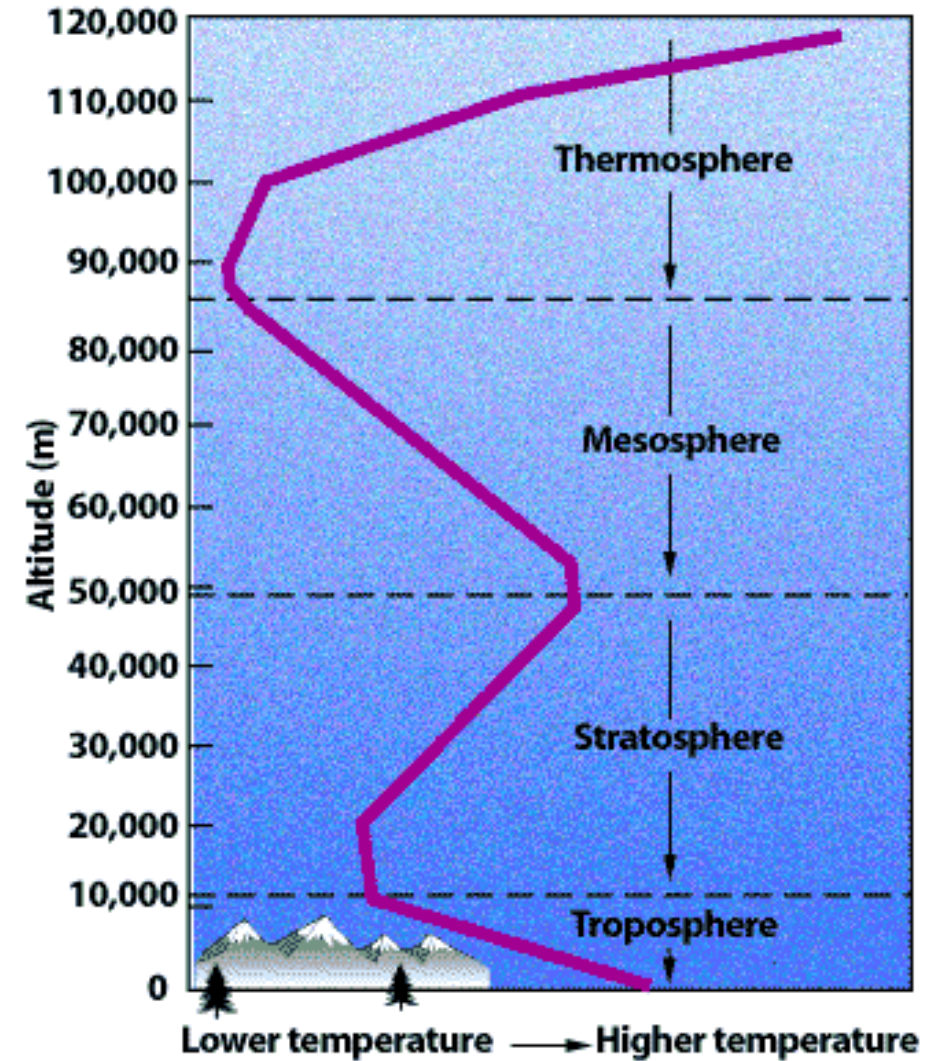


ATMOS_5300 (Fall2020)
Atmospheric Thermodynamics

Dry And Moist Adiabatic Lapse Rate

Xia Li
September 25, 2020

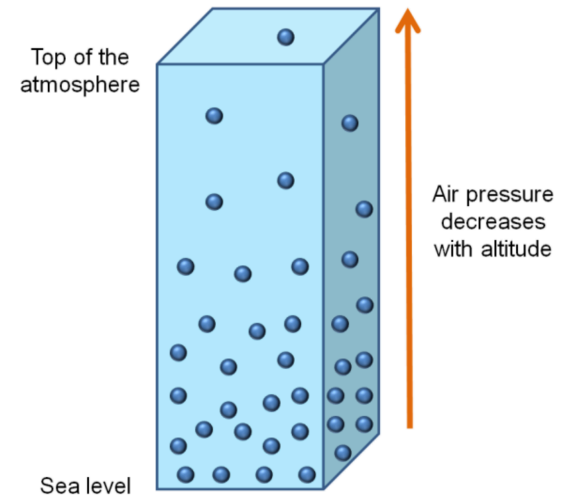


Lapse rate is the rate of temperature ~~change~~ ^{decrease} with a ~~change~~ ^{increase} in altitude

Three types of lapse rate:

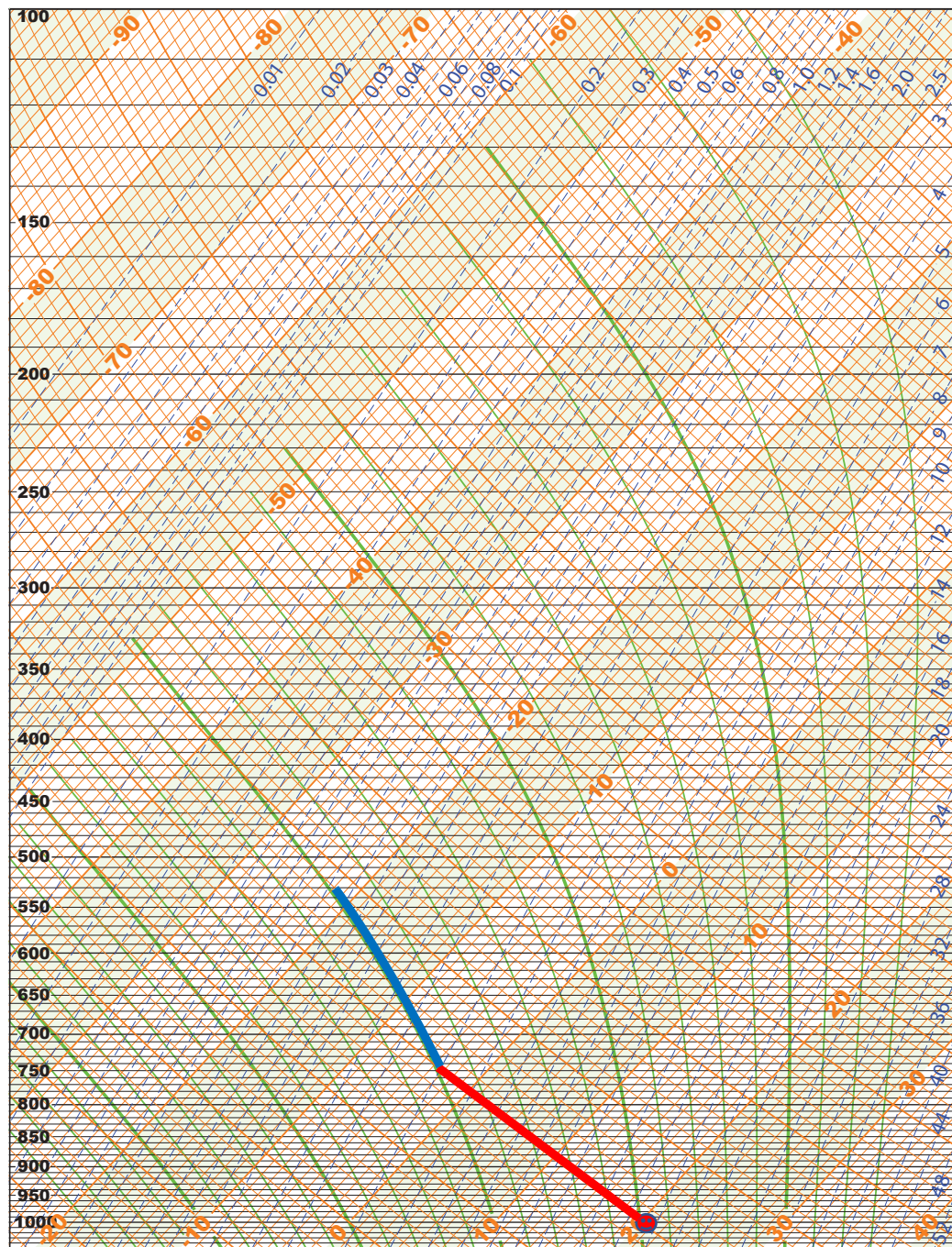
- *Dry adiabatic lapse rate*
unsaturated air ($RH < 100\%$)
- *Moist (saturation) adiabatic lapse rate*
- *Environmental (ambient) lapse rate*

