A Novel Approach for Simulating Droplet Microphysics in Entraining Clouds

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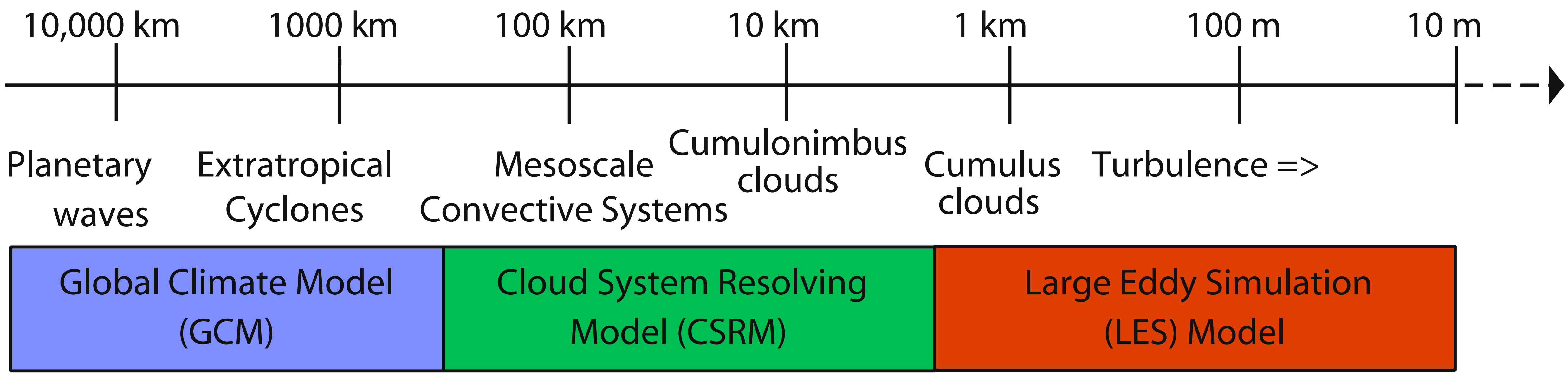
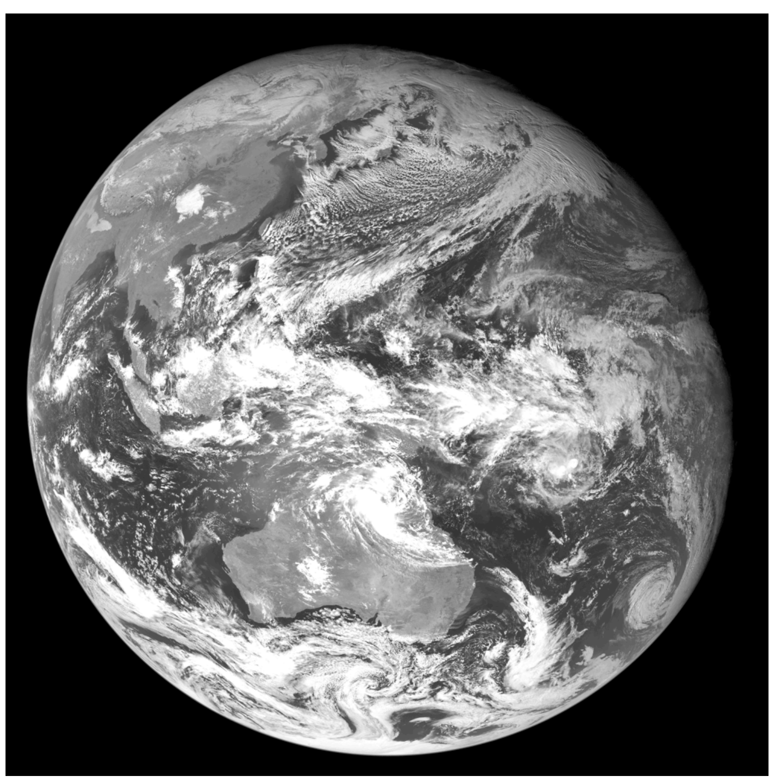
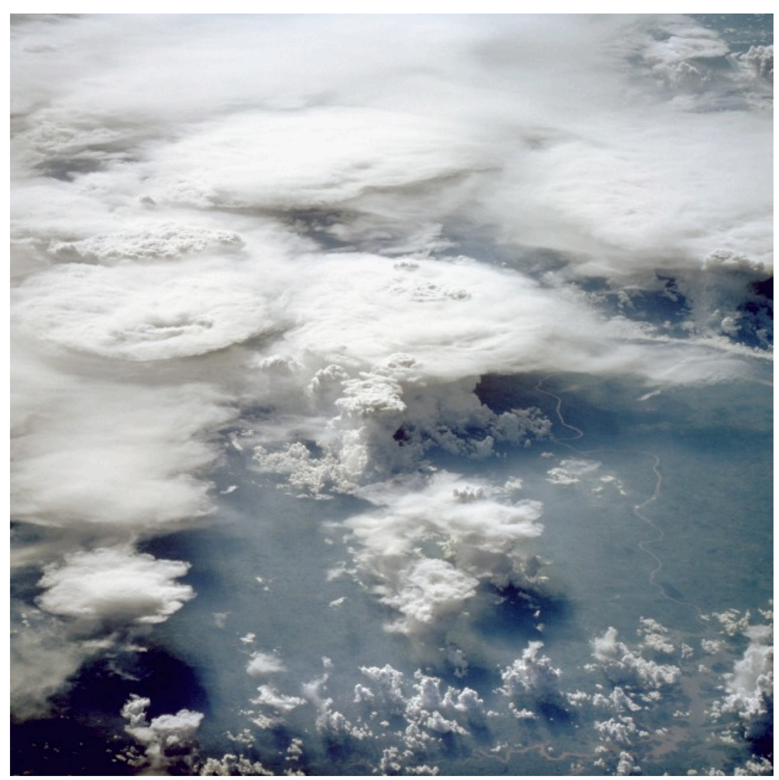
25 April 2012

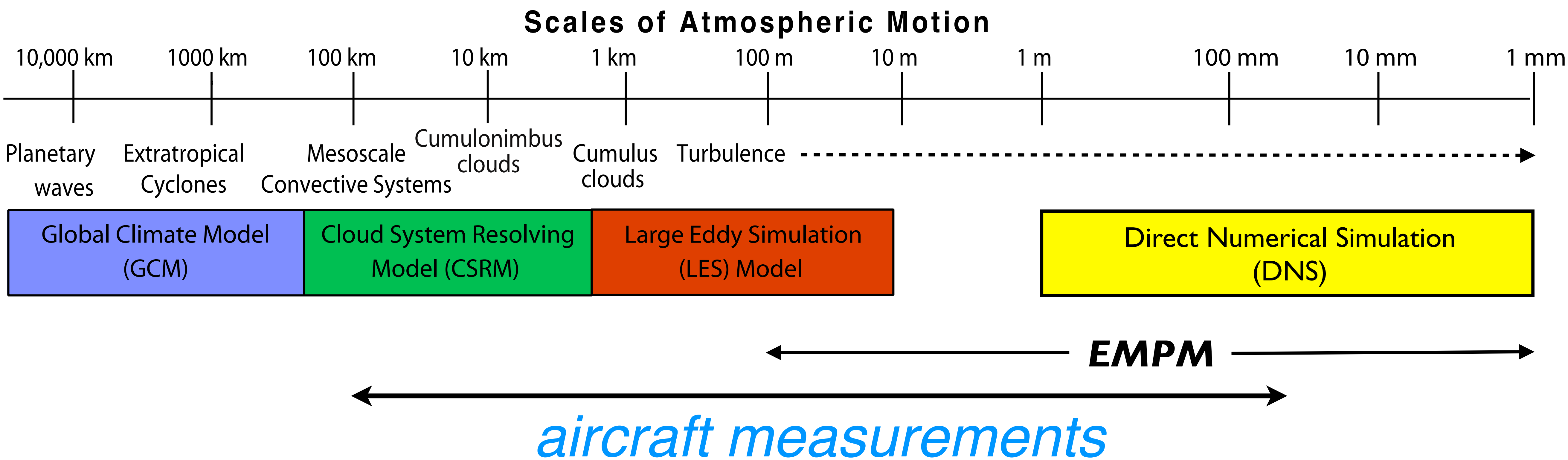
Jay Oh, Helena Schluter, Pam Lehr, Chwen-Wei Su, Phil Austin, Pat McMurtry

Scales of Atmospheric Motion









The smallest scale of turbulence is the Kolmogorov scale:

$$\eta \equiv (\nu^3/\epsilon)^{1/4}$$

For $\epsilon = 10^{-2}$ m^2 s^{-3} and $\nu = 1.5 \times 10^{-5}$ m^2 s^{-1}, $\eta = 0.7$ mm.

OUTLINE

- Cloud droplet microphysics
- Large-eddy simulation
- Linear Eddy Model (LEM)
- Explicit Mixing Parcel Model (EMPM)
- ClusColl (Clustering and Collision Model)

- **Cloud droplet microphysics**
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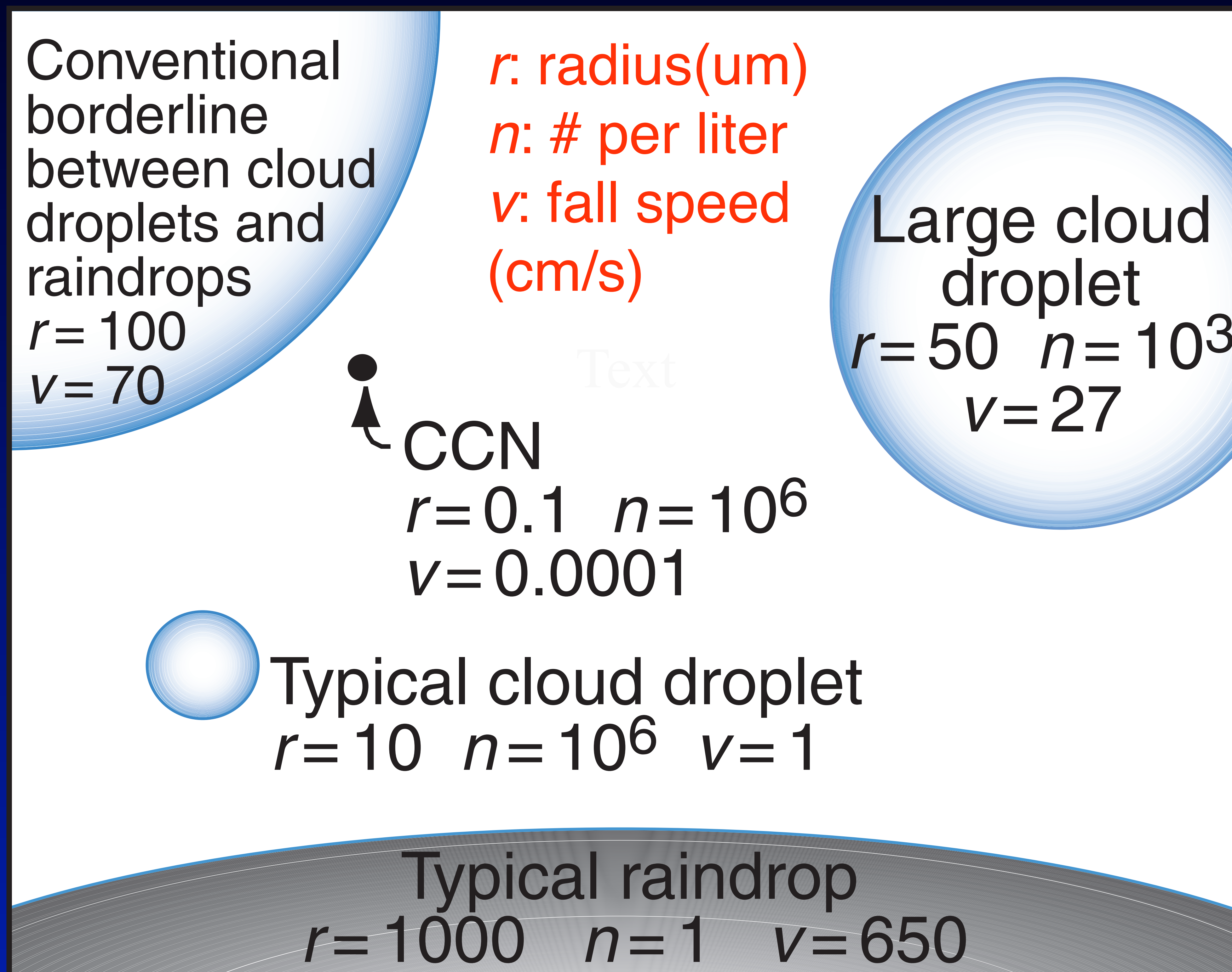
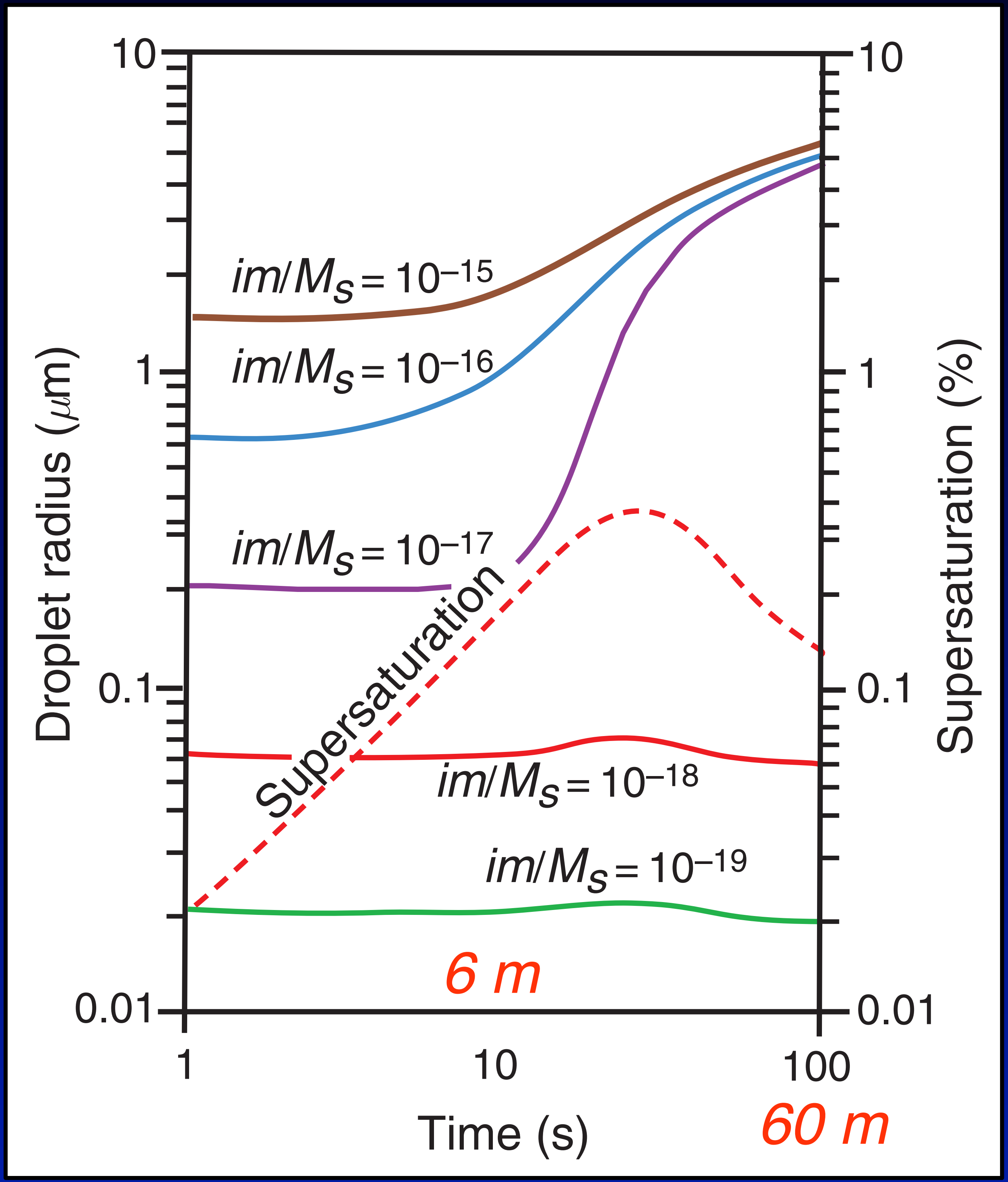
Growth of Cloud Droplets in Warm Clouds

