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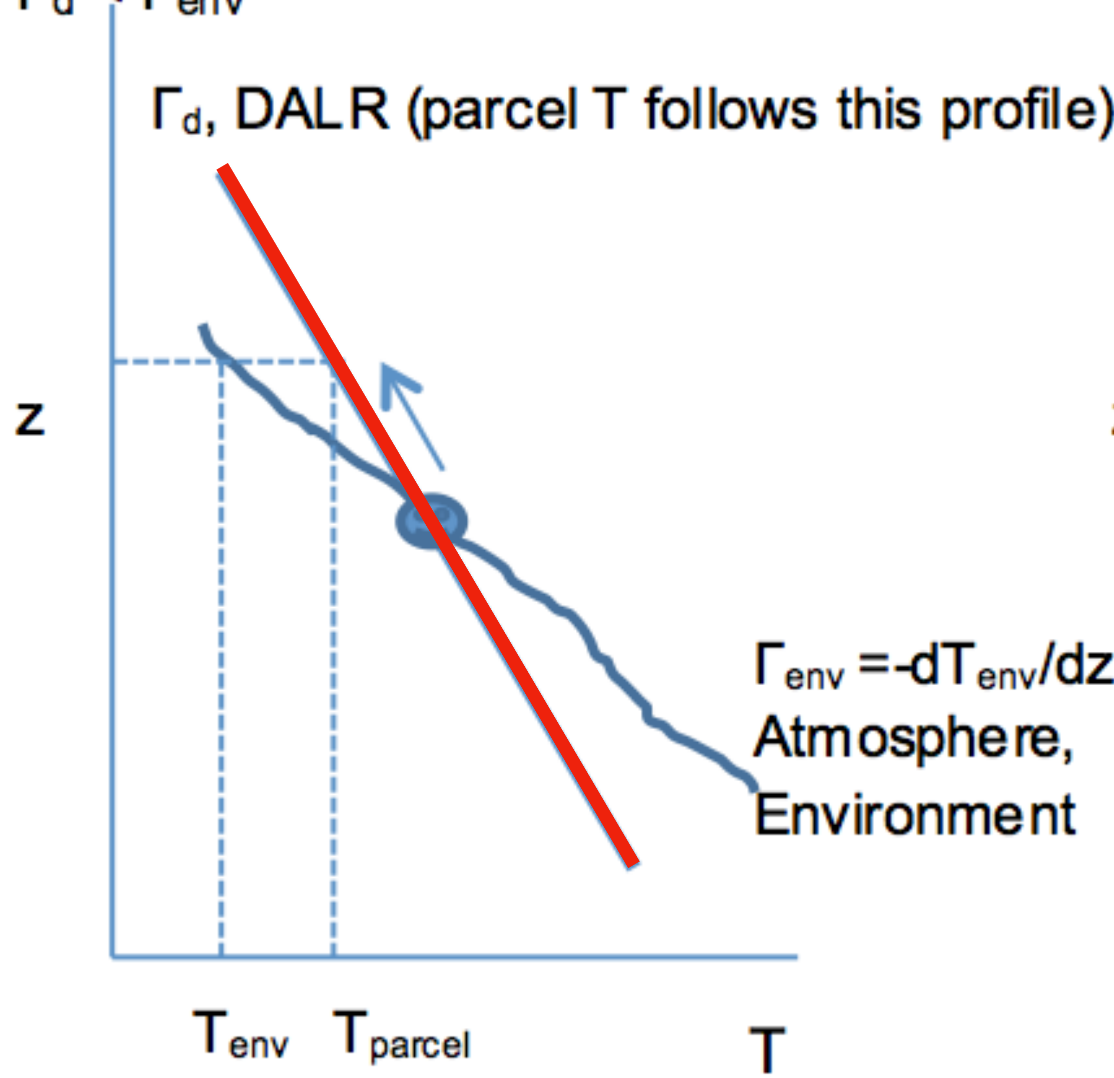
