Cloud microphysics E: 10.7

Warm rain processes: Kessler scheme

- No ice.
- Drop population distrib. is bimodal:
  - cloud droplets with $V_T \approx 0$.
  - rain drops with $V_T > 1\,m/s$,
    size distrib. is exponential:

    $N(r) = N_0 e^{-\lambda r}$, r: radius

    (Marshall - Palmer distribution)
- Predicted variables:
  - mixing ratio of cloud droplets, l_c.
  - mixing ratio of rain drops, l_R.

$$\frac{dl_c}{dt} = S - A - C + D_{l_c}$$

$$\frac{dl_R}{dt} = \frac{1}{\bar{\rho}}\frac{d}{dz}(\bar{\rho} V l_R) + A + C - E + D_{l_R}$$

Water vapor mixing ratio:

$$\frac{dr}{dt} = -S + E + D_r$$

D_x : turbulent diffusion

S: vapor → cloud droplets (condensation)

A: (autoconversion) cloud droplets → rain drops

C: (collection) cloud droplets + rain drops → larger rain drops

E: (rain evaporation) rain drops → water vapor.

V: (mass-weighted terminal velocity of rain drops)

A: Autoconversion

cloud droplets collide, coalesce to form (eventually) small rain drops. This process is still not well understood. Contributing factors may include:

- entrainment & mixing
- turbulence
- large aerosols

All of these ~~may broaden the DSD and increase~~ can increase collision rates.

$$A = \max[0, K_1(l_c - a)]$$

K_1: rate constant

a: critical value of l_c.

Neither is a constant in nature.

C: Collection

Cloud droplets are "collected" by rain drops. For a single drop (of diameter D), its mass M_D increases

$$\frac{dM_D}{dt} = \rho \frac{\pi}{4} D^2 \mathcal{E}_D V_D l_c$$

V_D: terminal velocity of diam. D drop.

$\mathcal{E}_D$: collection efficiency $(0 \leq \mathcal{E}_D \leq 1)$.

Integrate over size dist. of drops: to get

$$C = \frac{d l_R}{dz} = \int_0^\infty \frac{\pi}{4} D^2 \mathcal{E}_D V_D l_c N_0 e^{-\lambda D} dD .$$

Kessler : $\mathcal{E}_D = \bar{\mathcal{E}}$ (constant)

$$V_D \simeq k\left(g \frac{\rho_e}{\rho}\right)^{1/2} D^{1/2} \qquad (10.4.14)$$

ρ_e: density of liquid water

k: dimensionless constant

g: gravity

Subst. and integration gives:

$$C = k\left(\frac{\rho_e}{\rho}\right)^{1/2} g^{1/2} \frac{\pi}{4} \bar{\mathcal{E}} N_0 l_c \frac{\Gamma(7/2)}{\lambda^{7/2}} \qquad (10.4.16)$$

Γ : gamma function.

Need to express in terms of l_R:

$$l_R = \frac{\rho_e}{\rho} \frac{\pi}{6} N_0 \int_0^\infty D^3 e^{-\lambda D} dD$$

$$= \frac{\pi}{6} \frac{\rho_e}{\rho} N_0 \frac{\Gamma(4)}{\lambda^4} . \qquad (10.4.17)$$

Eliminate λ between (16), (17):

$$C = k_1 g^{1/2} \left(\frac{\rho}{\rho_e}\right)^{3/8} \bar{\mathcal{E}} N_0^{1/8} l_c l_R^{7/8} \qquad (10.4.18)$$

So $C \sim l_c l_R$

E: Evaporation of rain

See Emanuel 10.4c for details.

$$E \sim \underbrace{(r_a^* - r_a)}_{\text{subsaturation}} \times \left[l_R^{1/2} + \beta_5 \rho^{-1/8} l_R^{7/8} \right]$$

varies as $l_R^{1/2}$ for small l_R,
as $l_R^{7/8}$ for large l_R.

$V l_R$: Rain flux

For each drop size: rain flux is

$$V_D \rho_l \frac{\pi}{6} D^3 N(D).$$

Integrate over $N(D)$ using M-P dist,
and using (10.4.14) for D:

$$\rho k_4 g^{1/2} \left(\frac{\rho_l}{\rho}\right)^{3/8} N_0^{-1/8} l_R^{9/8} \equiv \rho V l_R$$

k_4: dimensionless constant,

Then

$$V \equiv k_4 g^{1/2} \left(\frac{\rho_l}{\rho}\right)^{3/8} N_0^{-1/8} l_R^{1/8}.$$

varies weakly with l_R, more so with ρ (air density).

See Table 10.1 for a summary of the Kessler scheme.