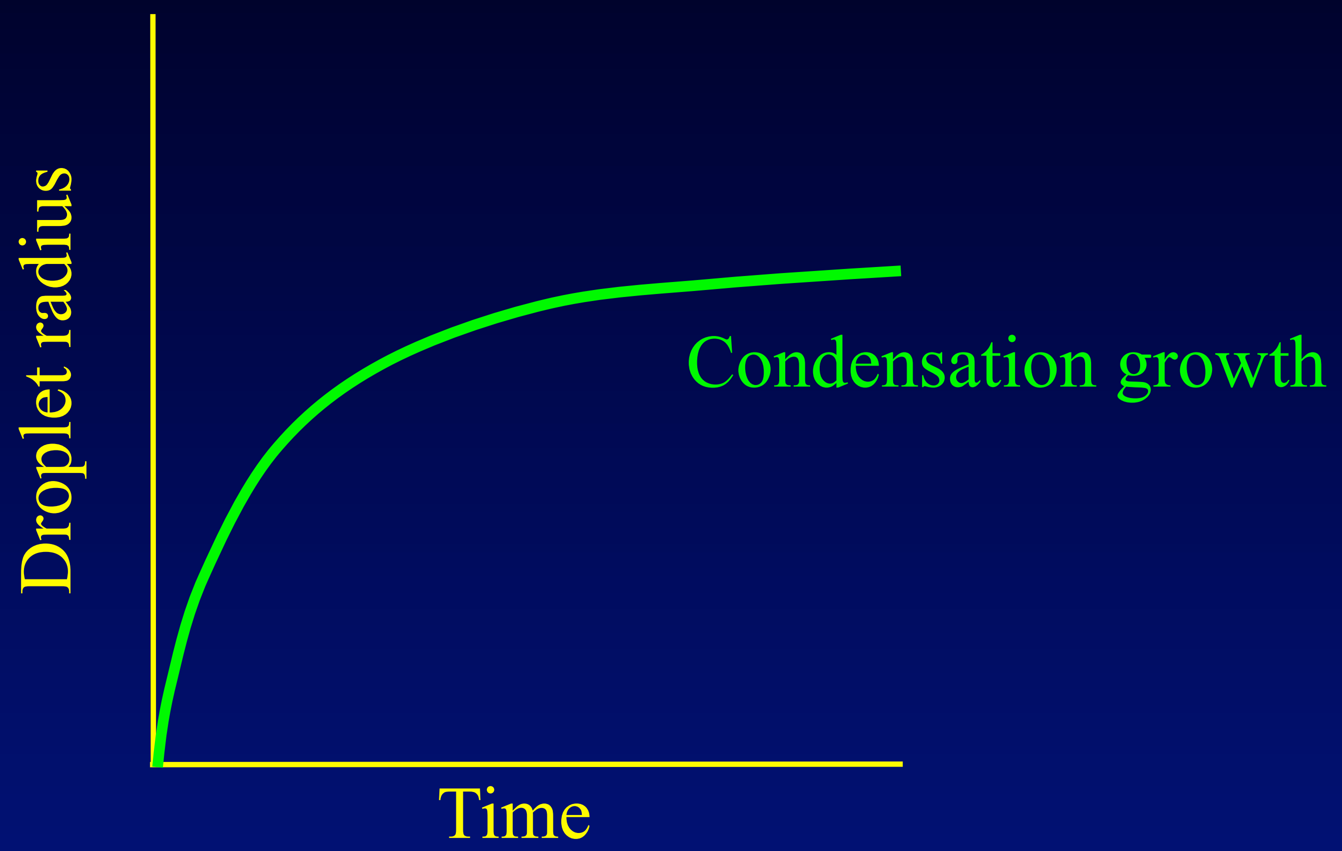


Cloud Condensation Nuclei (CCN)

- Calculation of the growth of cloud droplets from a CCN population (500/cm^3) by condensation in an updraft of 60 cm/s.
- Activated droplets are monodisperse by 100 s (60 m).



Condensation



- Increase of droplet radius by condensation is initially rapid, but diminishes as droplet grows.
- Condensational growth by itself cannot produce raindrops. (First noted by Osborne Reynolds in 1877.)

Collision-coalescence

VOL. 5, NO. 5 JOURNAL OF METEOROLOGY OCTOBER 1948

THE PRODUCTION OF RAIN BY A CHAIN REACTION IN CUMULUS CLOUDS AT TEMPERATURES ABOVE FREEZING

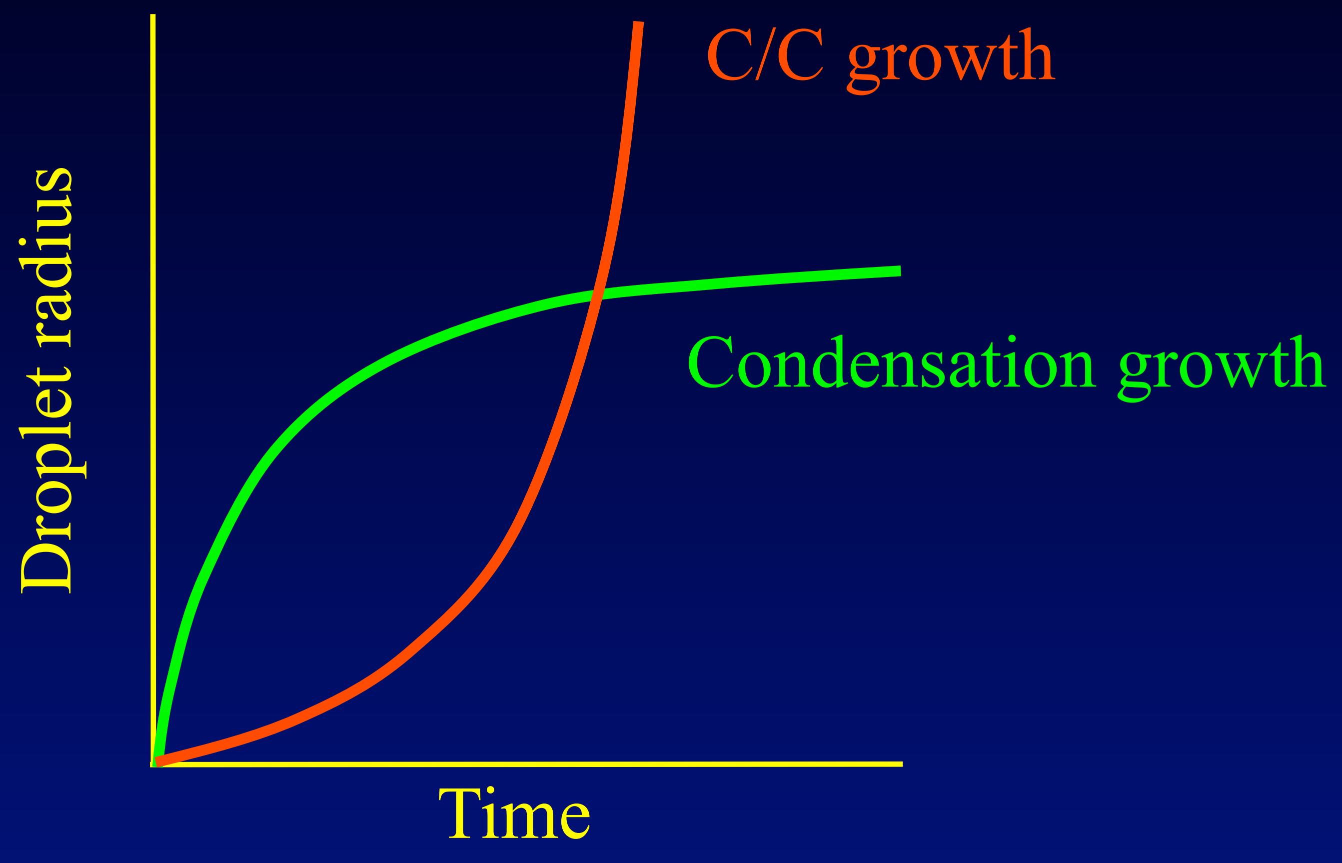
By Irving Langmuir

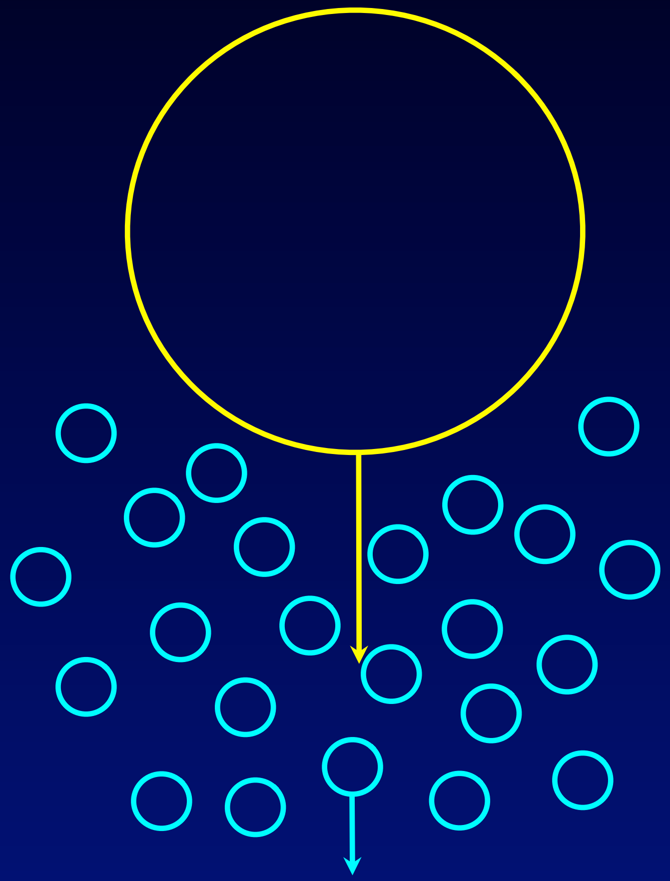
General Electric Research Laboratory, Schenectady, New York[1,2]

(Manuscript received 9 April 1948)

Since cumulus clouds often develop rain within less than thirty minutes after their formation, we see that some mechanism other than that assumed in the evaporation–condensation theory must be involved in rain formation.

Collision-coalescence





- Growth of droplets into raindrops is achieved by collision-coalescence.
- Fall velocity of a droplet increases with size.
- Larger drops collect smaller cloud droplets and grow.

Collision-coalescence

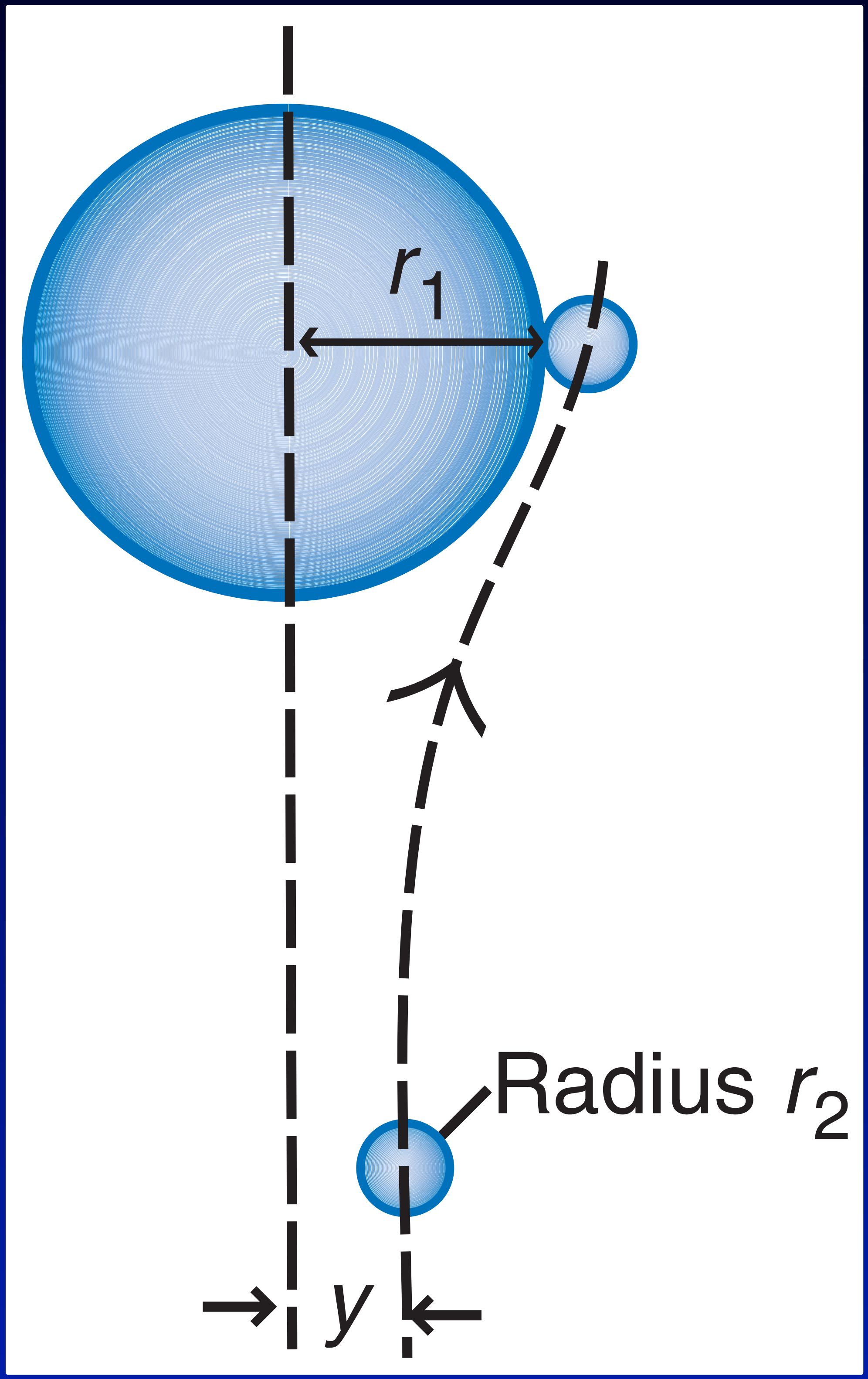
Relative motion of a droplet with respect to a collector drop. At the radius y the two make a grazing collision.