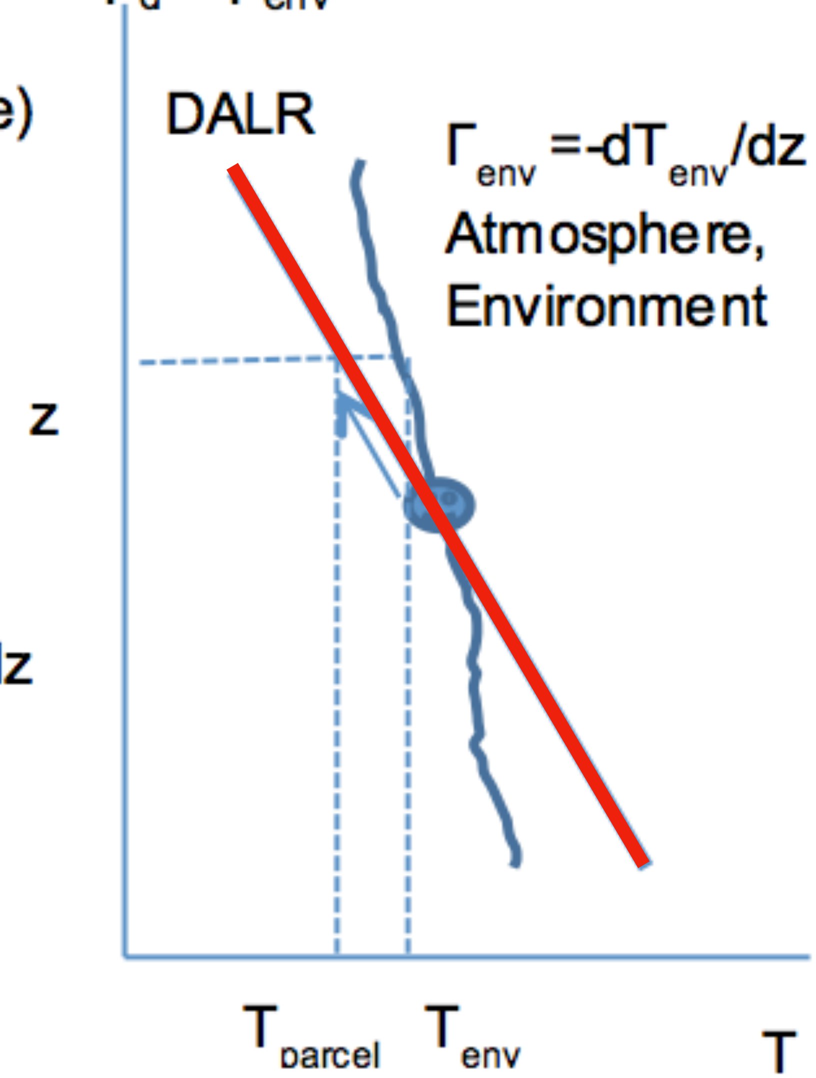
Unstable condition – negative static stability, $\Gamma_d < \Gamma_{env}$



