Convective Outflows

Atmospheric Sciences 6150

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Convective downdrafts, gust fronts, and cold pools are largely driven by rain evaporative cooling. The maximum impacts of rain evaporative cooling on these phenomena largely depend on boundary layer thermodynamic properties.

DCAPE (Downdraft Convective Available Potential Energy) is one of the most useful measures of the maximum possible convective downdraft speed. DCAPE is closely related to the surface pressure perturbation in "meso-highs", gust front propagation speed, and cold pool intensity (surface temperature deficit). DCAPE is defined as

$$\mathrm{DCAPE} \equiv -\int_{z_s}^{z_i} g \frac{T - \bar{T}}{\bar{T}} \, dz = -\int_{p_i}^{p_s} R(T - \bar{T}) \, d\ln p,$$

where T is the downdraft temperature, $\bar{T}$ is the environment temperature, z_i is the parcel's initial height, $z_s < z_i$ the parcel's final height (at the surface), p_i is the parcel's initial pressure, and $p_s > p_i$ is the parcel's final pressure (at the surface). At the parcel's initial height and pressure, $T = \bar{T}_w$, the wet-bulb temperature of the environment. Below that level, the parcel is cooled and moistened by rain evaporation so that it remains *saturated* as it descends. Because convective downdrafts are almost always *unsaturated,* we will henceforth define DCAPE so that it is based on the *actual* downdraft temperature, which depends on the amount of evaporation that actually occurs.

In an unsaturated mixed layer, the water vapor mixing ratio and potential temperature do not vary with height. In such a layer, DCAPE is the work done by buoyancy on a parcel that descends from the LCL (lifting condensation level), where it has the same properties as the mixed layer at that level, to the surface, while being cooled by evaporation of liquid hydrometeors.

1. Approximate the DCAPE for an unsaturated mixed layer in terms of the pressure at the LCL, p_t, or the height at the LCL, z_t, the temperature of the downdraft at the surface, T_s, and the temperature of the environment (the mixed layer) at the surface, $\bar{T}_s$. The required quantities can all be obtained from a skew T - log p diagram.

   Answer:
   Use $z_i = z_t$, $p_i = p_t$, so that

$$\text{DCAPE} \equiv -\int_{z_s}^{z_t} g\frac{T-\bar{T}}{\bar{T}}\,dz = -\int_{p_t}^{p_s} R(T-\bar{T})\,d\ln p.$$

   Then use $T = \bar{T}$ at the LCL, $T = T_s$ and $\bar{T} = \bar{T}_s$ at the surface, and linear in z and $\ln p$ in between, to obtain

$$\text{DCAPE} \equiv -\frac{g}{\bar{T}_s}\frac{T_s-\bar{T}_s}{2}(z_t - z_s) = -R\frac{T_s-\bar{T}_s}{2}\log(p_t/p_s).$$

2. (a) Mathematically, how is the maximum possible downdraft speed at the surface related to DCAPE?
   (b) Under what conditions would this maximum speed be most closely realized?
   Answer:
   (a)

$$w_{\max} = \sqrt{2\ DCAPE}.$$

   (b) A narrow downdraft would experience deceleration at a lower altitude than a wide downdraft. However, a very narrow downdraft would experience more slowing due to mixing with the environment than a wider downdraft.

3. (a) What happens to such a downdraft as it nears the surface?
   (b) How is the maximum possible horizontal outflow speed related to the maximum possible downdraft speed?
   (c) What is the physical mechanism (i.e., force) that turns the downdraft into a horizontal outflow?
   (d) How is the maximum downdraft speed related to the maximum surface pressure increase?
   Answer:
   (a) It decelerates.
   (b) The two are equal. This can be shown using Bernoulli's equation, which is

obtained for steady flow by integrating the equation of motion along a streamline (and trajectory) from the starting point of the downdraft. Integrate

$$u\,\frac{du}{dx} = -\frac{d}{dx}\left(\frac{p}{\rho_0}\right)$$

along the streamwise coordinate x to get

$$(\Delta u)^2/2 = -\Delta p/\rho_0,$$

where $\Delta u = u_2 - u_1$ and $\Delta p = p_2 - p_1$.
(c) The pressure gradient force decelerates the downdraft and accelerates the outflow. A region of high pressure is produced just above the surface below the downdraft. The pressure perturbation is a combination of hydrostatic and dynamic pressure perturbations.
(d) Bernoulli's equation shows that

$$w_{\max} = \sqrt{2\Delta p/\rho_0}.$$

4. A density current is a surface-based flow driven by horizontal density differences. Given a density current's depth h over which it is denser than its environment, and its vertically averaged temperature deficit compared to the environment, what is the speed of propagation of the front of a density current?
Answer:
The answer can be obtained from the previous question by recognizing that

$$DCAPE = gh\frac{\Delta T}{\bar{T}}$$

and that the propagation speed c is the same as the downdraft speed, so that

$$c = w_{\max} = \sqrt{2\ DCAPE} = \sqrt{2\ gh\frac{\Delta T}{\bar{T}}}.$$

5. (a) Use the observed maximum outflow speed, $u_{\max} = 30$ m/s, to estimate DCAPE.
(b) Use this estimate of DCAPE, along with the observed surface values of the downdraft temperature, $T_s = 25°$ C, the environment temperature, $\bar{T}_s = 30°$ C, the downdraft relative humidity, $r_s = 0.3$, the environment relative humidity, $r_s = 0.2$, and the (environment) pressure, $p_s = 800$ hPa, to estimate p_t, the (environment) pressure at the LCL.
(c) Discuss how your assumptions affect your estimate of p_t.