The *collision efficiency* is

$$E = \frac{\text{effective collision cross section}}{\text{geometrical collision cross section}}$$

therefore

$$E = \frac{y^2}{(r_1 + r_2)^2}$$



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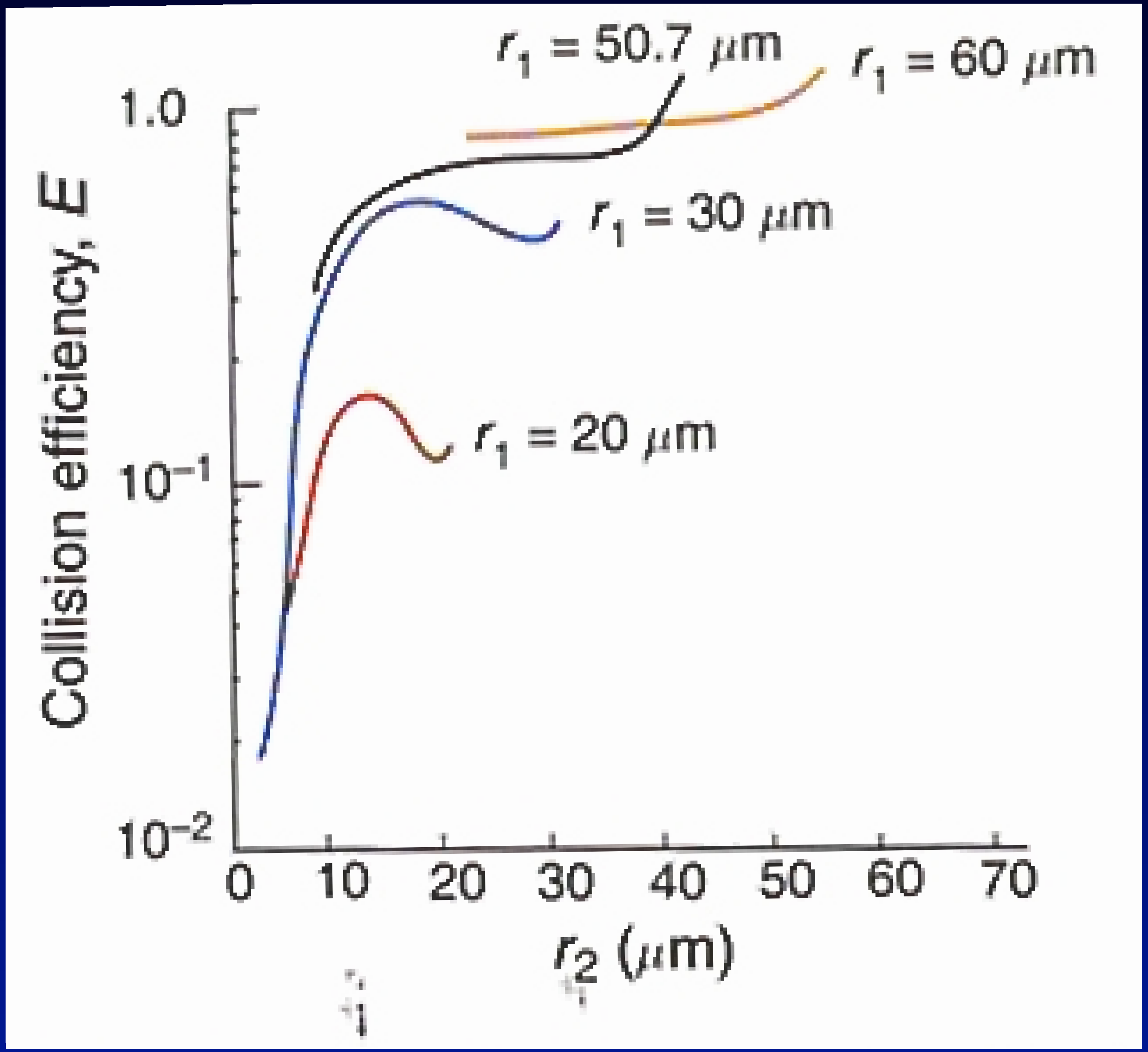
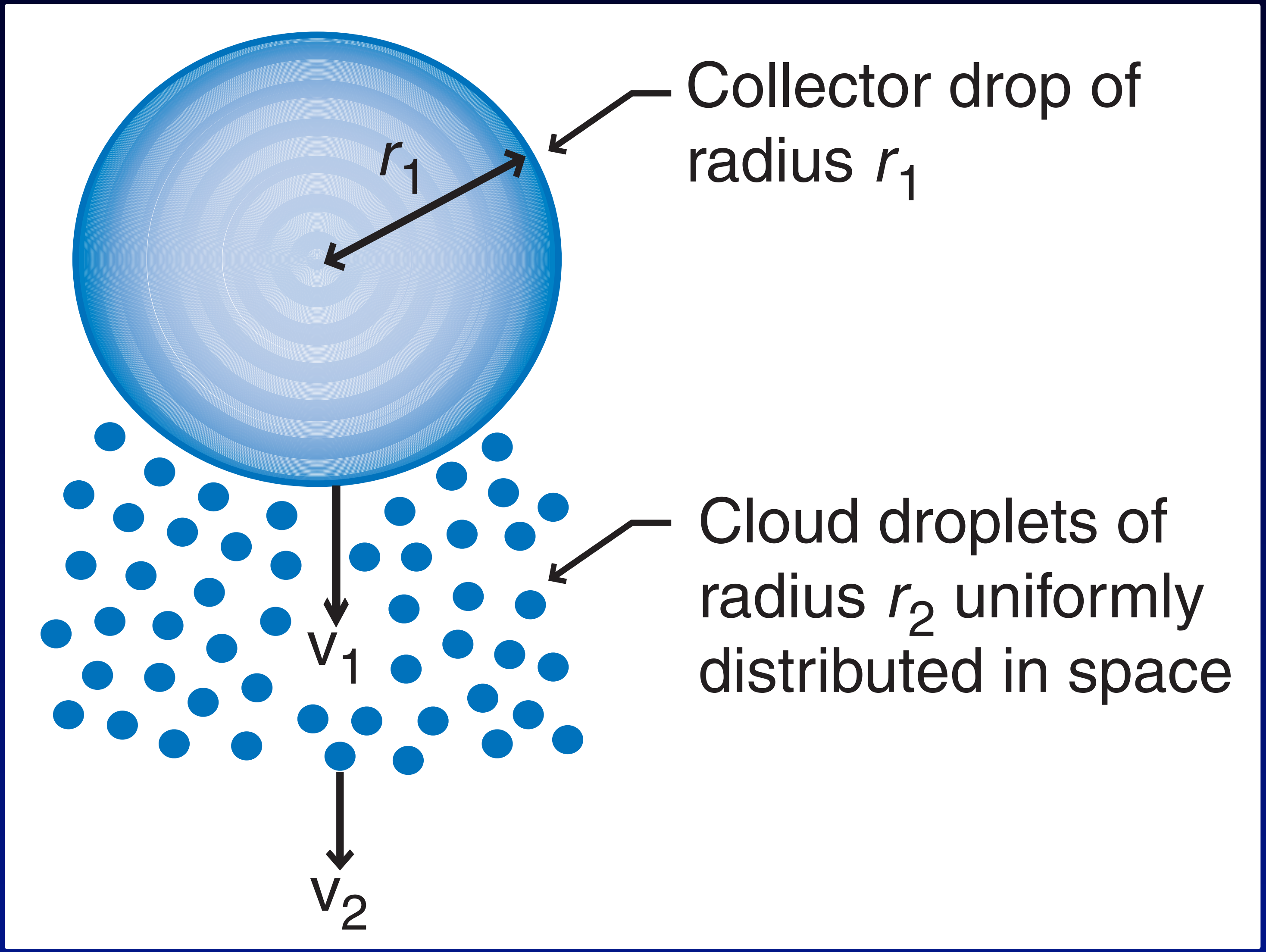


METEOROLOGY

Collision-coalescence

According to the *continuous collection model*, the rate of increase of the collector drop's mass M due to collisions is the volume of the cylinder swept out per unit time by the collector drop moving at the relative velocity $v_1 - v_2$ × LWC × collection efficiency:



$$\frac{dM}{dt} = \pi r_1^2 (v_1 - v_2) w_l E_c$$

where w_l is the LWC of the cloud droplets of radius r_2.

Collision-coalescence

VOL. 13, NO. 2 REVIEWS OF GEOPHYSICS AND SPACE PHYSICS MAY 1975

Theoretical Cumulus Dynamics

WILLIAM R. COTTON[1]

Experimental Meteorology Laboratory, NOAA Environmental Research Laboratories
Boulder, Colorado 80302

It is quite evident that the major uncertainties in modeling the collection process in a cumulus cloud are associated with its turbulent structure.

METEOROLOGY

- *Goal:* To develop an economical model that represents the essential processes that contribute to the rapid formation of rain drops by collision and coalescence of cloud droplets.

