

Water vapor is an exceptional gas in the earth's atmosphere because it can coexist with liquid water and solid water (ice).

We will assume that water vapor satisfies the equation of state of an ideal gas.

The specific heat capacities of water vapor at low concentrations (partial pressures) may vary appreciably with temperature, so that water vapor does not satisfy the second condition for an ideal gas (i.e., that the specific heat capacities are independent of temperature).

However, we will always be concerned with a mixture of dry air and a small amount of water vapor, so the specific heat capacities of the mixture will not be seriously affected by the variations in the vapor properties.

Moist air is a mixture of dry air and water vapor. Therefore, the **mean molecular weight of moist air** is

$$\frac{1}{\bar{m}} = \frac{1}{M_d + M_v} \left(\frac{M_d}{m_d} + \frac{M_v}{m_v} \right),$$

where subscript d indicates dry air and subscript v indicates water vapor. Expressing \bar{m} in terms of m_d gives

$$\frac{1}{\bar{m}} = \frac{1}{m_d} \frac{M_d}{M_d + M_v} \left(1 + \frac{M_v/M_d}{m_v/m_d} \right).$$

M_v/M_d is the water vapor *mixing ratio*. It is the mass of water vapor per unit mass of dry air. Using $w \equiv M_v/M_d$, we can write the equation above as

$$\frac{1}{\bar{m}} = \frac{1}{m_d} \frac{1 + w/\epsilon}{1 + w},$$

where $\epsilon = m_v/m_d$.

The **equation of state for a mixture of water vapor and dry air** may now be written as (see section 2e)

$$p\alpha = \frac{R^*}{m_d} \left(\frac{1 + w/\epsilon}{1 + w} \right) T.$$

Instead of using a variable gas constant for moist air, we define a new temperature,

$$T_v \equiv \left(\frac{1 + w/\epsilon}{1 + w} \right) T,$$

called the **virtual temperature**. It satisfies the equation of state for dry air,

$$p\alpha = \frac{R^*}{m_d} T_v = RT_v.$$

The **virtual temperature** is the temperature that dry air would have if its pressure and specific volume (or density) were equal to those of a given sample of moist air.

The virtual temperature is always greater than the actual temperature. Using $\epsilon = 0.622$, we may write

$$T_v = \left(\frac{1 + 1.609w}{1 + w} \right) T \approx (1 + 0.61w)T.$$

According to Dalton's law, each gas in a mixture of gases behaves as if it were the only gas present. Thus in a mixture of water vapor and dry air, water vapor obeys the ideal gas law,

$$e\alpha_v = \frac{R^*}{m_v}T,$$

where e is the partial pressure of water vapor, or the **vapor pressure**. Likewise in such a mixture, dry air obeys the ideal gas law,

$$(p - e)\alpha_d = \frac{R^*}{m_d}T,$$

where $p - e$ is the partial pressure of dry air. We can use these relations to express the **mixing ratio** in terms of e and p :

$$w \equiv \frac{M_v}{M_d} = \frac{\rho_v}{\rho_d} = \frac{\alpha_d}{\alpha_v} = \frac{m_v}{m_d} \frac{e}{p - e} \approx \frac{m_v e}{m_d p}.$$

Moisture variables

Vapor pressure, e The pressure exerted by water vapor in a mixture of air and water vapor.

Saturation vapor pressure, $e_s(T)$ The vapor pressure of water that is in equilibrium with a plane surface of water. It is a function of temperature only. Bolton's (1980) formula for $e_s(T)$ is sufficiently accurate for most purposes:

$$e_s(T) = 611.2 \exp\left(\frac{17.67 T_c}{T_c + 243.5}\right),$$

where e_s is in Pa and $T_c \equiv T - T_0$ is temperature in degrees Celsius.

Mixing ratio, w The ratio of the mass of water vapor present to the mass of dry air:

$$w = \frac{\rho_v}{\rho_d} = \frac{m_v}{m_d} \frac{e}{p - e} = \epsilon \frac{e}{p - e} \approx \epsilon \frac{e}{p}.$$

Saturation mixing ratio, $w_s(T, p)$ The mixing ratio for which water vapor is in equilibrium with a plane surface of water. It is a function of temperature and pressure:

$$w_s = \epsilon \frac{e_s(T)}{p - e_s(T)} \approx \epsilon \frac{e_s(T)}{p}.$$

Relative humidity, r The ratio of the actual vapor pressure to the saturation vapor pressure:

$$r = \frac{e}{e_s} \approx \frac{w}{w_s}.$$

Dew-point temperature, T_d The temperature at which moist air becomes just saturated after being cooled during a process in which p and w remain constant. Alternatively, the temperature at which the mixing ratio equals the saturation mixing ratio:

$$w = w_s(T_d, p).$$

The dew-point temperature T_d may be calculated from

$$T_d = \frac{T}{1 - \frac{TR_v}{L_e} \ln(e/e_s)},$$

where $R_v \equiv R^*/m_v = R/\epsilon$ and L_e is the latent heat of evaporation (see section 5.5).

Saturation pressure, p_s Also called the *lifting condensation level* or *LCL*. The pressure at which unsaturated air becomes saturated after a dry adiabatic ascent (expansion).

Saturation temperature, T_s (K) The temperature at the saturation pressure. At $p = p_s, T = T_s$, the mixing ratio becomes the saturation mixing ratio, so

$$w = w_s(T_s, p_s).$$

An approximation for T_s is

$$T_s = \left(\frac{1}{T - 55} - \frac{\log r}{2840} \right)^{-1} + 55.$$

Height of LCL, z_{LCL} (m) This is approximately

$$z_{\text{LCL}} - z \approx \frac{T(z) - T_d(z)}{8}.$$

A more accurate formula is Lawrence's formula:

$$z_{\text{LCL}} - z \approx 100 \left(20 + \frac{T_c(z)}{5} \right) (1 - r(z)).$$

M. G. Lawrence, 2005: The relationship between relative humidity and the dew point temperature in moist air: A simple conversion and applications, *Bull. Am. Meteorol. Soc.*, **86**, 225–233.

Virtual temperature, T_v The temperature at which dry air would have the same density as a sample of moist air, at the same pressure:

$$T_v = \left(\frac{1 + 1.609w}{1 + w} \right) T \approx (1 + 0.61w)T.$$