Useful criterion for atmospheric stability



Ascending: **expands**
Descending: **compresses**

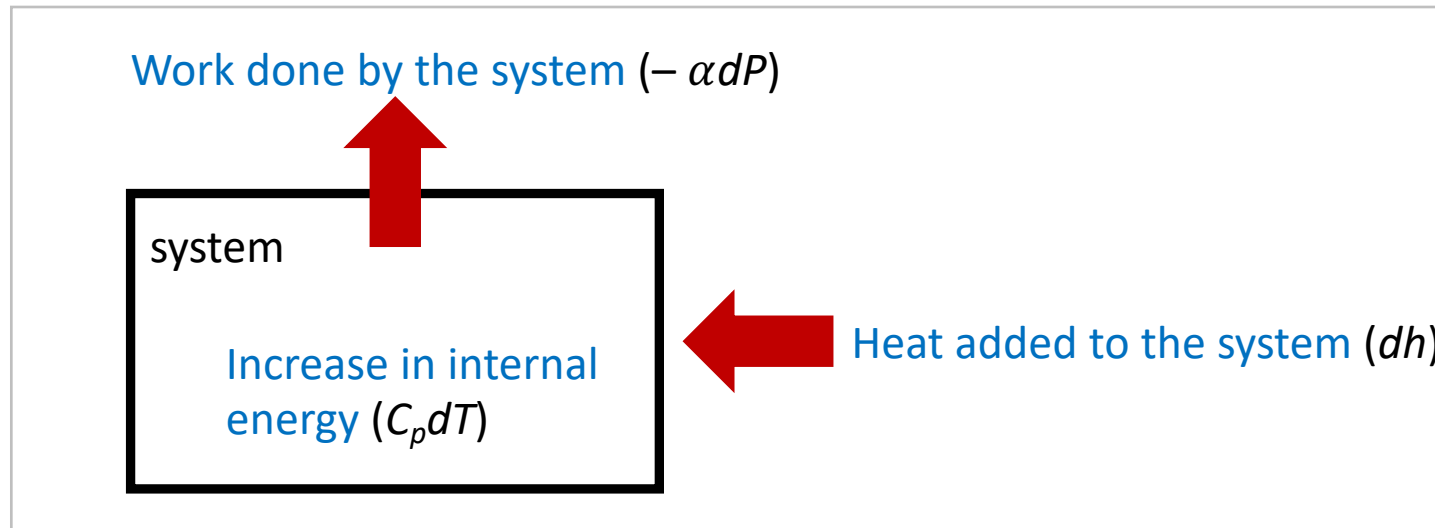
Check the isothermal lines
on Skew T- log P



Where should we start?

The fundamental law: **the first law of thermodynamics**

- Principle of conservation of energy



- One form is:

$$dh = c_p dT - \alpha dp$$

Theoretical dry adiabatic lapse rate

The first law of thermodynamics for a parcel is

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Our goal: $\frac{dT}{dz} = \dots$

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x : parcel properties

\bar{x} : environmental properties

First, let's try to convert p to z

Assumptions:

[1] We assume that the pressure of the parcel (p) is always the same as that of the environment (\bar{p}): $p = \bar{p}$

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\bar{x} : environmental properties

First, let's try to convert p to z

Assumptions:

[1] We assume that the pressure of the parcel (p) is always the same as that of the environment (\bar{p}): $p = \bar{p}$

[2] The environment is in hydrostatic equilibrium:

$$dp = d\bar{p} = -\frac{1}{\bar{\alpha}} g dz \quad (1)$$

Theoretical dry adiabatic lapse rate

Use Eq.1 in the first law to obtain

$$dh = c_p dT + \frac{\alpha}{\bar{\alpha}} g dz$$

Theoretical dry adiabatic lapse rate

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To further simply, considering that typically, temperature fluctuations in the horizontal are $O(1K)$, at most 10 K, so that

$$P\alpha = RT_v$$

$$\bar{P}\bar{\alpha} = R\bar{T}_v$$

$$\frac{\alpha}{\bar{\alpha}} = \frac{T_v}{\bar{T}_v} \approx 1$$

Now we have a good approximation the first law of thermodynamics for a parcel in a hydrostatic environment:

$$dh = c_p dT + g dz \quad (2)$$

Theoretical dry adiabatic lapse rate

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For unsaturated dry air, no heat is added or lost during the adiabatic process,

$$dh = 0$$

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Therefore, the dry adiabatic form of Eq. (2):

$$0 = c_p dT + g dz$$

The lapse rate for a dry-adiabatic process is thus,

$$\Gamma_d \equiv -\frac{dT}{dz} = \frac{g}{c_p}$$

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$$g = 9.81 \text{ m s}^{-2}$$

$$c_p = 1004 \text{ J kg}^{-1} \text{ K}^{-1}$$

Lets do the calculation!