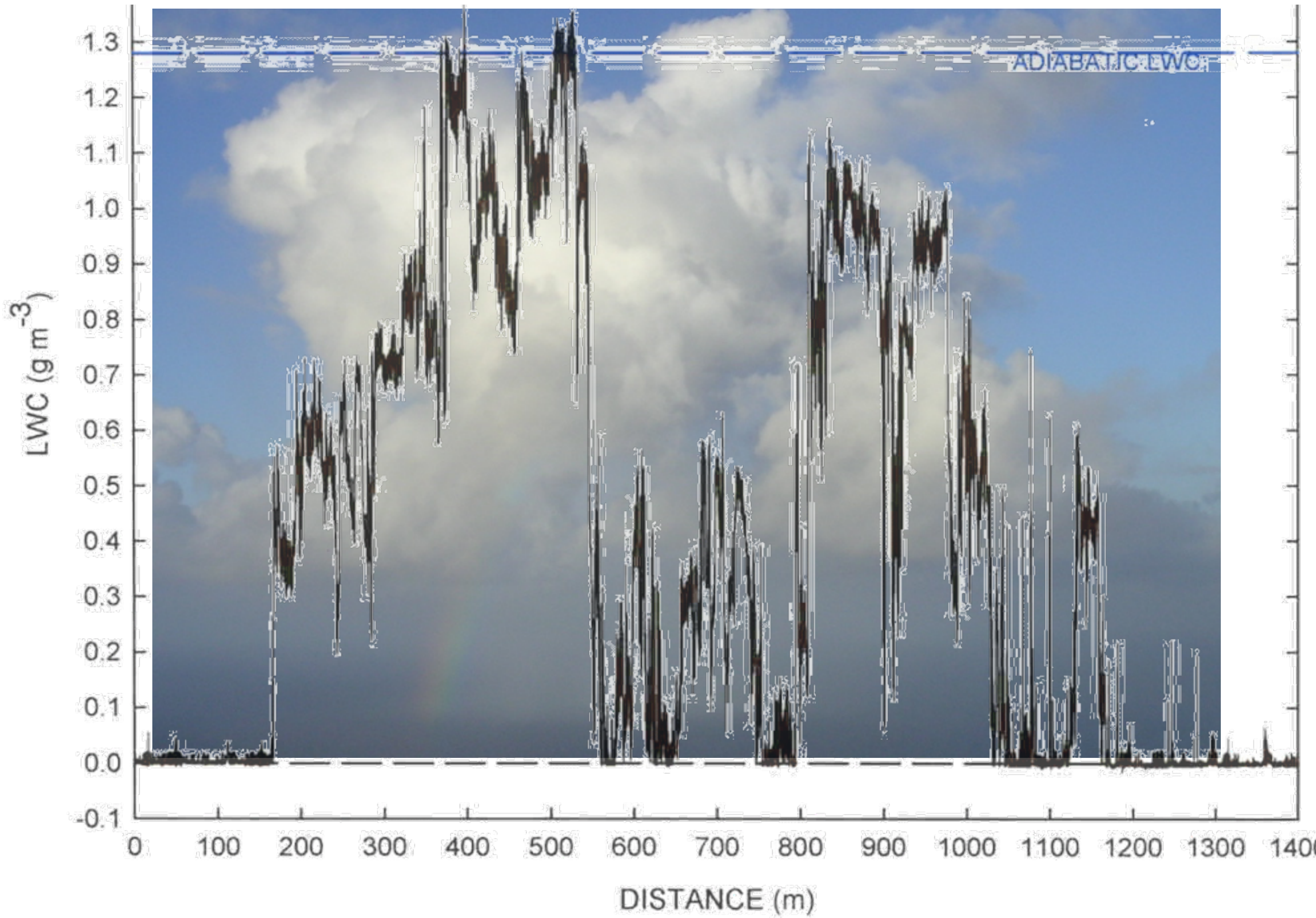
- *Processes that may contribute:*

    - Entrainment and mixing of unsaturated air

    - Droplet clustering due to turbulence

    - Giant aerosols

Small-scale variability in Cumulus mediocris



overlay is for illustration only

Small-scale variability in Cumulus fractus

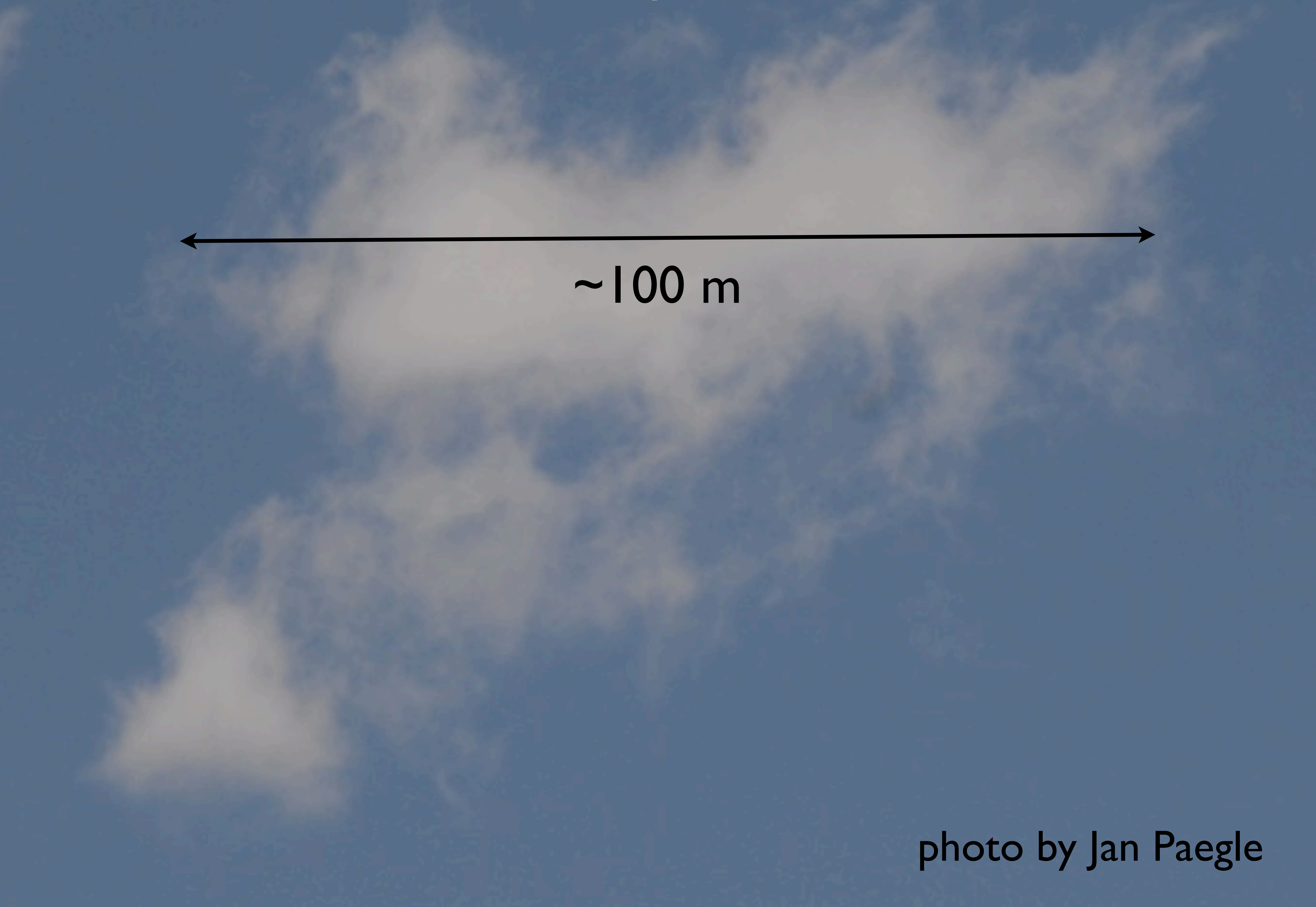


photo by Jan Paegle

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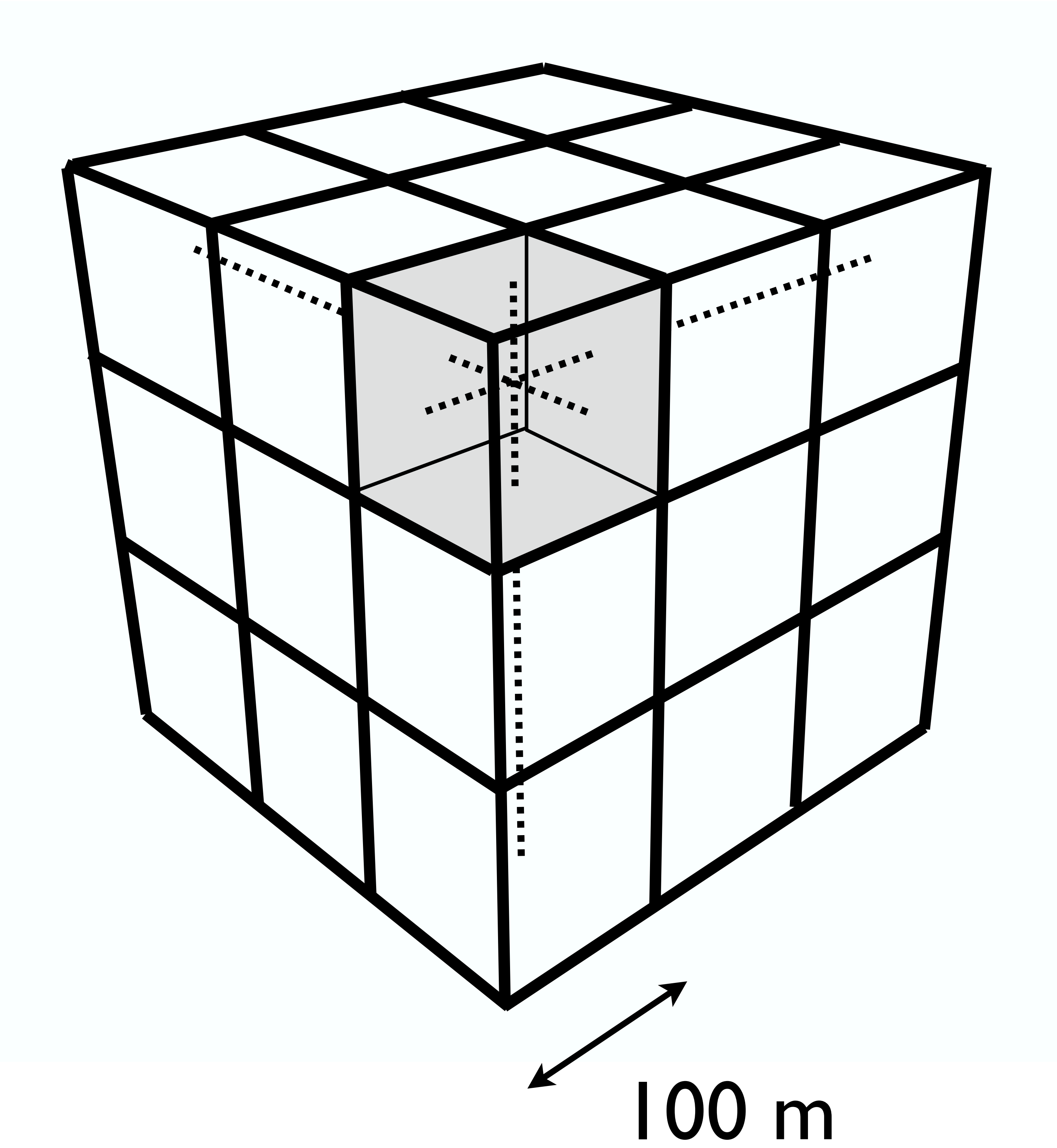
Large-Eddy Simulation (LES) model

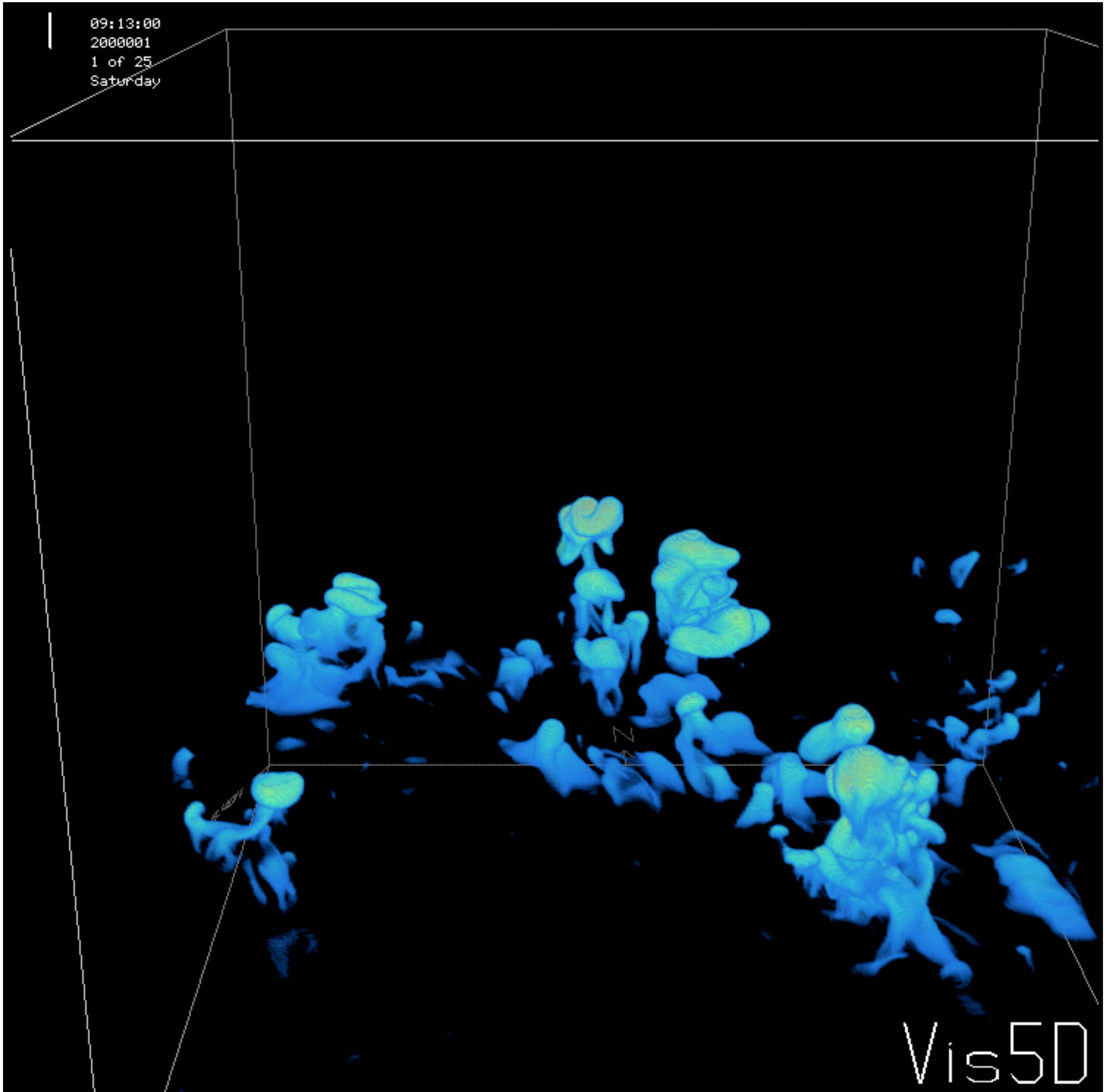


no subgrid-scale variability

LES Limitations

- The premise of LES is that only the large eddies need to be resolved.
- LES is appropriate if the important small-scale processes can be parameterized.
- Many cloud processes are subgrid-scale, yet can't (yet) be adequately parameterized.

Joseph Zehnder, Santa Cataline Mountain Project



Mike Zulauf, Univ of Utah

Subgrid-scale Cloud Processes

- Small-scale finite-rate **mixing** of clear and cloudy air determines evaporative cooling rate and affects buoyancy and cloud dynamics.
- Small-scale variability of water vapor due to entrainment and **mixing** broadens droplet size distribution (DSD) and increases droplet collision rates.
- Small-scale **turbulence** increases droplet collision rates.

Large droplets can initiate collision-coalescence growth

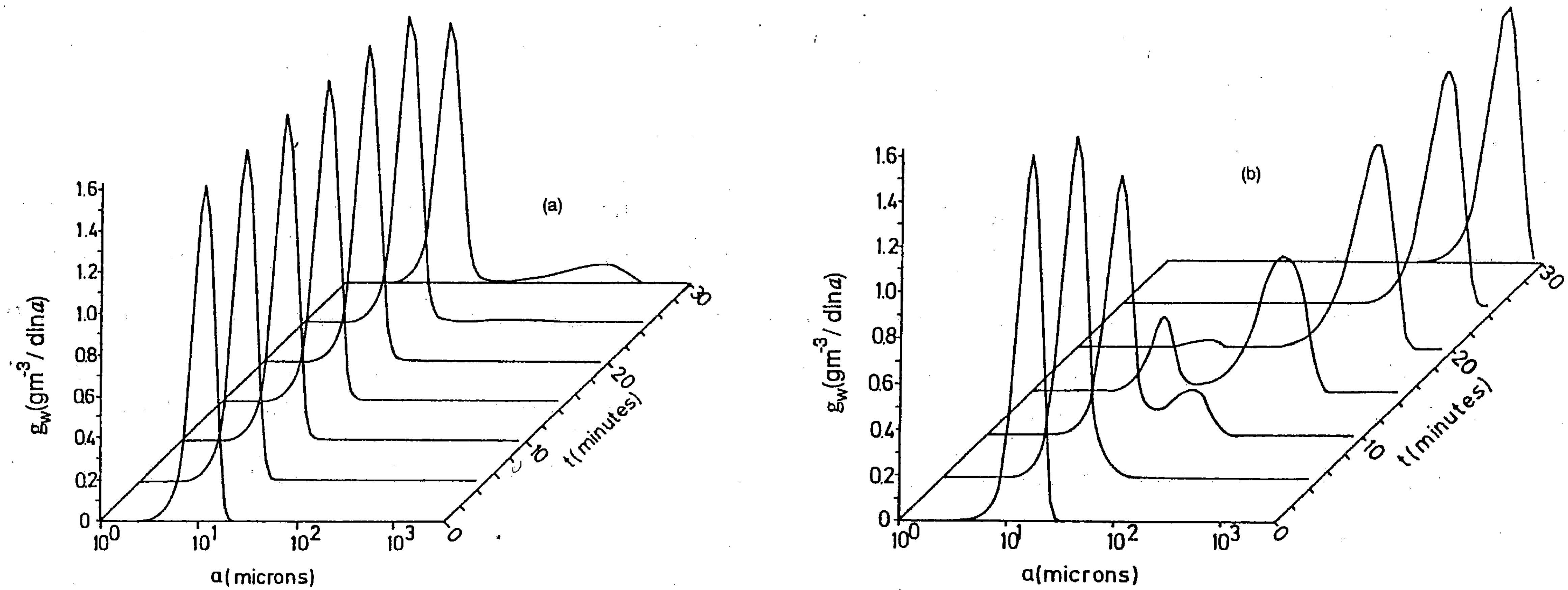


Fig. 15-8: Three-dimensional display of the time evolution of the drop mass distribution function as a function of drop radius, for an assumed initial spectrum of drops growing by collision and coalescence: (a) $\overline{a} = 9\ \mu$m, $N_d = 237$ cm^{-3}, $w_L = 1$ g m^{-3}; (b) $\overline{a} = 13\ \mu$m, $N_d = 108$ cm^{-3}, $w_L = 1$ g m^{-3}. Based on the Berry Reinhardt method. (From Flossmann *et al.*, 1985, with changes.)