$T_{env} < T_{parcel}$, $\rho_{env} > \rho_{parcel}$,

Parcel continues to rise

Stable condition – positive stability $\Gamma_d > \Gamma_{env}$



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Parcel sinks to original position

Credit: W. Brune

7.2 Stability criteria: The parcel method

If a parcel of air is displaced from its initial position in an atmosphere in hydrostatic balance, the parcel's resulting motion determines the atmospheric *stability* at that location.

- If the parcel moves further away from its initial position, then the atmosphere is *unstable.*
- If the parcel returns towards its initial position, then the atmosphere is *stable.*
- If the parcel does not move, then the atmosphere is *neutral.*

We will use the equation for the *v*ertical acceleration of an air parcel to calculate a displaced parcel's velocity and height as a function of time.

- In the following description, a variable with an overbar is a property of the environment; a variable without an overbar is a property of the parcel.

- Note that in this and the following sections, $w \equiv dz/dt$.

We assume that the environment of the parcel is in hydrostatic equilibrium:

$$\frac{d\bar{w}}{dt} = -g - \bar{\alpha}\frac{d\bar{p}}{dz} = 0.$$

The parcel itself will have a specific volume α and an acceleration dw/dt.

We also assume that the pressure of the parcel is the same as that of its environment so that

$$\frac{dw}{dt} = -g - \alpha\frac{d\bar{p}}{dz}.$$

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$$\frac{dw}{dt} = -g - \alpha\frac{d\bar{p}}{dz}.$$

We use the hydrostatic equation to eliminate $d\bar{p}/dz$ from this equation:

$$\frac{dw}{dt} = g\frac{\alpha - \bar{\alpha}}{\bar{\alpha}}.$$

The right hand side is called the *buoyancy* and is due to the difference in specific volume (or density) between the parcel and the environment.

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Substitute for α and $\bar{\alpha}$ from the equation of state for dry air, $p\alpha = RT$, (for simplicity, we ignore the effect of water vapor on density) to obtain

$$\frac{dw}{dt} = g\frac{T - \bar{T}}{\bar{T}}. \tag{41}$$

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Let $z = 0$ denote the parcel's equilibrium location. Then at $z = 0$, $T = \bar{T}$, and $dw/dt = 0$.

Assume that the temperature in the environment varies linearly with height. Then the temperature at any height z in the environment is

$$\bar{T}(z) = \bar{T}(0) - \gamma z,$$

where $\gamma = -d\bar{T}/dz$ is the *environmental lapse rate.* Similarly, the parcel temperature at any height z is

$$T(z) = T(0) - \Gamma_d z = \bar{T}(0) - \Gamma z,$$

where $\Gamma_= - dT/dz$ is the *parcel lapse rate*

$$\frac{dw}{dt} = g\frac{T - \bar{T}}{\bar{T}}.$$

When these expressions are substituted in Eq. (41), we obtain

$$\frac{dw}{dt} = \frac{g}{\bar{T}(0) - \gamma z}(\gamma - \Gamma)z \approx \frac{g}{\bar{T}(0)}(\gamma - \Gamma)z = bz. \tag{42}$$

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Eq. (42) describes how w changes with time. By definition,

$$\frac{dz}{dt} = w. \tag{43}$$

Eqs. (42) and (43) are coupled linear differential equations which are easy to solve analytically for $z(t)$.

They can also be combined into a single second-order differential equation:

$$\frac{d^2z}{dt^2} = bz.$$

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If the coefficient b is negative (i.e., $\gamma < \Gamma$), the solution $z(t)$ is sinusoidal. The parcel will oscillate about its original position with period

$$\tau = \frac{2\pi}{\sqrt{-b}} = \frac{2\pi}{\sqrt{\frac{g}{\bar{T}(0)}(\Gamma - \gamma)}}. \tag{44}$$

The atmosphere is thus *stable* in this case.

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If the coefficient b is positive (i.e., $\gamma > \Gamma$), the solution $z(t)$ is exponentially increasing, and the parcel moves away from its equilibrium level once displaced. The atmosphere is *unstable.*

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If the coefficient b is positive (i.e., $\gamma > \Gamma$), the solution $z(t)$ is exponentially increasing, and the parcel moves away from its equilibrium level once displaced. The atmosphere is *unstable.*

If $b = 0$ (i.e., $\gamma = \Gamma$), the atmosphere is *neutral.*

Recall that for unsaturated air, $\Gamma = \Gamma_d$, while for saturated air, $\Gamma = \Gamma_s$.

Because $\Gamma_s < \Gamma_d$, the atmosphere can have five responses with respect to parcel displacement in an atmospheric layer of lapse rate γ.

The atmosphere is said to be:

absolutely stable if	$\gamma < \Gamma_s$,
saturated neutral if	$\gamma = \Gamma_s$,
conditionally unstable if	$\Gamma_s < \gamma < \Gamma_d$,
dry neutral if	$\gamma = \Gamma_d$,
absolutely unstable if	$\gamma > \Gamma_d$.