Theoretical dry adiabatic lapse rate

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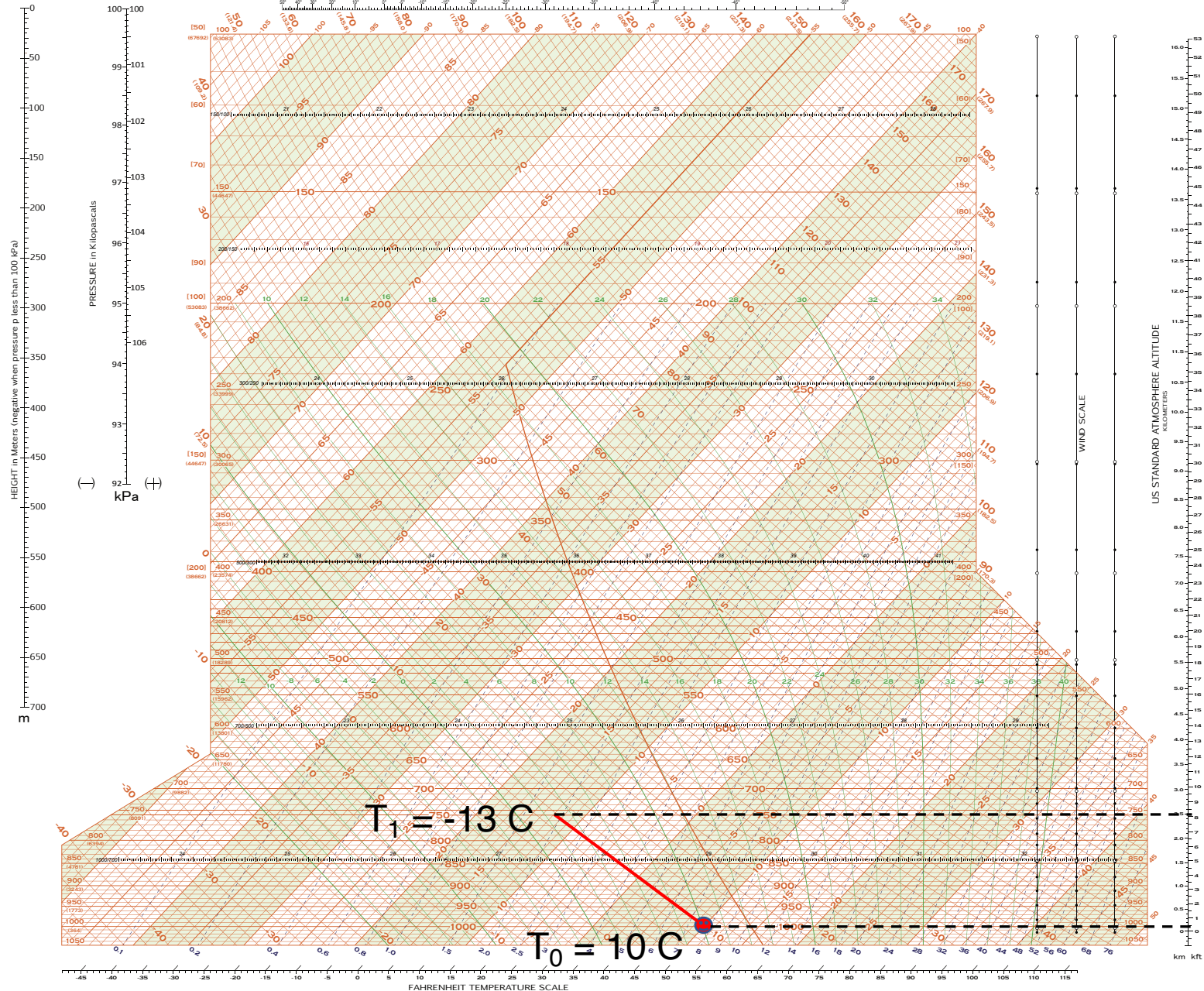
kft

Γ_d for dry air is **9.76 K km⁻¹** (5.4 F ~~feet~~^{kft}⁻¹)

Let's check it at the Skew T-^{log}~~op~~ P diagram

SKEW T ADIABATIC DIAGRAM

TEMPERATURE IN DEGREES CELSIUS



EXPLANATION

ISOBARS are straight horizontal brown lines. The heights in feet of the pressure surfaces in the U.S. Standard atmosphere are in parenthesis () below the pressure values on the left.

ISOTHERMS (°C) are the straight, equidistant brown lines running diagonally upward from the left to right.

DRY ADIABATS are the slightly curved brown lines that intersect the 1000 mb. isobar at intervals of 2°C, and run diagonally upward from right to left. The Dry Adiabats for the hatched portion of the pressure range are labeled with two (2) values. (See below.)

SATURATED ADIABATS are the curved green lines that intersect the 1000 mb. isobar at intervals of 2°C, diverging upward and tending to become parallel to the dry adiabats.

SATURATION MIXING RATIO (in gm. per kg.) is represented by dashed green lines. Their values appear at the bottom of diagram.

THICKNESS (in hundreds of geopotential meters) of the layers between the levels 1000, 700, 500, 300, 200, 150, and 100 mb. is represented by numbers and a graduation along the middle of each layer. The thicknesses are obtained from the virtual temperature curve by the equal-area method, using any straight line as a dividing line.

HEIGHT in geopotential meters above mean sea level, or station level, of the 100 kPa surface is obtained from the nomogram in the upper left-hand corner by drawing a straight line from the point on the temperature scale (°C) through the point ρ (mean sea level or station pressure) on the pressure scale, and reading the height on the height scale.

U.S. STANDARD ATMOSPHERE SOUNDING is indicated by a thick brown line.

The saturated adiabats and scoleths of saturation mixing ratio are computed by use of vapor pressure over a plane water surface at all temperatures.

Extensions of chart to 50 mb. have been accomplished by overlap with pressure indicated in brackets, [200] at 400 mb., and [50] at 100 mb. Dry adiabats for the overlap are labeled in parenthesis ().

APPROXIMATE VIRTUAL TEMPERATURE may be obtained from the formula $T_v = T + \frac{p_0 - p}{p_0} \frac{1000}{\alpha}$ where T_v is virtual temperature in °C, T is free air temperature in °C, and α is the mixing ratio in g/kg. For purposes of thickness computation, use the mean temperature of the layer for T , and use the mean mixing ratio of the layer for α .