Entrainment and mixing affect cloud droplet size distributions

An unsaturated blob is entrained at 375 s

some individual droplet radii

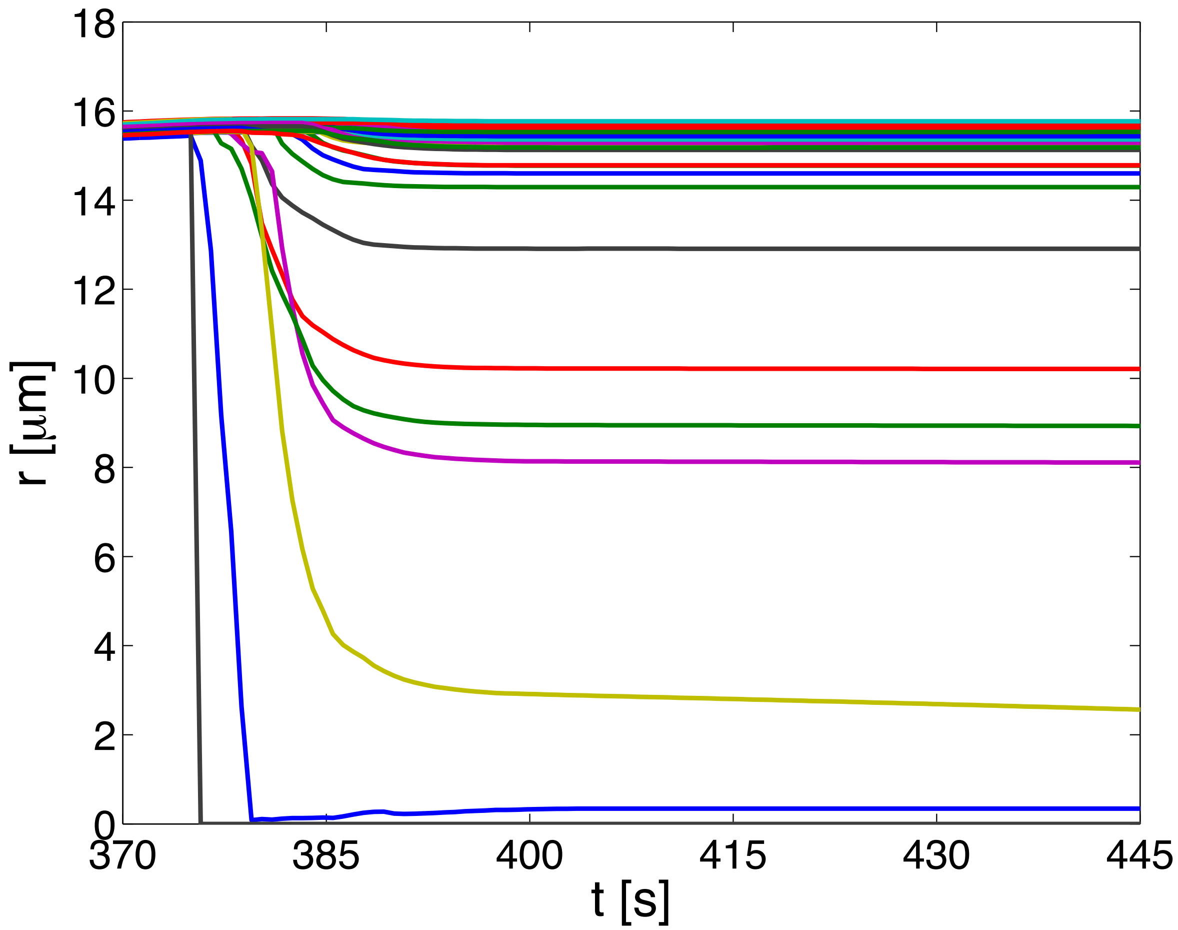


Figure 4.10: Radius histories of 30 droplets for $f = 0.1$ and $RH_e = 0.219$.

width of droplet size distribution

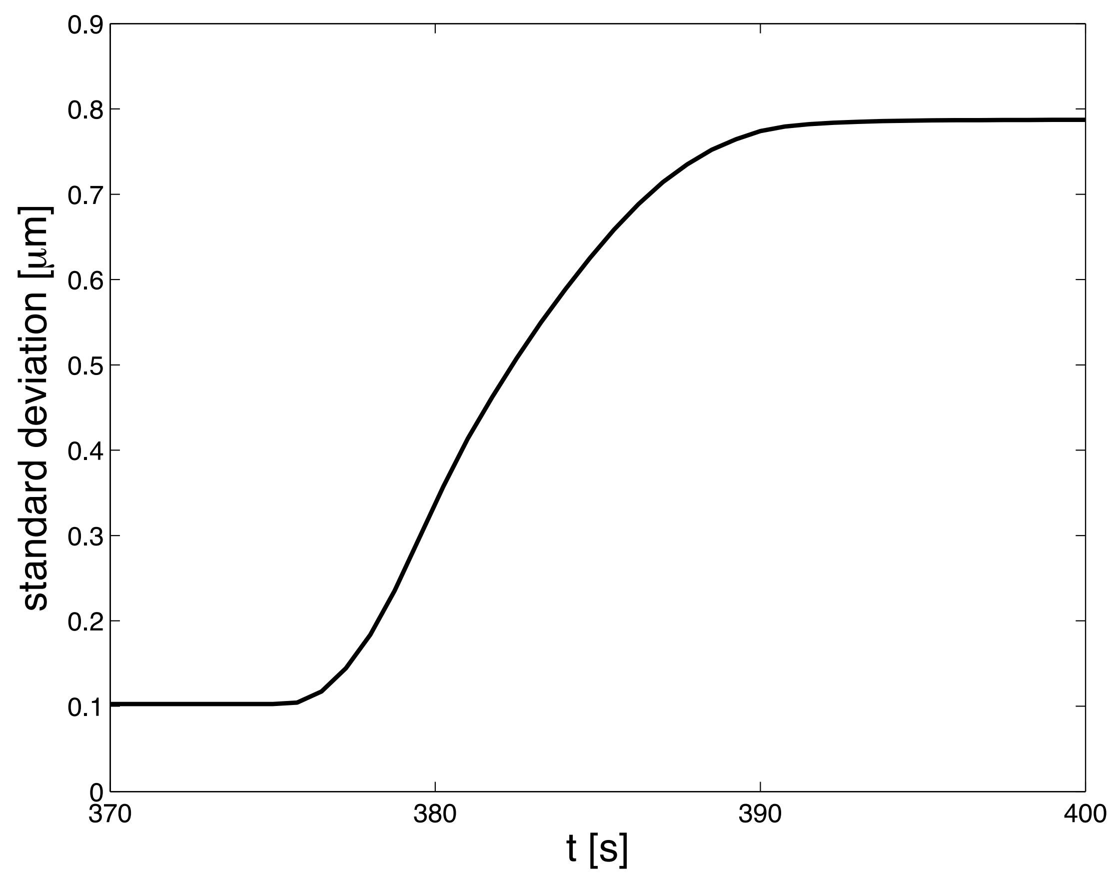
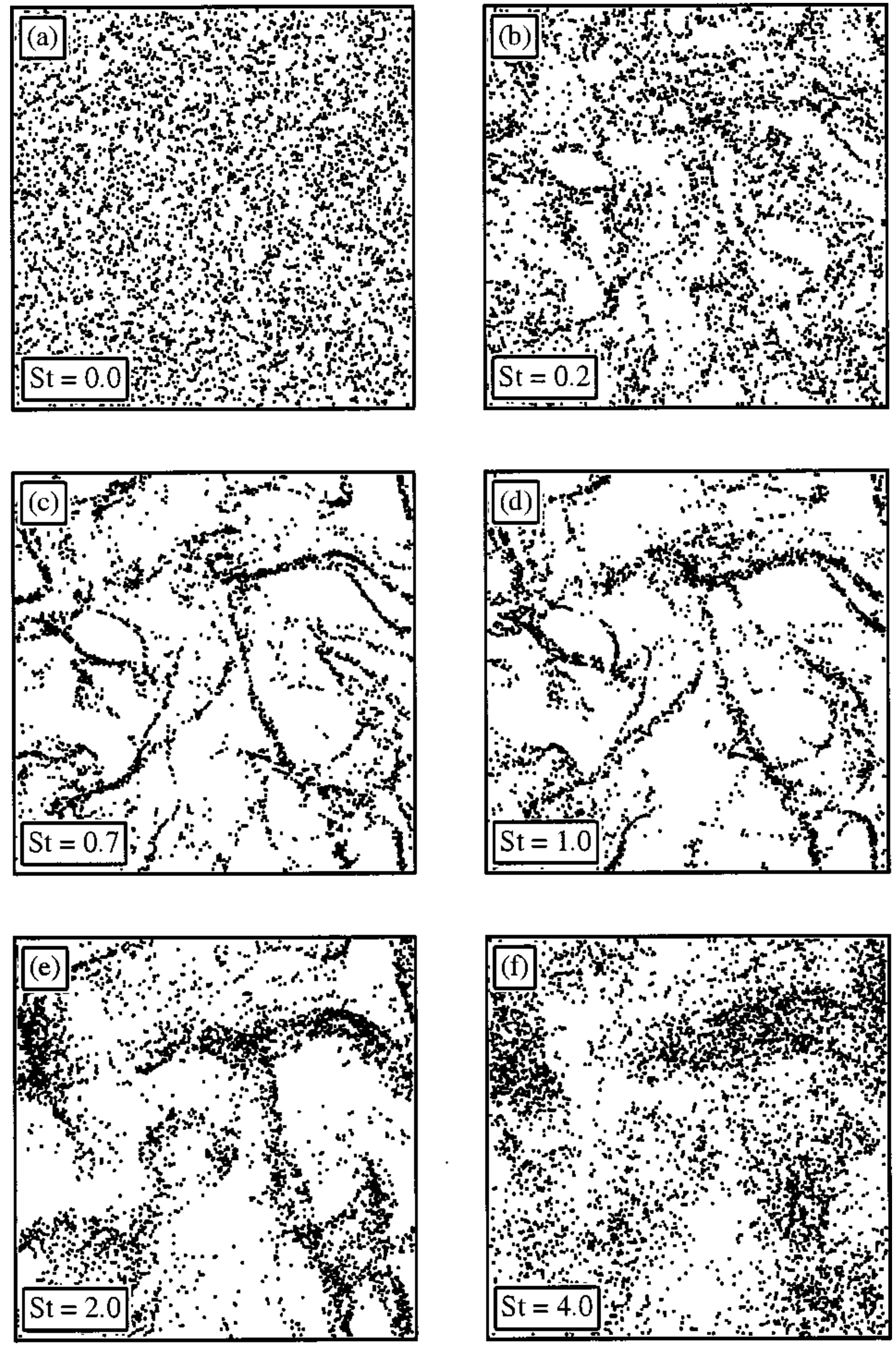


Figure 4.6: Standard deviation of the droplet radii just before entrainment until homogenization for entrainment fraction $f = 0.2$ for the control case.

Helena Schluter, Univ of Utah

Clustering of inertial particles in turbulence increases collision rates



Direct numerical simulation results from Reade & Collins (2000)

The *collision rate* between droplets with radii r_1 and r_2 is

$$\dot{C} \equiv \frac{\langle C \rangle}{\Delta t} = \frac{\Gamma N_1 N_2}{V},$$

where $\langle C \rangle$ is the number of collisions in volume V in time interval Δt, Γ is the *collision kernel*, and N_1 and N_2 are the number of droplets with radii r_1 and r_2 in volume V.

The mean collision rate in the presence of fluctuations of N_1 and N_2 is

$$\overline{\dot{C}} \equiv \frac{\overline{C}}{\Delta t} = \frac{\Gamma(\overline{N_1}\ \overline{N_2} + \overline{N_1' N_2'})}{V}.$$

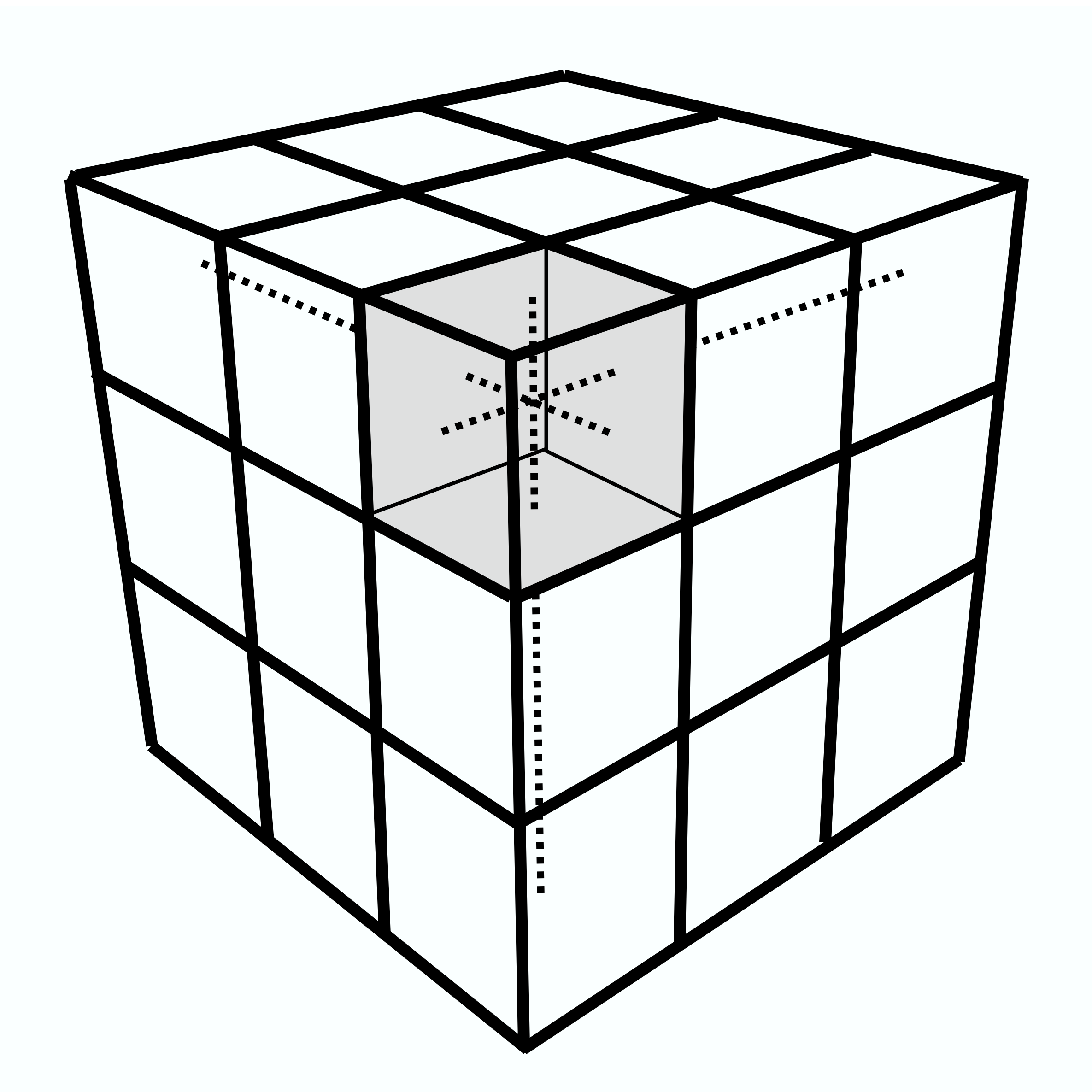
How to resolve the small-scale variability?

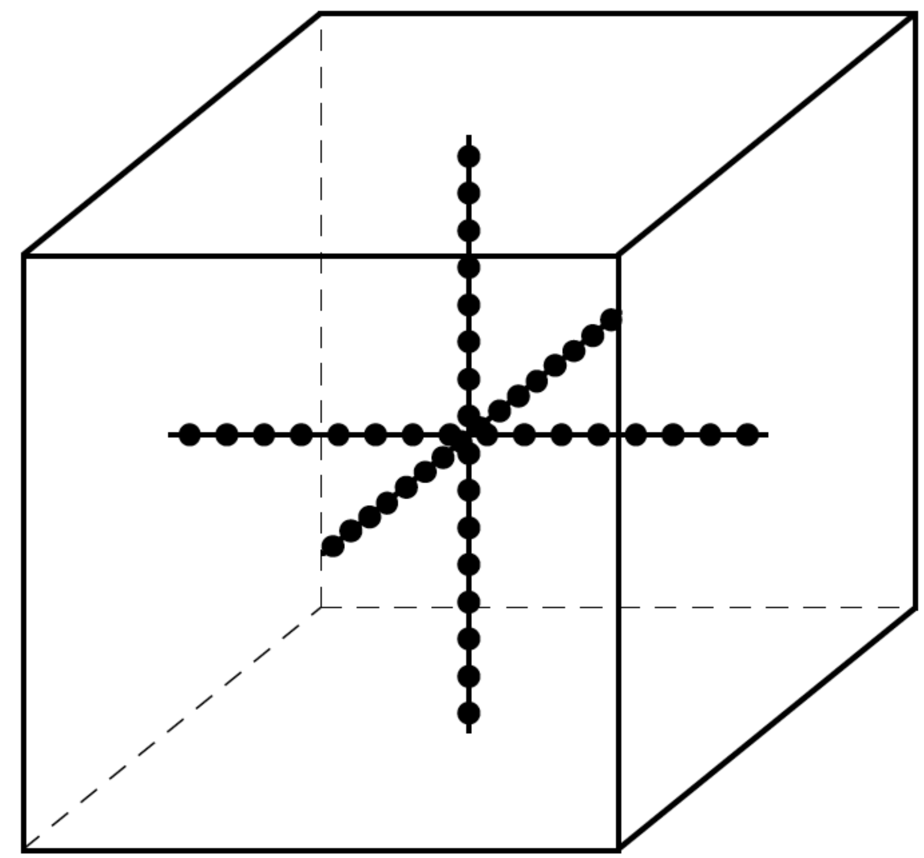
- Decrease LES grid size?
  - To decrease LES grid size from 10 m to 1 cm would require 10^9 grid points per $(10\ \mathrm{m})^3$ and an increase in CPU time of 10^{12}.
  - *This is not possible now or in the forseeable future.*

How to resolve the small-scale variability?