Answer:
(a) DCAPE $= u_{\max}^2/2 = 450$ J/kg.
(b) It is necessary to make an educated guess about the downdraft temperature profile. The downdraft is not saturated at the surface, so one could assume that the downdraft is saturated only from the LCL to a certain level (the sinking evaporation level, or SEL), then is unsaturated below that level. One would then need an estimate of the SEL, but this is simply the LCL of the downdraft air (easily obtained from a skew T-log p diagram). The LCL obtained using this $T(p)$ is about 540 hPa.

The simplest profile is a linear profile in z or $\log p$ with $T = \bar{T}$ at the LCL.

$$\mathrm{DCAPE} \approx -\frac{g}{\bar{T}_s}\frac{T_s - \bar{T}_s}{2}(z_t - z_s) = -R\frac{T_s - \bar{T}_s}{2}\log(p_t/p_s).$$

Solving this for p_t, we obtain

$$p_t = p_s \exp\left(-\frac{2\,\mathrm{DCAPE}}{\mathrm{R(T_s - \bar{T}_s)}}\right) \approx 430\,\mathrm{hPa}.$$

This would be a lower bound on acceptable values for p_t. If one uses linear in z, one must use the hypsometric equation to find p_t, and the result will be the same.

Another linear profile is one for which $T = \bar{T} - (T_s - \bar{T}_s)$ at the LCL. In this case,

$$\mathrm{DCAPE} \approx -\frac{g}{\bar{T}_s}(T_s - \bar{T}_s)(z_t - z_s) = -R(T_s - \bar{T}_s)\log(p_t/p_s).$$

Solving this for p_t, we obtain

$$p_t = p_s \exp\left(-\frac{\mathrm{DCAPE}}{\mathrm{R(T_s - \bar{T}_s)}}\right) \approx 585\,\mathrm{hPa}.$$

This would be an upper bound on acceptable values for p_t.
(c) When the same amount of evaporative cooling occurs at higher level, DCAPE is greater and the maximum downdraft speed is larger. This means that one can achieve the same DCAPE from a smaller amount of cooling aloft, or a greater amount near the surface. Similarly, a long gentle hill or a short or steep one can accelerate a cyclist or skier to the same speed at the bottom.

6. Consider a parcel of dry air with a mass of 1 kg and a volume V_1. A mass of rain r evaporates into the parcel (isobarically) and cools it. The volume of the air parcel decreases to V_2 as a result. Surrounding air converges (isobarically) into the original volume V_1. What is the increase of mass of air ΔM within the the original volume V_1? Mathematically relate ΔM to r. Use approximations as appropriate to get an answer that is accurate within 10 percent under typical lower tropospheric conditions.
Answer:
An approximate answer is

$$\Delta M \approx 9r$$

and can be obtained as follows. The mixing ratio of rain is r since the mass of air is 1 kg. The temperature change due to evaporation of M is $\Delta V = -L_e M/c_p$, where L_e is the latent heat of evaporation. The change in volume is

$$\Delta V = V_2 - V_1 = \Delta\alpha = \frac{R}{p}\Delta T = -\frac{R}{p}\frac{L_e r}{c_p}.$$

The change in mass is therefore

$$\Delta M = \rho\Delta V = \frac{p}{RT}\Delta V = \frac{p}{RT}\frac{R}{p}\frac{L_e r}{c_p} = \frac{L_e}{c_p T} r \approx 9r,$$

for $T = 273$ K.

Useful constants and formulae

0° C = 273 K
$g = 9.8$ m s^{-2} (acceleration of gravity)
$c_p = 1004$ J kg^{-1} K^{-1} (specific heat at constant pressure for dry air)
$c_v = 717$ J kg^{-1} K^{-1} (specific heat at constant volume for dry air)
$R = c_p - c_v = 287$ J kg^{-1} K^{-1} (gas constant for dry air)
$R^* = 8.31$ J mol^{-1} K^{-1} (universal gas constant)
$R_v = R^*/m_v$ (gas constant for water vapor)
$m_d = 28.97$ g mol^{-1} (mean molar mass of dry air)
$m_v = 18.02$ g mol^{-1} (molar mass of water vapor)
$L = 2.5 \times 10^6$ J kg^{-1} (latent heat of evaporation)

$$e_s(T) \approx A \exp(-B/T) \text{ (saturation vapor pressure),}$$

$$\text{where } A = 2.53 \times 10^{11} \text{ Pa}, B = 5420 \text{ K}.$$

$$\frac{de_s}{dT} = \frac{Le_s}{R_v T^2} \text{ (Clausius-Clapeyron equation).}$$

$$w_s \approx \frac{\epsilon e_s}{p} \text{ (saturation mixing ratio),}$$

$$\text{where } \epsilon \equiv m_v/m_d.$$