Black dots along wind scale lines indicate the levels for which wind data is reported and plotted. The open circles indicate the mandatory pressure level at which wind data is also entered.

| SKEW T ANALYSIS | | | |
|----------------------------------|-----------|-----------|------|
| TIME | DATE | STATION | NO. |
| AIRMASS ANALYSIS | | | |
| WIND | DIR. | VEG. | W. |
| TEMP. | REL. HUM. | WIND | W. |
| WIND | DIR. | VEG. | W. |
| TEMP. | REL. HUM. | WIND | W. |
| SIGNIFICANT WIND | | | |
| DIR. | VEG. | W. | W. |
| TEMP. | REL. HUM. | WIND | W. |
| LEVELS OF SHEAR | | | |
| TO | FROM | STABILITY | WIND |
| TO | FROM | STABILITY | WIND |
| CLOUDS | | | |
| TYPE | BASE | TOP | WIND |
| TYPE | BASE | TOP | WIND |
| TEMPERATURES | | | |
| MAX. | MIN. | WIND | W. |
| MAX. | MIN. | WIND | W. |
| STANDARD QUANTIFICATION AT TOP | | | |
| STATION ID | DATE | TIME | TIME |
| COMPARISON OF LOW LEVEL WINDS AT | | | |
| STATION | DATE | TIME | TIME |

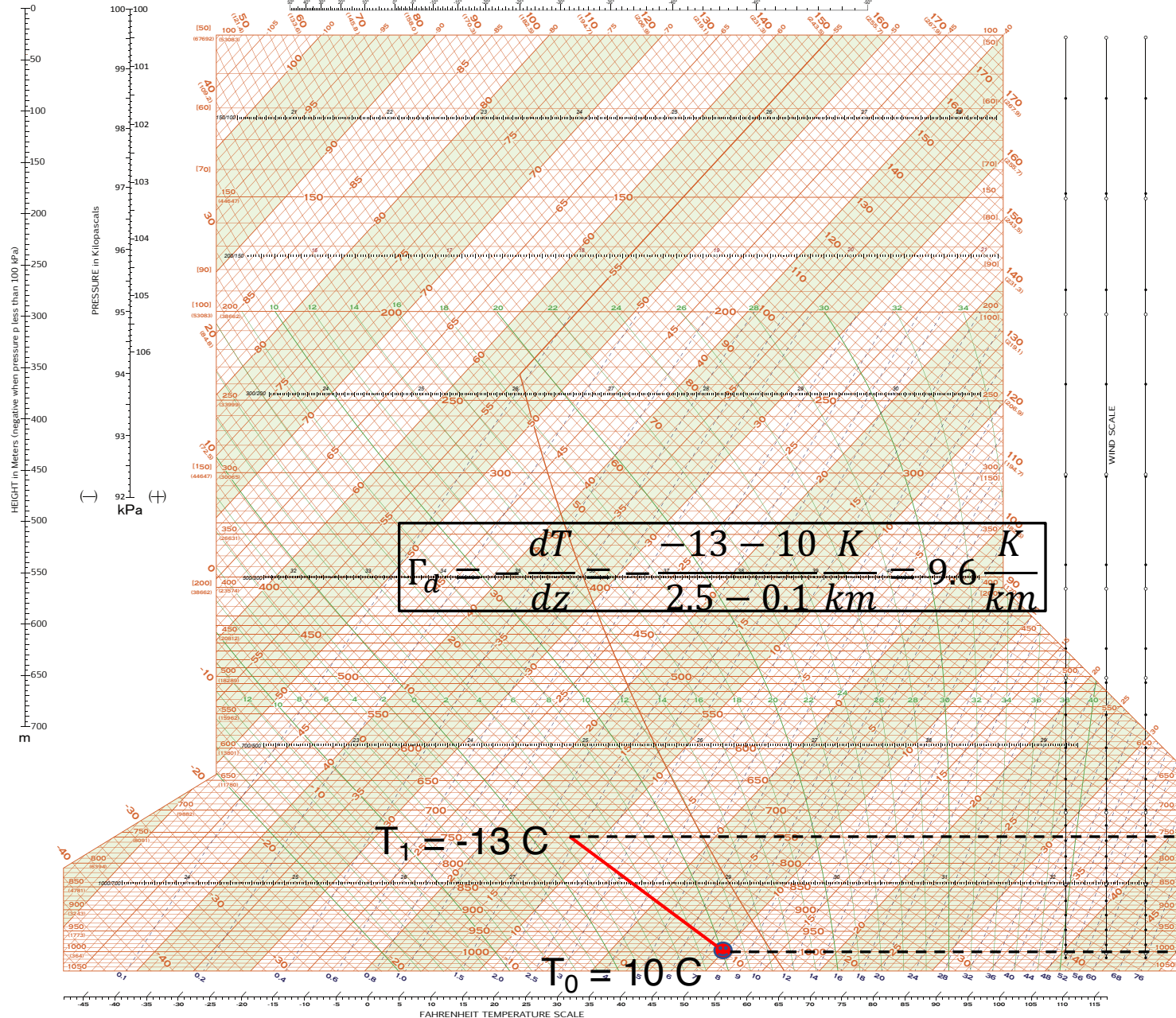
$Z_1 = 2.5 \text{ km}$

$Z_0 = 0.1 \text{ km}$

| | | |
|------------|------------|------------|
| STATION ID | STATION ID | STATION ID |
| DATE | DATE | DATE |
| TIME (GMT) | TIME (GMT) | TIME (GMT) |

SKREW T ADIABATIC DIAGRAM

TEMPERATURE IN DEGREES CELSIUS



$$\Gamma_d = \frac{dT}{dz} = \frac{-13 - 10 \text{ K}}{2.5 - 0.1 \text{ km}} = 9.6 \frac{\text{K}}{\text{km}}$$

