3.5 Non-linear parallel-plate convection

1. In atmos., Ra >> Ra_c. Under these conditions, which features of linear convection are relevant?

2. Look at lab. exps. that cover a range of Ra and Pr.

3. Highest attained in lab. ~ 10^{11} << 10^{17}-10^{20} characteristic of atmospheric convecting layers.

4. B.C. for atmos. is more nearly a constant flux condition, not const. T. Also, boundary is rough. Expect atmos. convection to be quite different from lab. exp. convection.

Krishnamurti (1970a,b)

Rayleigh-Bénard convection in a lab. apparatus.
Ra : up to 10^6.
Pr : vary from 0.7 to 10^4 (by changing fluid)

As Ra increases, so does non-dimensional heat flux (Nusselt no.): linearly.
But slope changes at certain Ra.
⊕ See Fig. 3.11 (Nu vs Ra).
(As many a 7 such transitions have been obs.)

Krishnamurti's exps. showed that these transitions are associated with changes in the character of the convection.
⊕ See Fig. 3.12

Describe diagram:

As Ra increases, at fixed Pr, convection begins

I. as steady, 2D rolls, at Ra_c predicted by linear theory.

II. Next: 2D → steady 3D. This is transition R_{II} (see Fig. 3.11). Heat flux incr. more rapidly with Ra now.

III. Convection oscillates at a definite freq.

IV. Freq. doubles

Through a series of freq. doublings, convection becomes chaotic (aperiodic, turbulent), This, was the subject of Lorenz's study in 1963, chaotic regime

Busse (1967)

Predicted transition from 2D to 3D flow, by applying weakly non linear theory.

Showed that only 2D rolls are stable, and only in a certain range of k-Ra

⊛ space ("Busse's bubble"), see Fig. 3.13. Tested theory with exp.; also shown in Fig.

Describe diagram: critical Ra is curve at bottom. Curve upward/left: critical curve for "zigzag" instability; upward/right: "cross-roll" instability.

⊛ Temperature profile vs Ra. Fig. 3.14

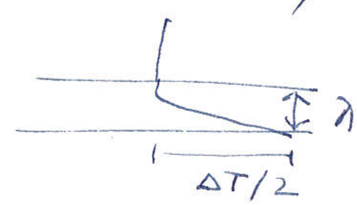
conductive : linear
Ra > Ra_c : S-shape
Ra >> Ra_c : all gradients retreat into boundary
⊛ layers (show WD results)
⊛ Show Moeng & Rotunno LES of RB convection.
In turbulent convection: interior has neutral stratification. This is also a property of atmospheric convective layers.

Boundary layer depth?

One idea is that for this thickness, $Ra \simeq Ra_c$,

that is

$$Ra_c \simeq \frac{\frac{1}{2} g \beta \Delta T \lambda^3}{\nu \kappa}.$$



Since

$$Ra = \frac{g \beta \Delta T H^3}{\nu \kappa},$$

we have

$$\frac{\lambda^3}{H^3} \sim \frac{Ra_c}{Ra}, \text{ or } \frac{\lambda}{H} \sim Ra^{-1/3}. \quad (3.5.2)$$

A consequence of this is that nondim. heat flux varies as $Ra^{-1/3}$.

We show this now.

Note that total heat flux is constant, and that the heat flux in boundary layers is principally conductive.

Define Nusselt no.

$$Nu \equiv \frac{\text{heat flux}}{\underbrace{\kappa \Delta T / H}_{\text{heat flux if purely conductive}}} \sim \frac{\frac{1}{2}\kappa \Delta T / \lambda}{\kappa \Delta T / H} \simeq \frac{H}{\lambda}.$$

From (3.5.2),

$$\boxed{Nu \sim Ra^{1/3}.}$$

For many years, this was considered to be the asymptotic limit for large Ra.

Recent exp. work shows that this applies quite well for $5 \times 10^5 \lesssim Ra \lesssim 4 \times 10^7$.

Above 4×10^7, another transition occurs, for which

$$Nu \sim Ra^{2/7 \pm 0.006}.$$

In addition, exps. show

$$\left(\overline{T'^2}\right)^{1/2} / \Delta T \sim Ra^{-1/7}$$

$$\left(\overline{w'^2}\right)^{1/2} H/\nu \sim Ra^{3/7}$$

so $\overline{w'T'} \sim \left(\overline{T'^2}\right)^{1/2} \left(\overline{w'^2}\right)^{1/2} \sim Ra^{2/7} \frac{\Delta T}{H} \nu.$

Since $Ra \sim \nu^{-2}$,

$$\overline{w'T'} \sim \nu^{1-2(2/7)}.$$

A similar result holds for the 1/3 power law regime ($2/7 \to 1/3$),

Note that heat flux depends on ν even at high Ra!

<u>For geophysical flows</u>, other processes will determine the convective heat flux.

Near boundary, turbulence, and hence turbulent fluxes, are determined more by mean wind speed than by convection, so a fixed flux b.c. is more appropriate.

(Little lab. work has been done with this condition, unfortunately.)

Also, boundary layer depth λ (not δ) is much smaller than roughness elements (rocks, bushes, etc.), so their scale is important.

Atmospheric convective boundary layers are considered in sec. 3.6.