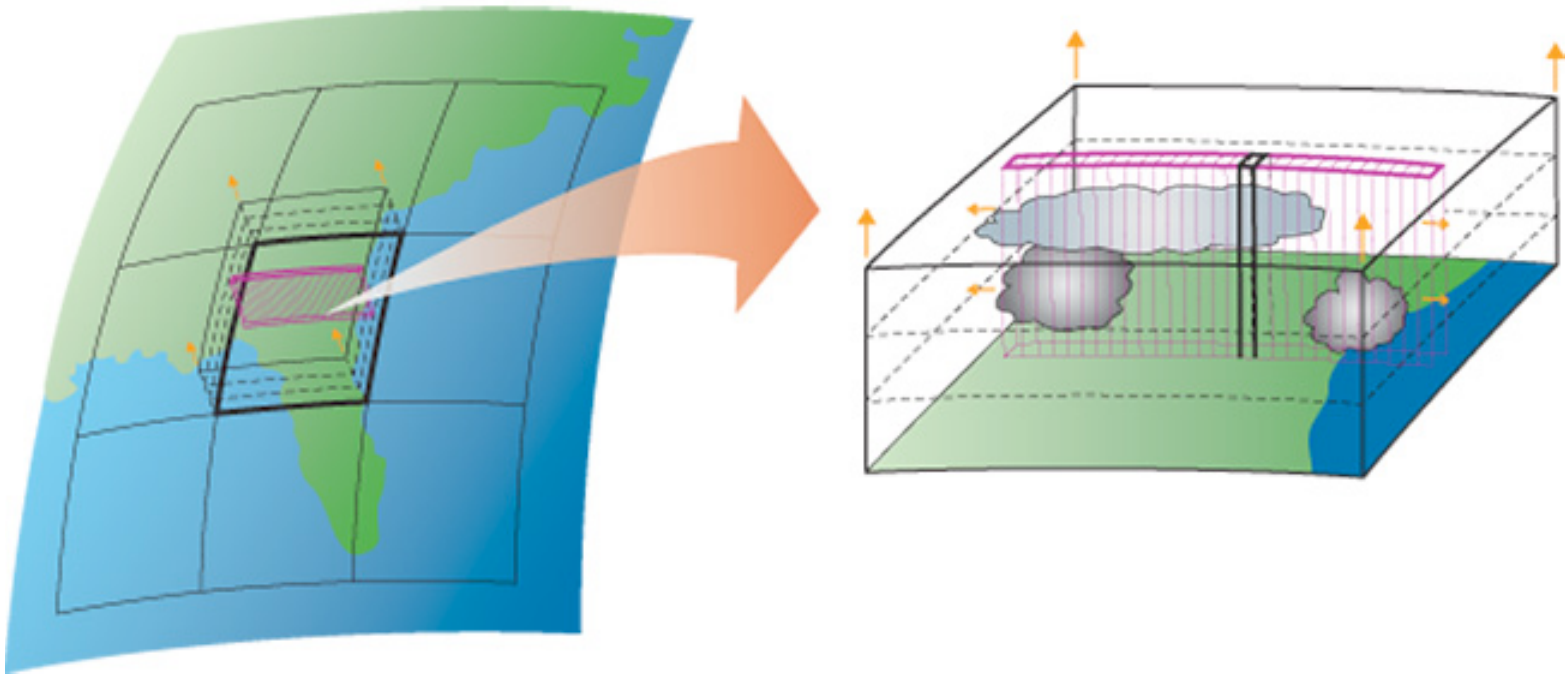
- Decrease dimensionality from 3D to 1D?
  - To decrease grid size from 10 m to 1 cm would require only 10^3 grid points per $(10\text{ m})^3$.
  - *This is feasible now.*

LES with 1D subgrid-scale model





Multiscale Modeling Framework (MMF)



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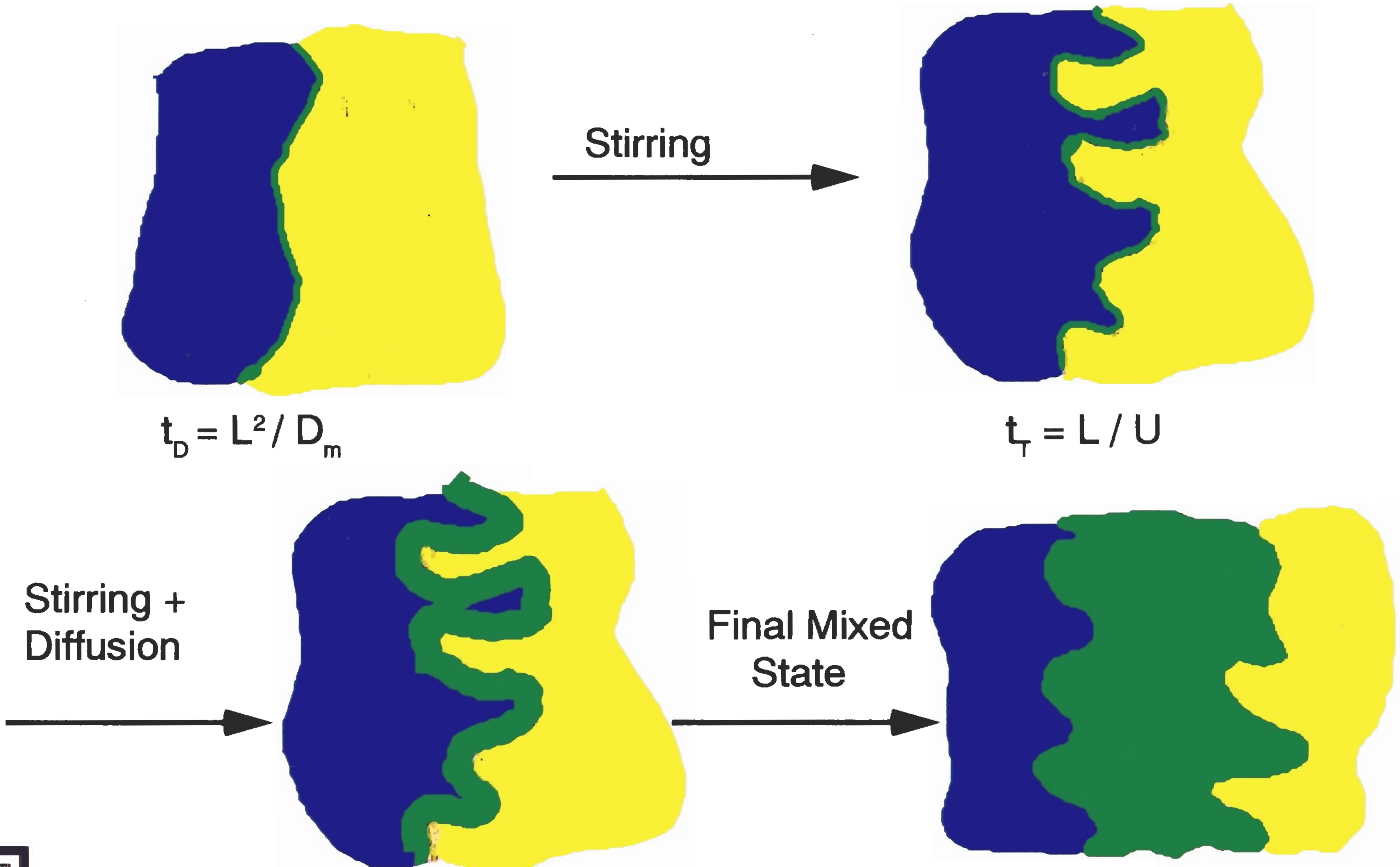
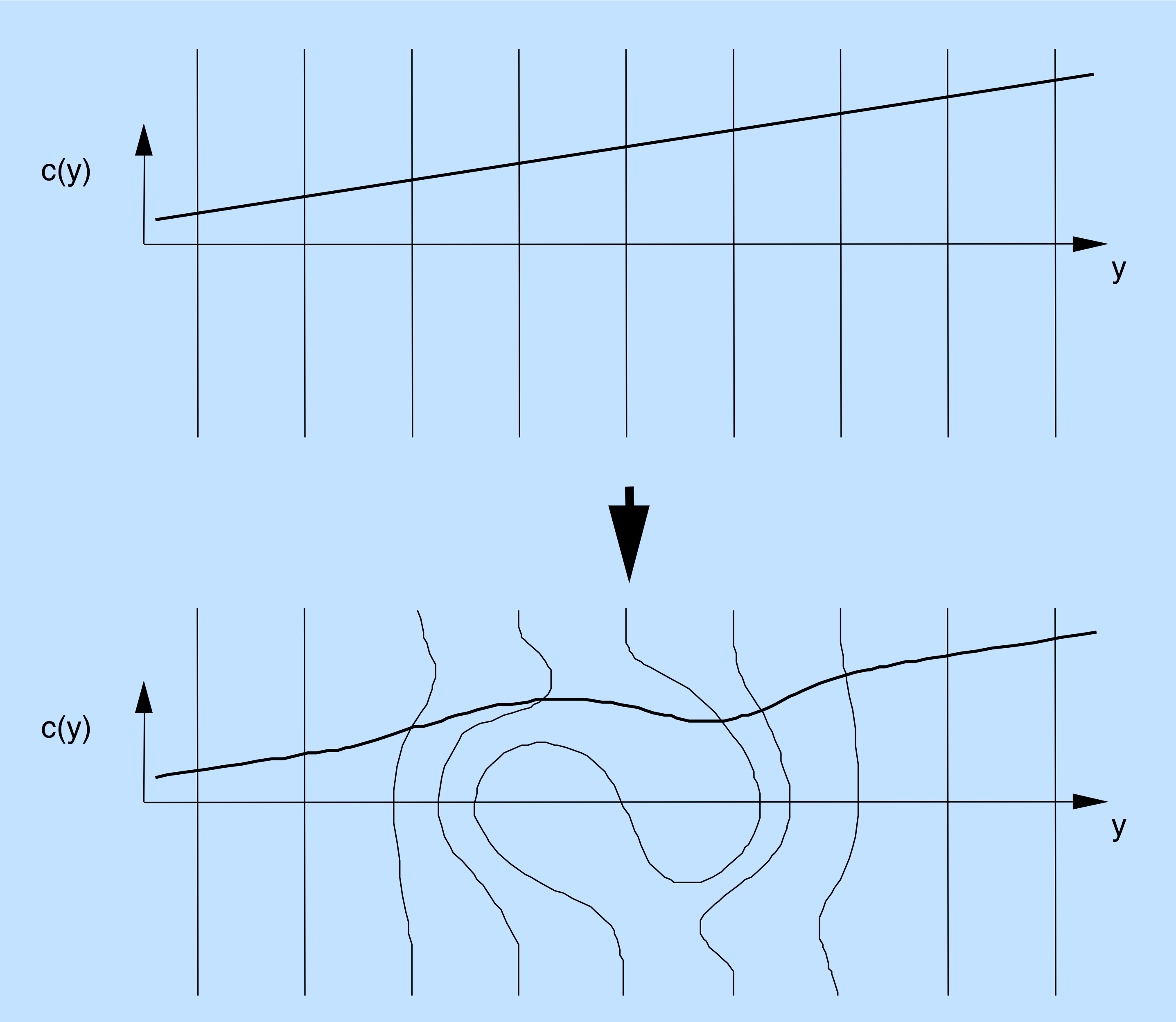
- **Linear Eddy Model (LEM)**
  - Evolves **scalar** spatial variability on all relevant turbulence scales using one dimension.
  - Distinguishes turbulent deformation and molecular diffusion.
  - Turbulence properties are **specified**.

Turbulent Mixing: Process by which a fluid with two initially segregated scalar properties mix at the molecular level



Turbulent motion of *fluid elements* is modeled as a sequence of ***triplet maps*** that preserve desired advection properties, even in 1D



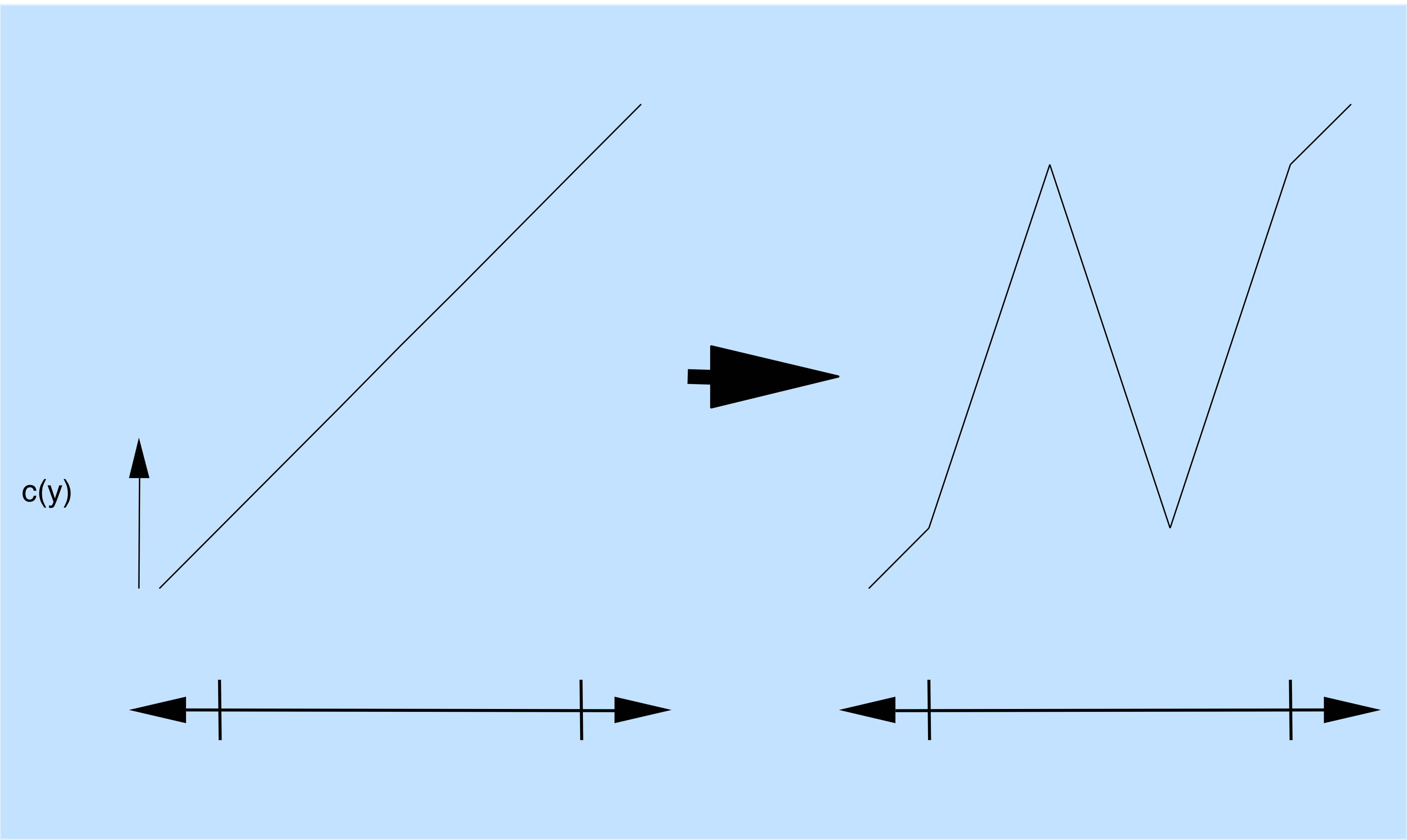
Alan Kerstein

The ***triplet map*** imitates the effect of a 3D eddy on property profiles along a line of sight.

