} \text{ K}^{-1}$.

Solve (1) for the temperature change ΔT, with $c = c_p$:

$$\Delta T = \frac{\Delta H}{c_p M}. \tag{2}$$

2. An insulated room is heated by an electric heater. The room is 3 m high, 4 m wide, and 5 m across. The heat output rate of the heater is 1500 W. The air density, ρ, is 1 kg m^{-3}. After 1000 s, how much has the room warmed?

dH =? (J)
M = ? (kg)
dT = dH / (cp * M)
cp = 1000 J/(kg-K)

2. An insulated room is heated by an electric heater. The room is 3 m high, 4 m wide, and 5 m across. The heat output rate of the heater is 1500 W. The air density, ρ, is 1 kg m^{-3}. After 1000 s, how much has the room warmed?

Solution:

From (2):

$$\Delta T = \frac{\Delta H}{c_p M}.$$

$$M = \rho V = 1 \text{ kg m}^{-3} \times 3 \text{ m} \times 4 \text{ m} \times 5 \text{ m} = 60 \text{ kg}.$$

$$\Delta H = P \times t = 1500 \text{ W} \times 1000 \text{ s} = 1.5 \times 10^6 \text{ J}.$$

2. An insulated room is heated by an electric heater. The room is 3 m high, 4 m wide, and 5 m across. The heat output rate of the heater is 1500 W. The air density, ρ, is 1 kg m^{-3}. After 1000 s, how much has the room warmed?

Solution:

From (2):

$$\Delta T = \frac{\Delta H}{c_p M}.$$

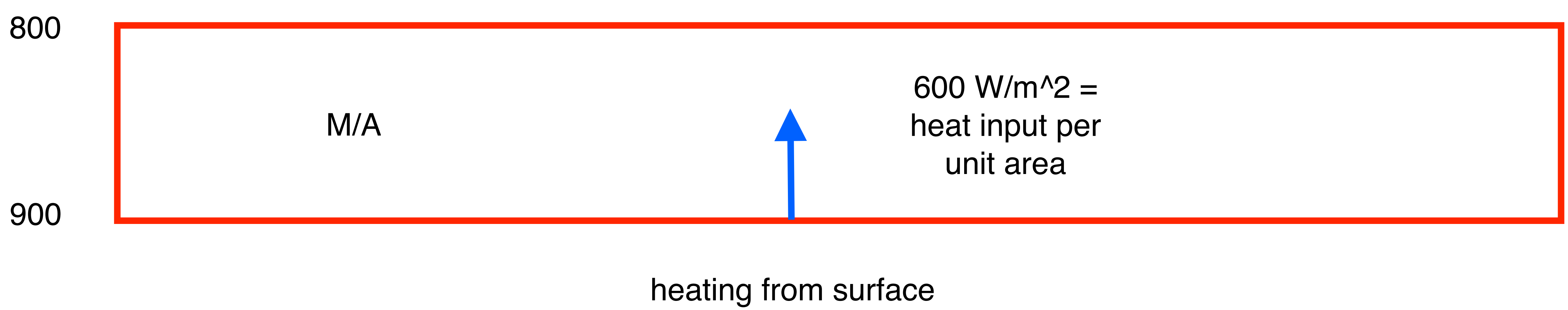
$$M = \rho V = 1 \text{ kg m}^{-3} \times 3 \text{ m} \times 4 \text{ m} \times 5 \text{ m} = 60 \text{ kg}.$$

$$\Delta H = P \times t = 1500 \text{ W} \times 1000 \text{ s} = 1.5 \times 10^6 \text{ J}.$$

$$\Delta T = \frac{\Delta H}{c_p M} = \frac{1.5 \times 10^6 \text{ J}}{1004 \text{ J kg}^{-1} \text{ K}^{-1} \text{ 60 kg}} = 25 \text{ K}.$$

3. On a clear summer afternoon, the atmosphere is receiving an energy flux of 600 W m^{-2} from the underlying land surface, due to conduction and thermal radiation from the ground, which is being warmed by solar radiation.

If this energy flux is uniformly distributed through the layer of air between the surface, at $p_1 = 900$ hPa, and $p_2 = 800$ hPa, how much will the average temperature of this layer change over 3 hours?



3. On a clear summer afternoon, the atmosphere is receiving an energy flux of 600 W m^{-2} from the underlying land surface, due to conduction and thermal radiation from the ground, which is being warmed by solar radiation.

If this energy flux is uniformly distributed through the layer of air between the surface, at $p_1 = 900$ hPa, and $p_2 = 800$ hPa, how much will the average temperature of this layer change over 3 hours?