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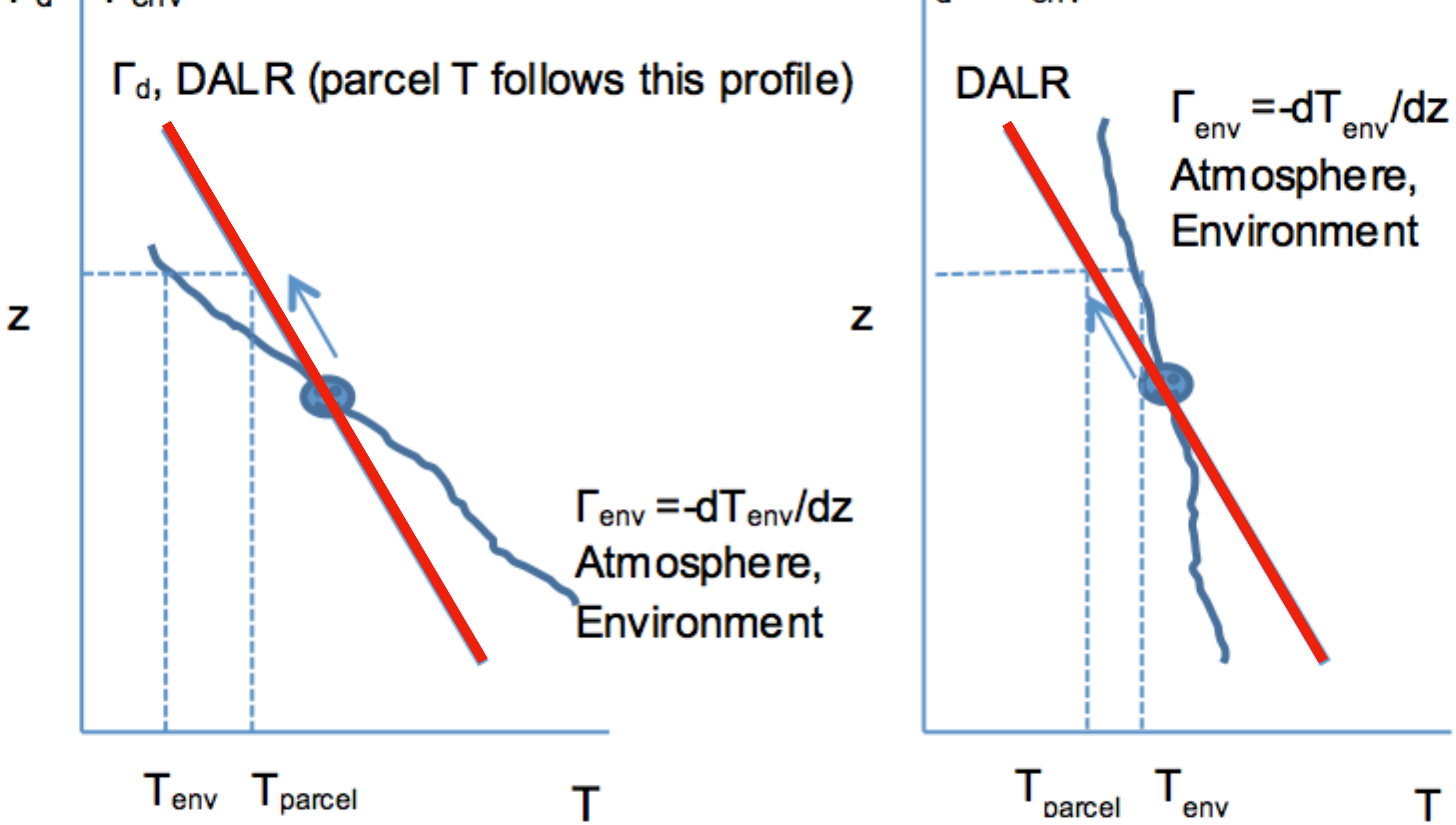

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