The triplet map (1D eddy)

- <u>moves</u> fluid parcels <u>without intermixing</u> their contents
- <u>conserves</u> fluid properties
- does not cause property discontinuities
- reduces fluid separations by at most a factor of 3

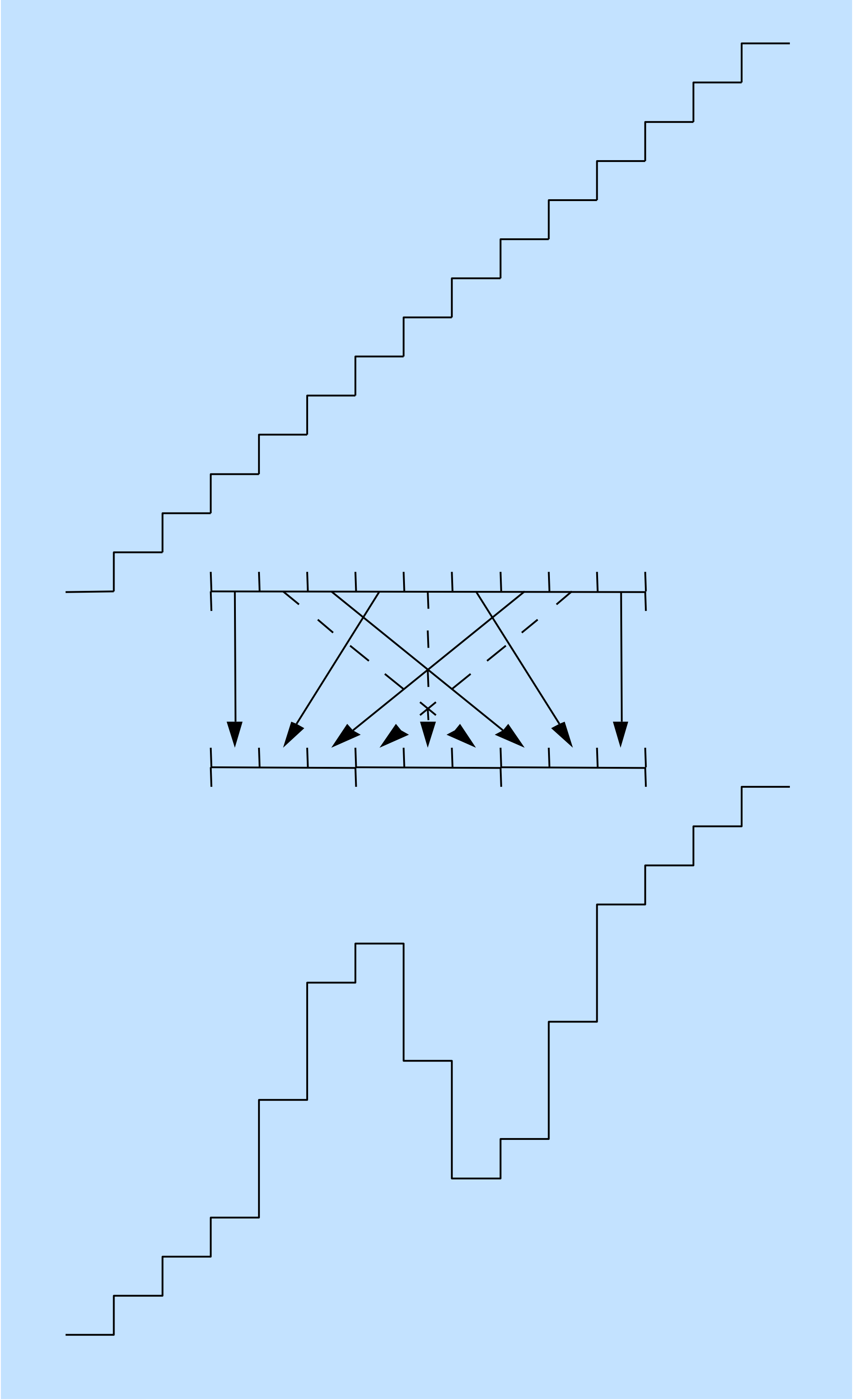
Turbulent motion of *fluid elements* is modeled as a sequence of ***triplet maps*** that preserve desired advection properties, even in 1D



Alan Kerstein

The triplet map captures compressive strain and rotational folding effects, and causes no property discontinuities.

Turbulent motion of *fluid elements* is modeled as a sequence of ***triplet maps*** that preserve desired advection properties, even in 1D



The triplet map is implemented numerically as a permutation of fluid cells.

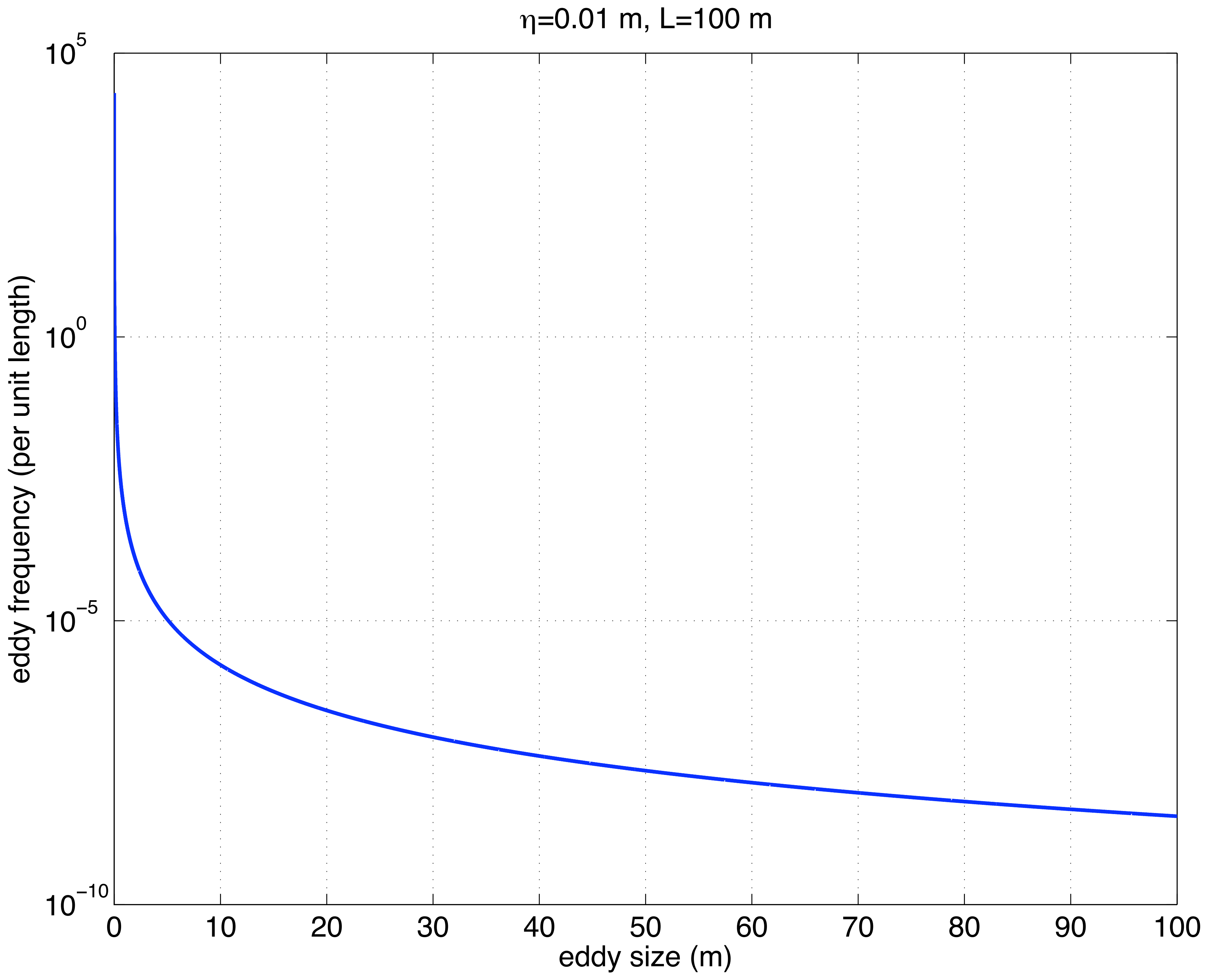
Alan Kerstein

Triplet Map for Fluid Elements

Each triplet map has a location, size, and time.

- *Location* is randomly chosen.
- *Size* l is randomly chosen from a distribution that matches inertial range scalings.
  - Smallest map (eddy) is Kolmogorov scale, η.
  - Largest eddy is L, usually domain size.
- Eddies occur at a *rate* determined by the large eddy time scale and eddy size range.

Eddy Size Distribution



- Cloud droplet microphysics
- Large-eddy simulation
- Parcel model
- Linear Eddy Model (LEM)
- **Explicit Mixing Parcel Model (EMPM)**
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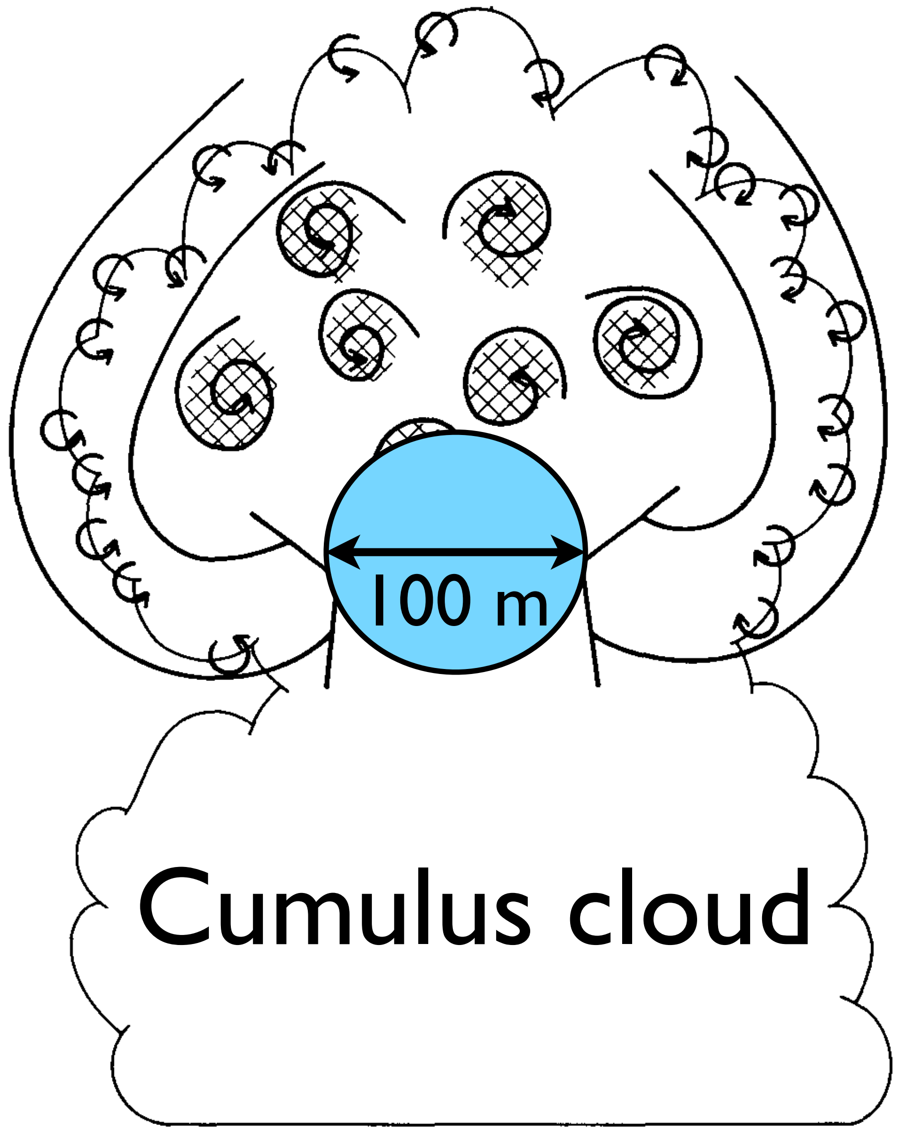
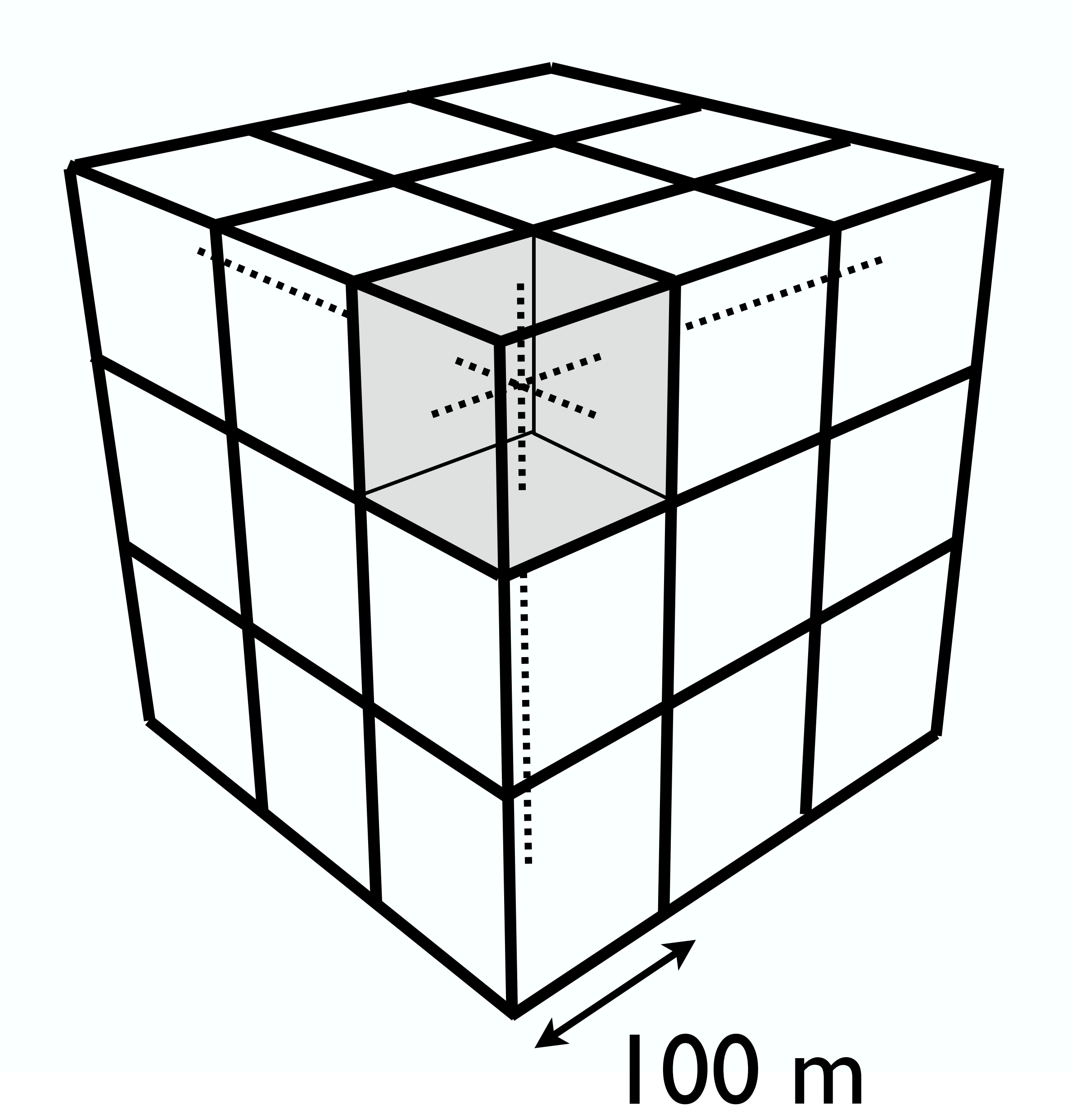
- **Explicit Mixing Parcel Model (EMPM)**
  - Combines the Linear Eddy Model with:
    - A *parcel model.*
    - Stochastic entrainment events.
    - Bulk or droplet microphysics.
    - Specified ascent speed.
  - Cloud droplets can grow or evaporate according to their local environments.