Solution:

From (2):

$$\Delta T = \frac{\Delta H}{c_p M}.$$

From the information given, we can obtain ΔH per unit area and M per unit area. Therefore we rewrite (2) as

$$\Delta T = \frac{\Delta H/A}{c_p M/A}. \tag{3}$$

$$\Delta T = \frac{\Delta H/A}{c_p M/A}. \tag{3}$$

Calculate

$$\Delta H/A = P/A \times t = 600 \text{ W m}^{-2} \times 3 \text{ h} \times 3600 \text{ s h}^{-1} = 6.48 \text{ MJ m}^{-2},$$

where P/A is the power per unit area (energy flux) and t is the time interval, and

$$M/A = \frac{p_1 - p_2}{g} = \frac{(900 - 800) \times 100 \text{ Pa}}{9.8 \text{ m s}^{-2}} = 1020 \text{ kg m}^{-2}.$$

$$\Delta T = \frac{\Delta H/A}{c_p M/A}. \tag{3}$$

Calculate

$$\Delta H/A = P/A \times t = 600 \text{ W m}^{-2} \times 3 \text{ h} \times 3600 \text{ s h}^{-1} = 6.48 \text{ MJ m}^{-2},$$

where P/A is the power per unit area (energy flux) and t is the time interval, and

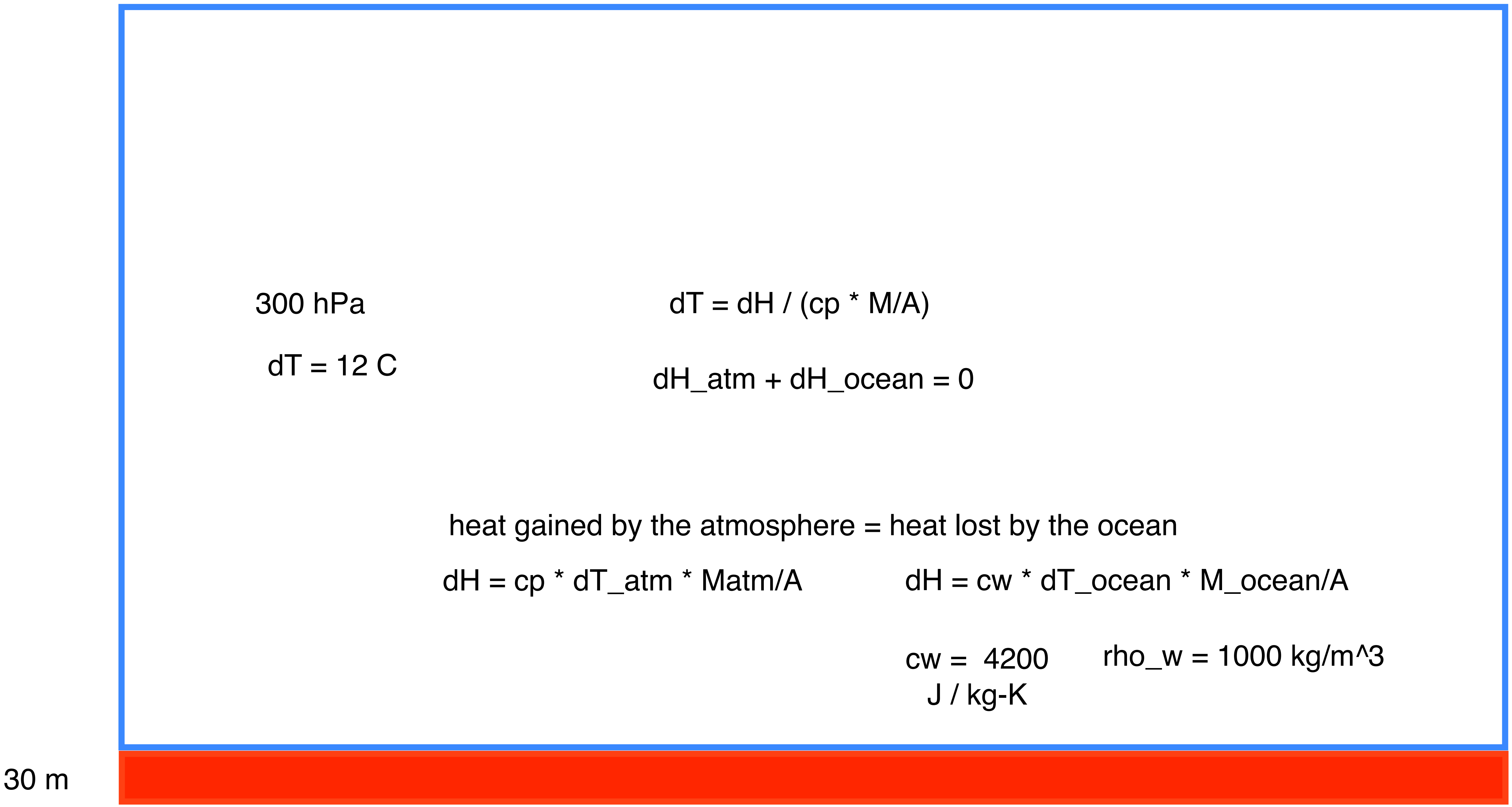
$$M/A = \frac{p_1 - p_2}{g} = \frac{(900 - 800) \times 100 \text{ Pa}}{9.8 \text{ m s}^{-2}} = 1020 \text{ kg m}^{-2}.$$

