Meteorology 6150: Exercise 4
Nonlinear Rayleigh (parallel plate) convection

EXPERIMENT DESIGN

Equations: Include non-linear advection terms (i.e., full equations).

Compilation: You may need to use

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pgf90 -r8 qcom.f
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to maintain precision of potential temperature field. An alternative is to replace the equation for θ with one for

$$\theta_1(y,z) \equiv \theta(y,z) - \theta_0(z)$$

or

$$\theta_2(y,z) \equiv \theta(y,z) - \theta_0(0).$$

Domain, grid size, and initial conditions: Same as test case, except use $\Delta\theta = 1.2, 2.4$, and 4.8 K (three cases).

Molecular diffusivities: $K_v = K_w = K_\theta = 50$ m^{-2} s^{-1}. For $\Delta\theta = 2.4$ K, this increases the Rayleigh number by a factor of 4 over the linear case.

Sound speed: $c_s = 25$ to 50 m s^{-1} (should be at least twice speed of fastest signal in the simulation)

Time step: $\Delta t = 0.1$ s.

Simulation time: Run until a steady state is achieved.

ANALYSIS

1. Calculate (and plot) the average kinetic energy,

$$[E] = \frac{1}{N}\left(\sum_v v^2 + \sum_w w^2\right),$$

where $\sum_v$ is the sum over all interior (or predicted) v-points, $\sum_w$ is the sum over all interior (or predicted) w-points, and $N = \sum_\theta$, the number of interior θ-points. Note that $\sum_\theta = \sum_v$.

Plot $[E]$ versus time to determine when a steady state is reached for each case.

2. Once steady state is reached for each case, plot the horizontally averaged potential temperature profile, $\bar{\theta}$. The overbar indicates a horizontal average. The profiles should resemble the curves shown in Fig. 3.14 of Emanuel.

3. For each case, calculate and plot the steady-state profiles of the horizontally averaged conductive vertical heat flux,

$$(F_\theta)_{\text{cond}} \equiv \overline{-K_\theta \frac{d\theta}{dz}} = -K_\theta \frac{d\bar{\theta}}{dz},$$

   the horizontally averaged convective vertical heat flux,

$$(F_\theta)_{\text{conv}} \equiv \overline{w'\theta'},$$

   and the total heat flux,

$$F_\theta \equiv (F_\theta)_{\text{cond}} + (F_\theta)_{\text{conv}},$$

   which is the sum of the conductive and convective heat fluxes. The overbar indicates a horizontal average, and a prime a departure from the horizontal average.

   Note that $\bar{w} = 0$ so $w' \equiv w - \bar{w} = w$. Then

$$\overline{w'\theta'} = \overline{w\theta'} = \overline{w(\theta - \bar{\theta}} = \overline{w\theta} - \overline{w\bar{\theta}} = \overline{w\theta}.$$

4. Calculate the Rayleigh number and Nusselt number for each case. Discuss the dependence of $\bar{\theta}$ and the Nusselt number on the Rayleigh number.