$T_1 = -13 \text{ C}$

$T_0 = 10 \text{ C}$

$Z_1 = 2.5 \text{ km}$

$Z_0 = 0.1 \text{ km}$

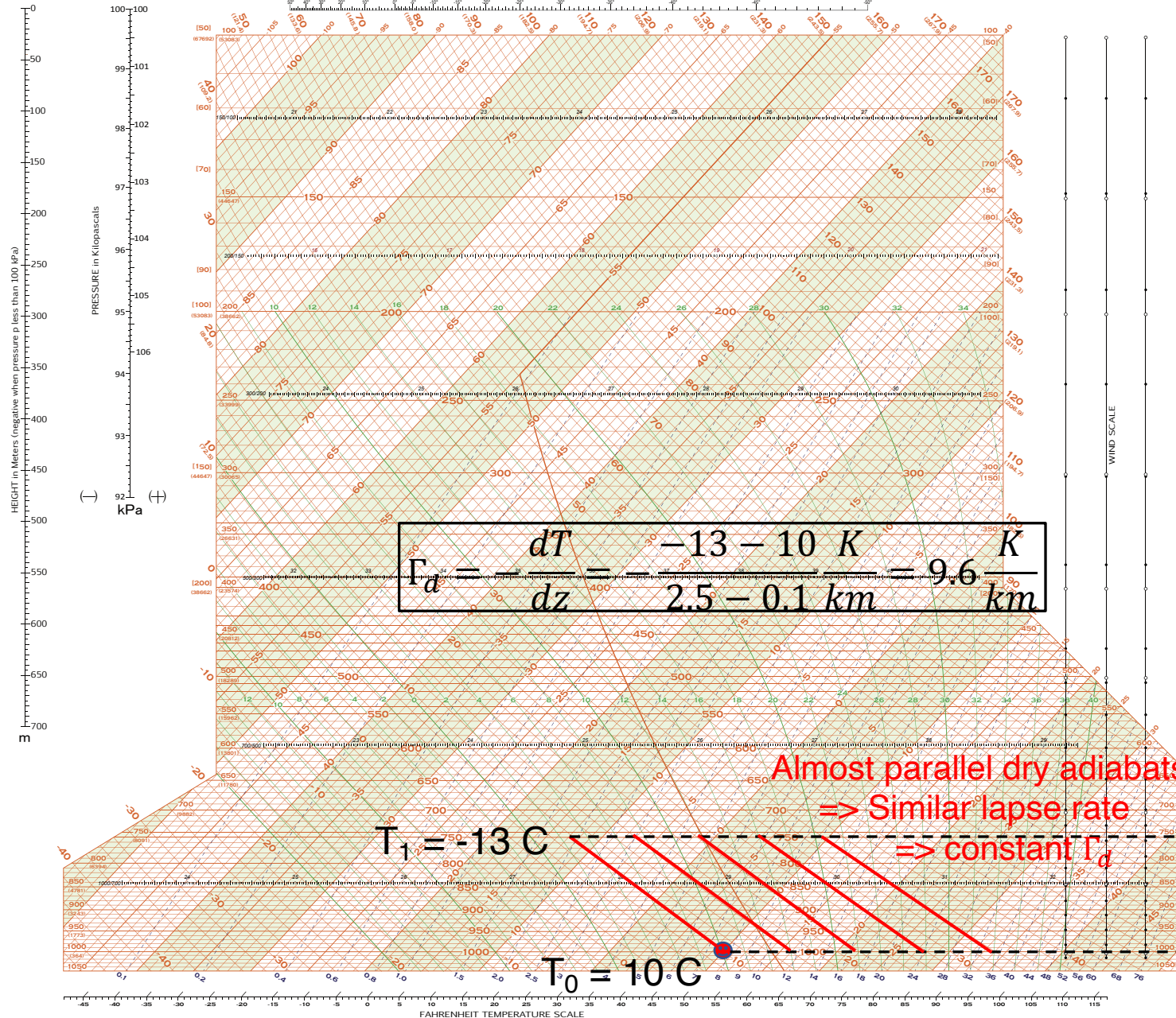
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| SKREW T ANALYSIS | | |
|---------------------------------------|-----------|------|
| TIME | TIME | |
| AIRMASS ANALYSIS | | |
| WIND | W | W |
| TEMPERATURE | W | W |
| MOISTURE | W | W |
| PRESSURE ANALYSIS | | |
| HEIGHT | INDICATOR | |
| STATION | | |
| LOCATION | | |
| INSTRUMENTS | | |
| OPERATOR | | |
| SIGNIFICANT WIND | | |
| TIME | | |
| LEVELS OF SHEAR | | |
| | WIND | WIND |
| TO | TO | TO |
| FR | FR | FR |
| CLOUDS | | |
| TIME | | |
| BASE | | |
| TOP | | |
| TYPE | | |
| VISIBILITY | | |
| REMARKS | | |
| CONTROLS | | |
| REMARKS | | |
| TEMPERATURES | | |
| TIME | | |
| TEMP | | |
| STANDARD QUANTIFICATION AT TOP | | |
| TEMPERATURE | TIME | TIME |
| CORRECTION OF LOW LEVEL INDICATION AT | | |
| REMARKS | | |

| STATION ID | STATION ID | STATION ID |
|------------|------------|------------|
| DATE | DATE | DATE |
| TIME (GMT) | TIME (GMT) | TIME (GMT) |

SKREW T ADIABATIC DIAGRAM

TEMPERATURE IN DEGREES CELSIUS



$$\Gamma_d = \frac{dT}{dz} = \frac{-13 - 10 \text{ K}}{2.5 - 0.1 \text{ km}} = 9.6 \frac{\text{K}}{\text{km}}$$

Almost parallel dry adiabats
=> Similar lapse rate
=> constant Γ_d

$T_1 = -13 \text{ C}$

$T_0 = 10 \text{ C}$

$Z_1 = 2.5 \text{ km}$

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| SKREW T ANALYSIS | |
|-------------------|------------|
| TIME | DATE |
| STATION ANALYSIS | |
| STATION ID | STATION ID |
| DATE | DATE |
| TIME (GMT) | TIME (GMT) |
| SOUNDING ANALYSIS | |
| STATION ID | STATION ID |
| DATE | DATE |
| TIME (GMT) | TIME (GMT) |



Moist adiabatic lapse rate

Theoretical moist adiabatic lapse rate

Back to the first law of thermodynamics for a parcel in a hydrostatic environment:

$$dh = c_p dT + g dz$$

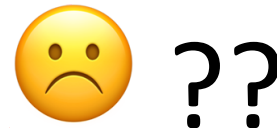
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As saturated air ascends \Rightarrow water vapor begins to condense and release the latent heat of condensation \Rightarrow rate of cooling changes

Therefore, $dh \neq 0$, and released latent heat is added to the parcel:

$$dh = -Ldw = -Ldw_s$$

Theoretical moist adiabatic lapse rate

$$dh = c_p dT + g dz \quad (2)$$

Therefore, $dh \neq 0$, and released latent heat is added to the parcel:

$$dh = -Ldw = -Ldw_s$$

$$L = 2.5 \times 10^6 \text{ J kg}^{-1}$$

dw mass changes in water vapor

Eq. (2) changes to:

$$-L dw_s = c_p dT + g dz. \quad (3)$$

Theoretical moist adiabatic lapse rate

$$-L dw_s = c_p dT + g dz. \quad (3)$$

Now, the main problem is dw_s

We know that w_s is a function of pressure and temperature, $w_s = w_s(p, T)$.

Theoretical moist adiabatic lapse rate

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We know that w_s is a function of pressure and temperature, $w_s = w_s(p, T)$.

Let's first expand dw_s to:

$$dw_s = \left(\frac{\partial w_s}{\partial p} \right)_T dp + \left(\frac{\partial w_s}{\partial T} \right)_p dT$$

Theoretical moist adiabatic lapse rate

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Substitute this into (3),

$$-L \left[\left(\frac{\partial w_s}{\partial p} \right)_T dp + \left(\frac{\partial w_s}{\partial T} \right)_p dT \right] = c_p dT + g dz$$

Theoretical moist adiabatic lapse rate

Recall that

$$dp = d\bar{p} = -\frac{1}{\bar{\alpha}}g dz$$

$$\frac{\alpha}{\bar{\alpha}} \approx 1$$

Substitute the above two and divide the result by $c_p dz$ to obtain:

$$-L \left[\left(\frac{\partial w_s}{\partial p} \right)_T dp + \left(\frac{\partial w_s}{\partial T} \right)_p dT \right] = c_p dT + g dz$$

Need your help!