Then

$$\Delta T = \frac{\Delta H/A}{c_p M/A} = \frac{6.48 \text{ MJ m}^{-2}}{1004 \text{ J kg}^{-1} \text{ K}^{-1} \times 1020 \text{ kg m}^{-2}} = 6.3 \text{ K}.$$

4. During a cold air outbreak from Siberia over the Sea of Japan, the temperature of the lowest 300 hPa of the atmosphere rises by 12° C, due to heating by the upper 30 m of the ocean.

How much does this ocean layer cool as a result?



dT_ocean=?

4. During a cold air outbreak from Siberia over the Sea of Japan, the temperature of the lowest 300 hPa of the atmosphere rises by 12° C, due to heating by the upper 30 m of the ocean.

   How much does this ocean layer cool as a result?

   Solution: In this case, (3) applies to the atmospheric layer:

$$\Delta T_a = \frac{\Delta H_a / A}{c_p M_a / A},$$

   and to the ocean layer:

$$\Delta T_o = \frac{\Delta H_o / A}{c_w M_o / A},$$

   where $c_w = 4186$ J kg^{-1} K^{-1} is the specific heat capacity of water.

4. During a cold air outbreak from Siberia over the Sea of Japan, the temperature of the lowest 300 hPa of the atmosphere rises by 12° C, due to heating by the upper 30 m of the ocean.

How much does this ocean layer cool as a result?

Solution: In this case, (3) applies to the atmospheric layer:

$$\Delta T_a = \frac{\Delta H_a/A}{c_p M_a/A},$$

and to the ocean layer:

$$\Delta T_o = \frac{\Delta H_o/A}{c_w M_o/A},$$

where $c_w = 4186$ J kg^{-1} K^{-1} is the specific heat capacity of water.

The key to solving this problem is recognizing that the energy gained by the atmosphere is equal to that lost by the ocean layer:

$$\Delta H_a/A = -\Delta H_o/A.$$

We can now solve for ΔT_o:

the set of 3 eqs.

$$\Delta T_o = -\Delta T_a \frac{c_p M_a/A}{c_w M_o/A}. \quad (4)$$

We can now solve for ΔT_o:

$$\Delta T_o = -\Delta T_a \frac{c_p M_a/A}{c_w M_o/A}. \tag{4}$$

Calculate the mass per unit of the atmospheric layer:

$$M_a/A = \frac{\Delta p}{g} = \frac{300 \times 100 \text{ Pa}}{9.8 \text{ m s}^{-2}} = 3061 \text{ kg m}^{-2}$$

and of the ocean layer:

$$M_o/A = \rho_w \, \Delta z = 1000 \text{ kg m}^{-3} \times 300 \text{ m} = 30000 \text{ kg m}^{-2},$$

where ρ_w is the density of water.

We can now solve for ΔT_o:

$$\Delta T_o = -\Delta T_a \frac{c_p M_a/A}{c_w M_o/A}. \tag{4}$$

Calculate the mass per unit of the atmospheric layer:

$$M_a/A = \frac{\Delta p}{g} = \frac{300 \times 100 \text{ Pa}}{9.8 \text{ m s}^{-2}} = 3061 \text{ kg m}^{-2}$$

and of the ocean layer:

$$M_o/A = \rho_w \, \Delta z = 1000 \text{ kg m}^{-3} \times 300 \text{ m} = 30000 \text{ kg m}^{-2},$$

where ρ_w is the density of water.

Substitute into (4):

$$\Delta T_o = -12 \text{ K} \times \frac{1004 \text{ J kg}^{-1} \text{ K}^{-1} \times 3061 \text{ kg m}^{-2}}{4186 \text{ J kg}^{-1} \text{ K}^{-1} \times 30000 \text{ kg m}^{-2}} = -0.29 \text{ K}.$$