QCOM 2D Convection

$$\frac{\partial v}{\partial t} = -v\frac{\partial v}{\partial y} - w\frac{\partial v}{\partial z} - c_p\theta_0\frac{\partial \pi_1}{\partial y} + D_v,$$

$$\frac{\partial w}{\partial t} = -v\frac{\partial w}{\partial y} - w\frac{\partial w}{\partial z} - c_p\theta_0\frac{\partial \pi_1}{\partial z} + g(\frac{\theta}{\theta_0} - 1) + D_w,$$

$$\frac{\partial \theta}{\partial t} = -v\frac{\partial \theta}{\partial y} - w\frac{\partial \theta}{\partial z} + D_\theta,$$

$$\frac{\partial \pi_1}{\partial t} = -\frac{c_s^2}{c_p\theta_0^2}\left[\frac{\partial}{\partial y}(\theta_0 v) + \frac{\partial}{\partial z}(\theta_0 w)\right].$$

The right-hand sides of (1)-(4) will be denoted f_v, f_w, f_θ, and f_π. Their centered, second-order accurate finite-difference forms are

$$f_v = -v\delta_{2y}v - \overline{\overline{w}^y\delta_z v}^z - c_p\theta_0\delta_y\pi_1 + D_v, \tag{6}$$

$$f_w = -\overline{\overline{v}^z\delta_y w}^y - w\delta_{2z}w - c_p\overline{\theta_0}^z\delta_z\pi_1 + g(\frac{\bar{\theta}^z}{\overline{\theta_0}^z} - 1) + D_w, \tag{7}$$

$$f_\theta = -\overline{v\delta_y\theta}^y - \overline{w\delta_z\theta}^z + D_\theta, \tag{8}$$

$$f_\pi = -\frac{c_s^2}{c_p\theta_0^2}\left[\delta_y(\theta_0 v) + \delta_z(\overline{\theta_0}^z w)\right]. \tag{9}$$

Each of (1)-(4) can be written as

$$\frac{\partial\phi}{\partial t} = f_\phi,$$

4 Turbulence closure

For simplicity, we will use the eddy viscosity approach. Then terms D_v, D_w, and D_θ each have the form