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$$\frac{\alpha}{\bar{\alpha}} \approx 1$$

Substitute the above two and divide the result by $c_p dz$ to obtain:

$$-\frac{L}{c_p} \left[-\left(\frac{\partial w_s}{\partial p}\right)_T \rho g + \left(\frac{\partial w_s}{\partial T}\right)_p \frac{dT}{dz} \right] = \frac{dT}{dz} + \frac{g}{c_p}$$

$$-L \left[\left(\frac{\partial w_s}{\partial p}\right)_T dp + \left(\frac{\partial w_s}{\partial T}\right)_p dT \right] = c_p dT + g dz$$

Theoretical moist adiabatic lapse rate

Then solve for dT/dz to obtain the lapse rate for a saturation-adiabatic process,

$$\Gamma_s \equiv -\frac{dT}{dz} = \frac{g}{c_p} \frac{1 - \rho L \left(\frac{\partial w_s}{\partial p}\right)_T}{1 + \frac{L}{c_p} \left(\frac{\partial w_s}{\partial T}\right)_p} \quad (4)$$

Theoretical moist adiabatic lapse rate

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Theoretical moist adiabatic lapse rate

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$$w_s \approx \epsilon \frac{e_s}{p}$$

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Use assumption for w_s :

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and the *Clausius-Clapeyron equation* for e_s ,

$$\frac{de_s}{dT} = \frac{Le_s}{R_v T^2}$$

Only a function of Temperature
Nice!

Theoretical moist adiabatic lapse rate

where R_v is the gas constant of water vapor ($461.5 \text{ J kg}^{-1} \text{ K}^{-1}$), $R = R_d$ is gas constant of dry air ($287 \text{ J kg}^{-1} \text{ K}^{-1}$)

$$\epsilon = R_d/R_v \approx 0.622$$

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The lapse rate for moist-adiabatic process is

$$\Gamma_s \equiv -\frac{dT}{dz} = \frac{g}{c_p} \frac{1 + \frac{L w_s}{R T}}{1 + \frac{\epsilon L^2 w_s}{c_p R T^2}}$$

Then solve for dT/dz to obtain the lapse rate for a saturation-adiabatic process,

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Γ_s is not a constant

It is equal to Γ_d multiplied by a factor that depends on temperature and pressure.

Then solve for dT/dz to obtain the lapse rate for a saturation-adiabatic process,

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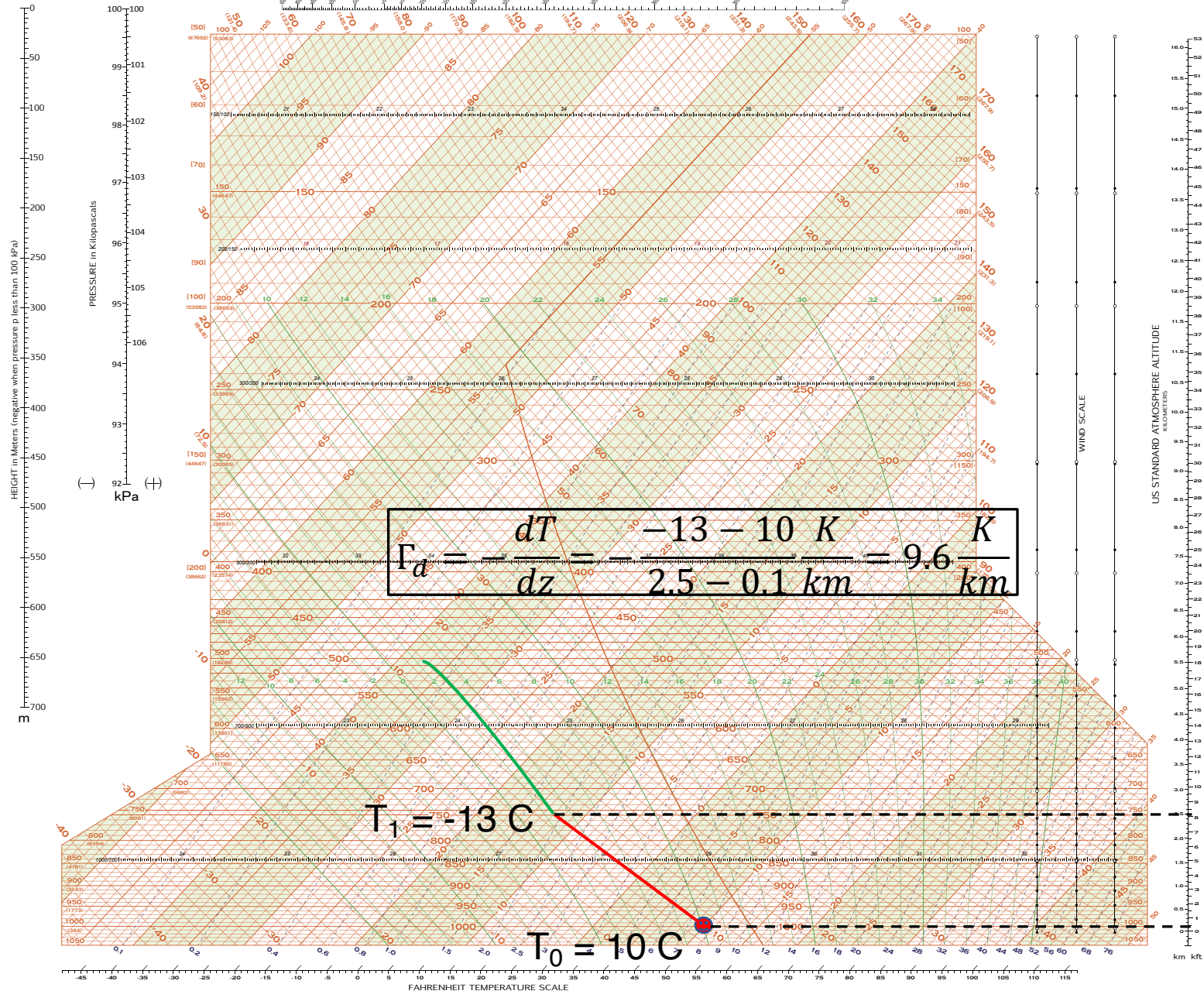
and the *Clausius-Clapeyron equation* for e_s ,

$$\frac{de_s}{dT} = \frac{L e_s}{R_v T^2}$$

Again, let's check it at the Skew $T\text{-log}P$ diagram

SKREW T ADIABATIC DIAGRAM

TEMPERATURE IN DEGREES CELSIUS



EXPLANATION

ISOBARS are straight horizontal brown lines. The heights in feet of the pressure surfaces in the U.S. Standard atmosphere are in parenthesis () below the pressure values on the left.

ISOTHERMS (°C) are the straight, equidistant brown lines running diagonally upward from the left to right.

DRY ADIABATS are the slightly curved brown lines that intersect the 1000 mb. isobar at intervals of 2°C, and run diagonally upward from right to left. The Dry Adiabats for the lidded portion of the pressure range are labeled with two (2) values. (See below.)

SATURATED ADIABATS are the curved green lines that intersect the 1000 mb. isobar at intervals of 2°C, diverging upward and tending to become parallel to the dry adiabats.

SATURATION MIXING RATIO (in gm. per kg) is represented by dashed green lines. Their values appear at the bottom of diagram.

THICKNESS (in hundreds of geopotential meters) of the layers between the levels 1000, 700, 500, 300, 200, 150, and 100 mb. is represented by numbers and a graduation along the middle of each layer. The thicknesses are obtained from the virtual temperature curve by the equal-area method, using any straight line as a dividing line.

HEIGHT in geopotential meters above mean sea level, or station level, of the 100 kPa surface is obtained from the nomogram in the upper left-hand corner by drawing a straight line from the point on the temperature scale (°C) through the point () (mean sea level or station pressure) on the pressure scale, and reading the height on the height scale.

U.S. STANDARD ATMOSPHERE SOUNDING is indicated by a thick brown line.

The saturated adiabats and scoplets of saturation mixing ratio are computed by use of vapor pressure over a plane water surface at all temperatures.

Extensions of chart to 50 mb. have been accomplished by overlap with pressure indicated in brackets, [200] at 400 mb., and [50] at 100 mb. Dry adiabats for the overlap are labeled in parenthesis () .

APPROXIMATE VIRTUAL TEMPERATURE may be obtained from the formula $T_v = T + \frac{p_0 - p}{1000}$ where T_v is virtual temperature in °C, T is free air temperature in °C, and w is the mixing ratio in g/kg. For purposes of thickness computation, use the mean temperature of the layer for T , and use the mean mixing ratio of the layer for w .

Black dots along wind scale lines indicate the levels for which wind data is reported and plotted. The open circles indicate the mandatory pressure level at which wind data is also entered.

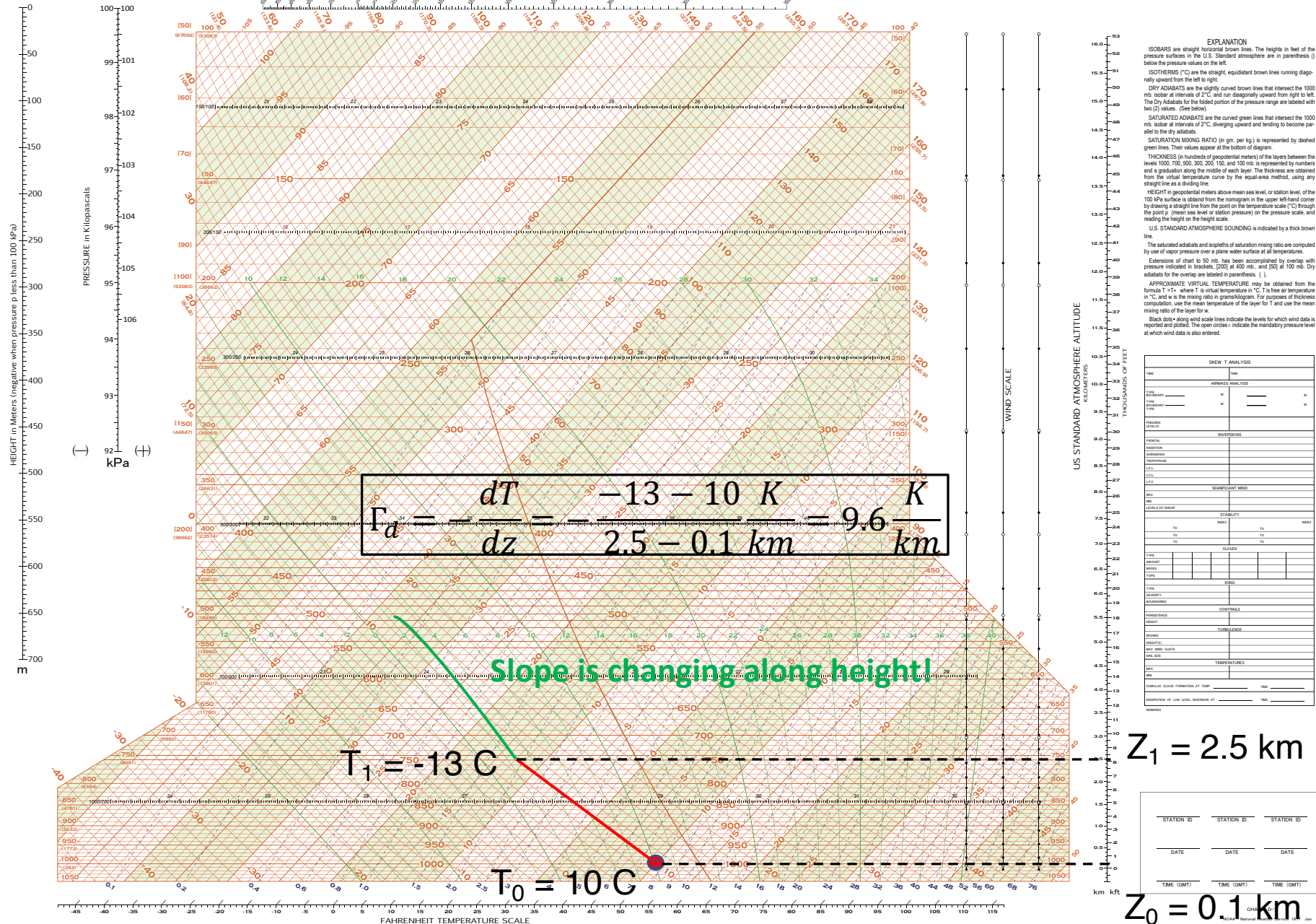
| SKREW T ANALYSIS | |
|----------------------------------|---------|
| TIME | TIME |
| STATION ANALYSIS | |
| STATION | STATION |
| DATE | DATE |
| TIME | TIME |
| PRESSURE ANALYSIS | |
| STATION | STATION |
| DATE | DATE |
| TIME | TIME |
| WIND ANALYSIS | |
| STATION | STATION |
| DATE | DATE |
| TIME | TIME |
| TEMPERATURE ANALYSIS | |
| STATION | STATION |
| DATE | DATE |
| TIME | TIME |
| SATURATION MIXING RATIO ANALYSIS | |
| STATION | STATION |
| DATE | DATE |
| TIME | TIME |

Z₁ = 2.5 km

Z₀ = 0.1 km

SKREW T ADIABATIC DIAGRAM

TEMPERATURE IN DEGREES CELSIUS



$$\Gamma_d = \frac{dT}{dz} = \frac{-13 - 10\text{ K}}{2.5 - 0.1\text{ km}} = 9.6 \frac{\text{K}}{\text{km}}$$

Slope is changing along height!