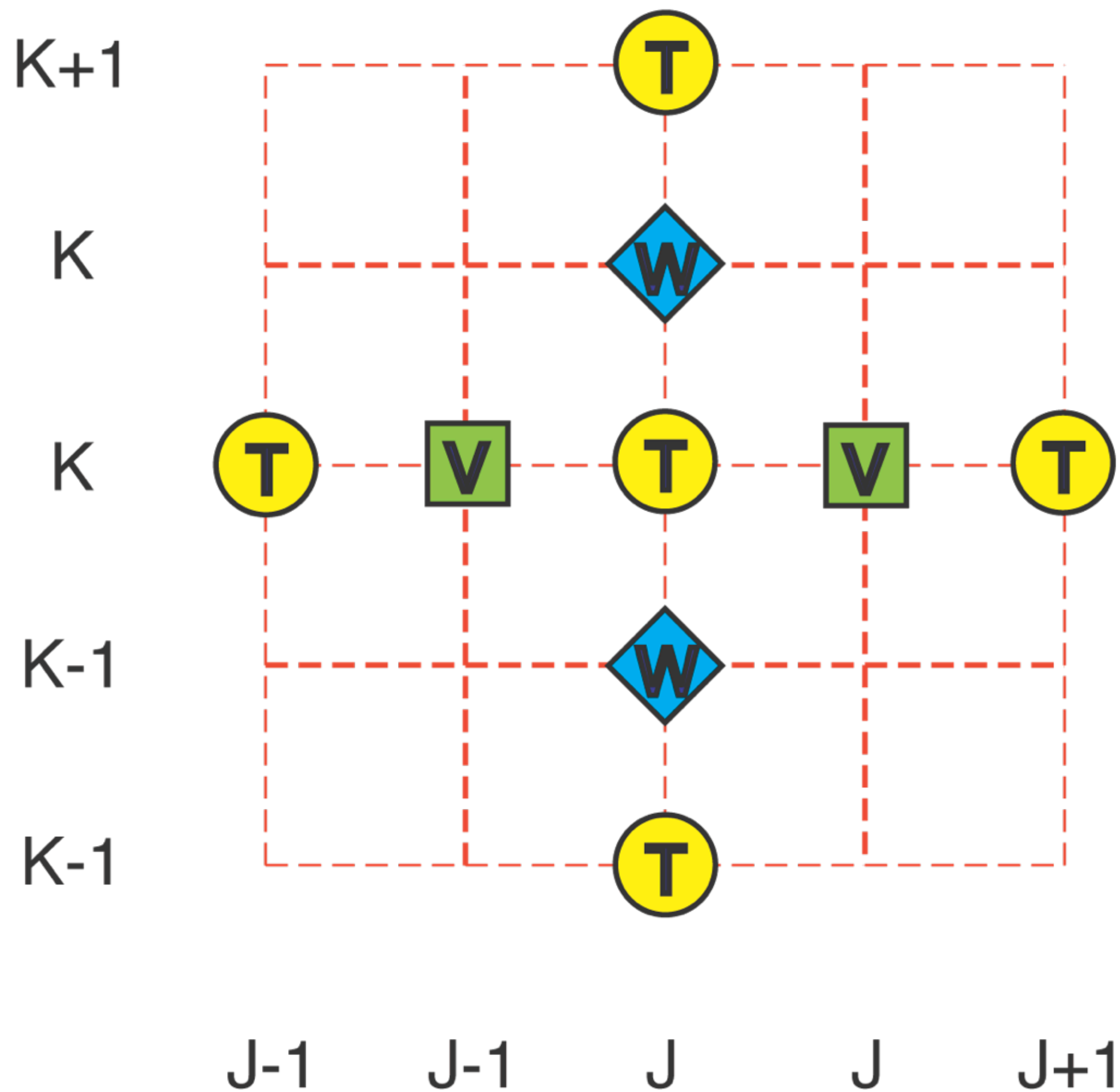
$$K_\phi \nabla^2 \phi,$$

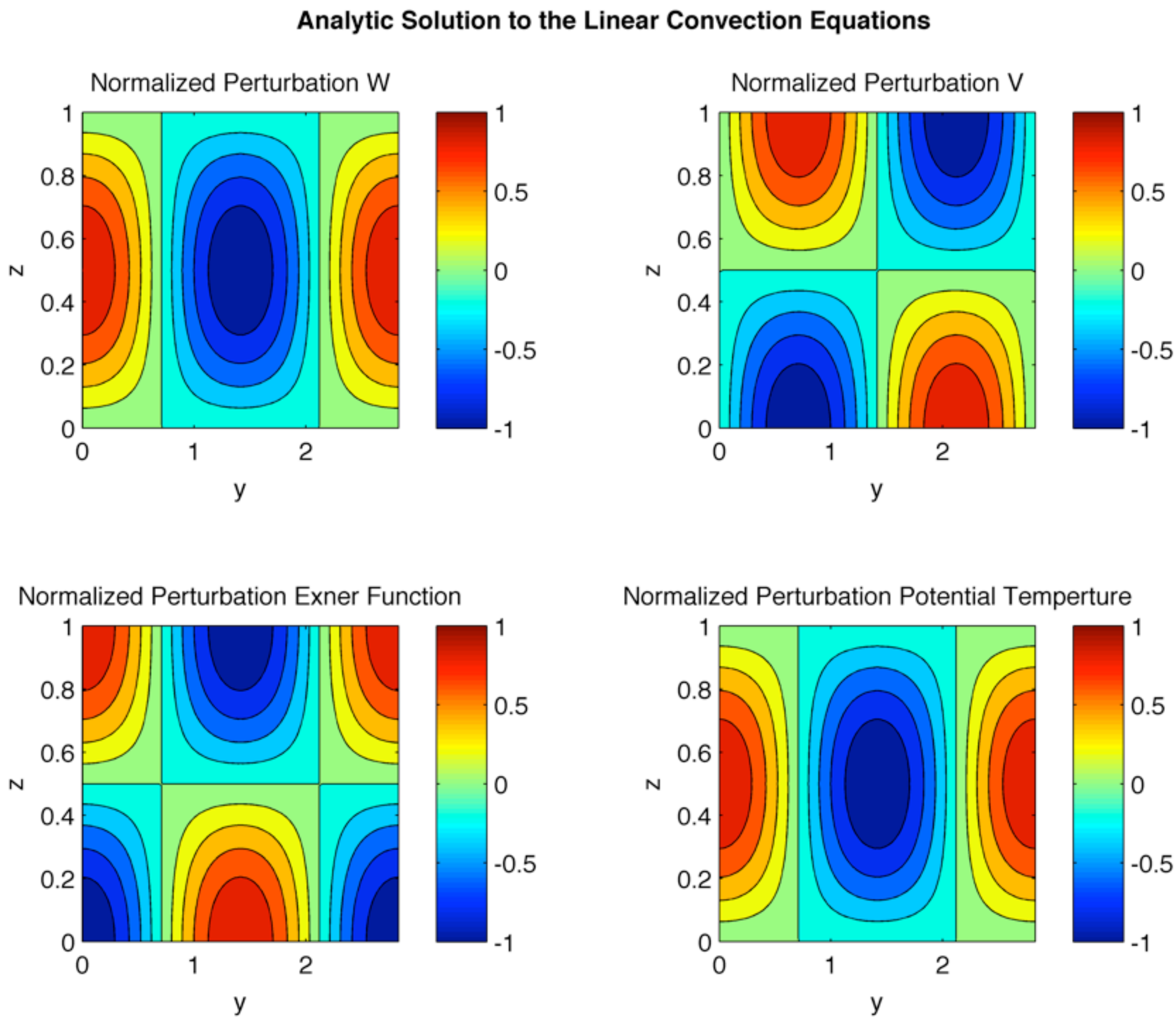
or, in finite-difference form,

$$K_\phi[\delta_y(\delta_y \phi) + \delta_z(\delta_z \phi)],$$

where K_ϕ is the eddy diffusivity.

Array subscripting convention




Analytic Solution to the Linear Convection Equations
Normalized Perturbation W
Normalized Perturbation V
Normalized Perturbation Exner Function
Normalized Perturbation Potential Temperture
z
y
0
0.2
0.4
0.6
0.8
1
2
-1
-0.5
0.5