EXPLANATION

ISOBARS are straight horizontal brown lines. The heights in feet of the pressure surfaces in the U.S. Standard atmosphere are in parenthesis () below the pressure values on the left.

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Extensions of chart to 50 mb. have been accomplished by overlap with pressure indicated in brackets, [200] at 400 mb., and [50] at 100 mb. Dry adiabats for the overlap are labeled in parenthesis ().

APPROXIMATE VIRTUAL TEMPERATURE may be obtained from the formula $T^* = T + \frac{p - p_0}{1000}$ where T^* is virtual temperature in °C, T is free air temperature in °C, and w is the mixing ratio in grams/kg. For purposes of thickness computation, use the mean temperature of the layer for T , and use the mean mixing ratio of the layer for w .

Black dots along wind scale lines indicate the levels for which wind data is reported and plotted. The open circles indicate the mandatory pressure level at which wind data is also entered.

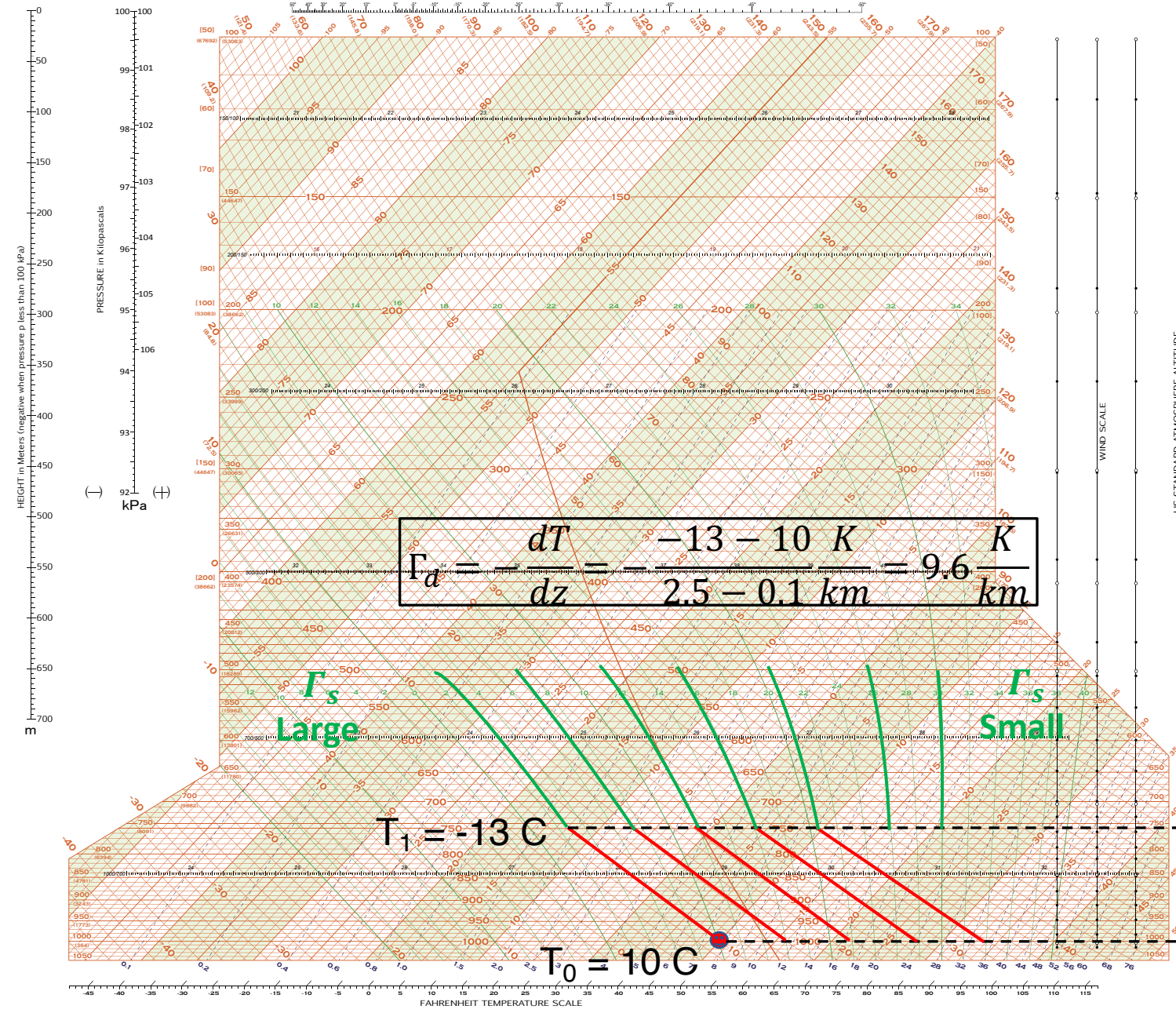
| SKREW T ANALYSIS | | | |
|------------------|------------|------------|------------|
| TIME | TIME | TIME | TIME |
| STATION ANALYSIS | | | |
| STATION ID | STATION ID | STATION ID | STATION ID |
| DATE | DATE | DATE | DATE |
| TIME (GMT) | TIME (GMT) | TIME (GMT) | TIME (GMT) |

$Z_1 = 2.5\text{ km}$

$Z_0 = 0.1\text{ km}$

SKREW T ADIABATIC DIAGRAM

TEMPERATURE IN DEGREES CELSIUS



EXPLANATION

ISOBARS are straight horizontal brown lines. The heights in feet of the pressure surfaces in the U.S. Standard atmosphere are in parenthesis () below the pressure values on the left.

ISOTHERMS (°C) are the straight, equidistant brown lines running diagonally upward from the left to right.

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Black dots along wind scale lines indicate the levels for which wind data is reported and plotted. The open circles indicate the mandatory pressure level at which wind data is also entered.

| STATION ID | | STATION ID | | STATION ID | |
|------------|------------|------------|------------|------------|------------|
| DATE | DATE | DATE | DATE | DATE | DATE |
| TIME (GMT) | TIME (GMT) | TIME (GMT) | TIME (GMT) | TIME (GMT) | TIME (GMT) |

Z₁ = 2.5 km

Z₀ = 0.1 km