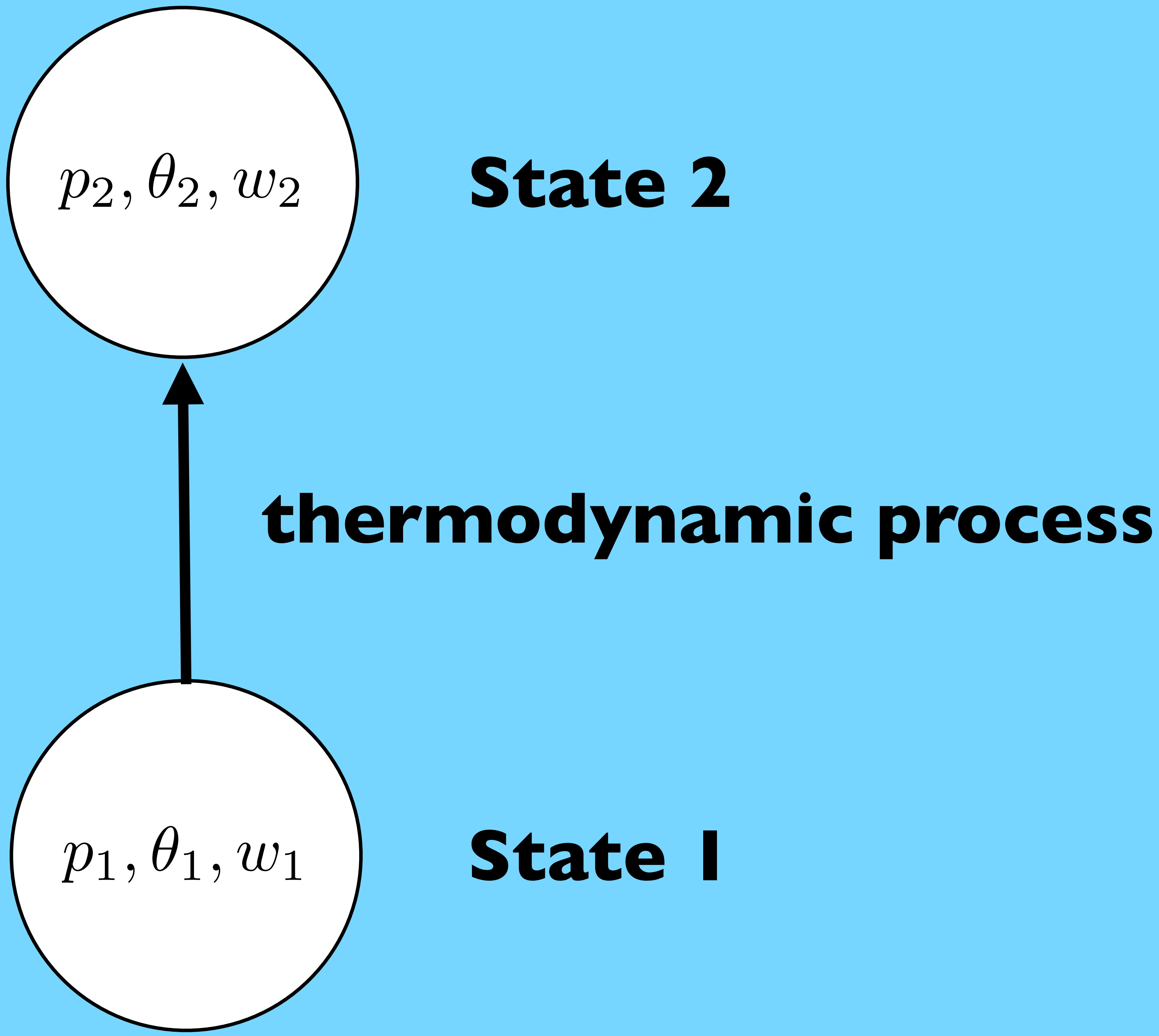
Parcel model



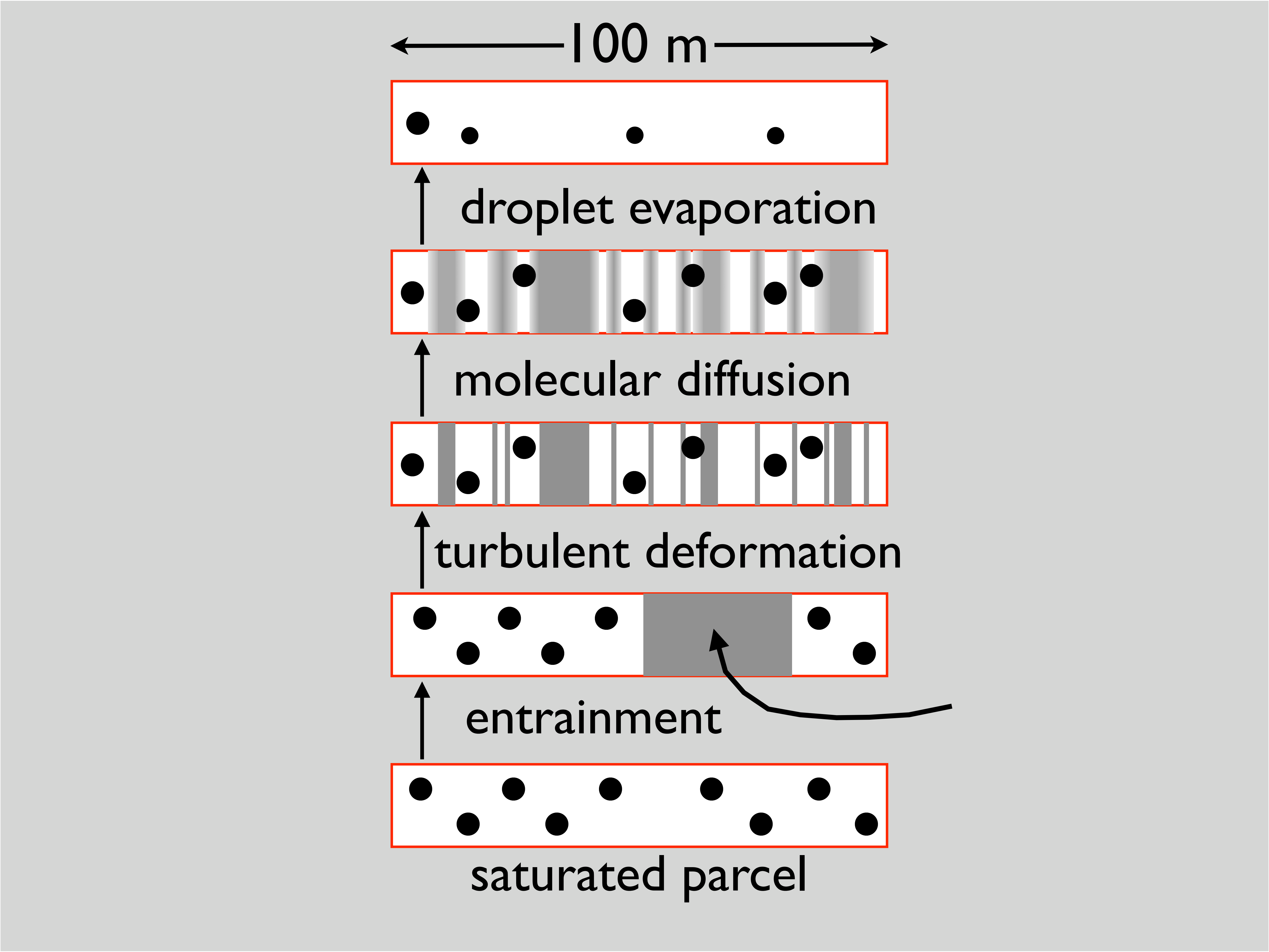
Large-Eddy Simulation (LES) model



Parcel Model



EMPM with droplets and entrainment



Explicit Mixing Parcel Model (EMPM)

- The EMPM predicts the evolving in-cloud variability due to entrainment and finite-rate turbulent mixing using a 1D representation of a rising cloudy parcel.

- The 1D formulation allows the model to resolve fine-scale variability down to the smallest turbulent scales ($\sim$ 1 mm).

- The EMPM can calculate the growth of several thousand individual cloud droplets based on each droplet s local environment.

Krueger, S. K., C.-W. Su, and P. A. McMurtry, 1997: Modeling entrainment and fine-scale mixing in cumulus clouds. J. Atmos. Sci., 54, 2697–2712.

Su, C.-W., S. K. Krueger, P. A. McMurtry, and P. H. Austin, 1998: Linear eddy modeling of droplet spectral evolution during entrainment and mixing in cumulus clouds. Atmos. Res., 47–48, 41–58.

EMPM Fluid Variables

- Bulk microphysics:
  - Liquid water static energy
  - Total water mixing ratio
- Droplet microphysics:
  - Temperature
  - Water vapor mixing ratio

EMPM Droplet Variables

- Location (in one coordinate)
- Radius
- CCN properties

In the EMPM, droplets move relative to the fluid at their terminal velocities.

Droplet Microphysics

droplet radius:

$$r_j \frac{dr_j}{dt} = \frac{S - A_1 + A_2}{A_3 + A_4},$$

supersaturation

$$S = \frac{q_v}{q_{vs}} - 1,$$

water vapor:

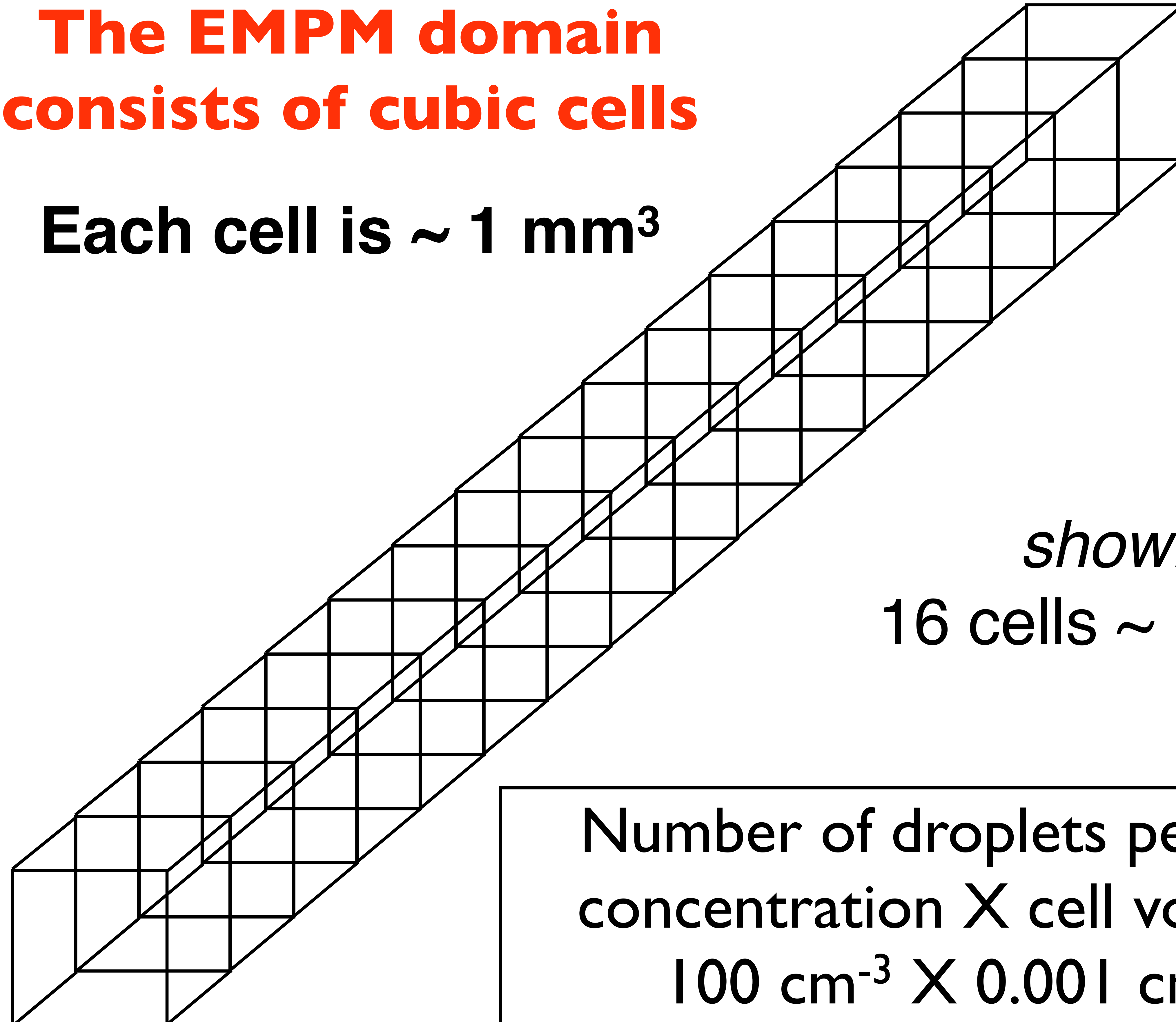
$$\left.\frac{dq_v}{dt}\right|_{phase\ \ change} = -\sum_j 4\pi N_j \rho_w r_j{}^2 \frac{dr_j}{dt},$$

temperature:

$$\left.\frac{dT}{dt}\right|_{phase\ \ change} = -\frac{L_v}{c}\left.\frac{dq_v}{dt}\right|_{phase\ \ change} - w\frac{g}{c},$$

The EMPM domain consists of cubic cells

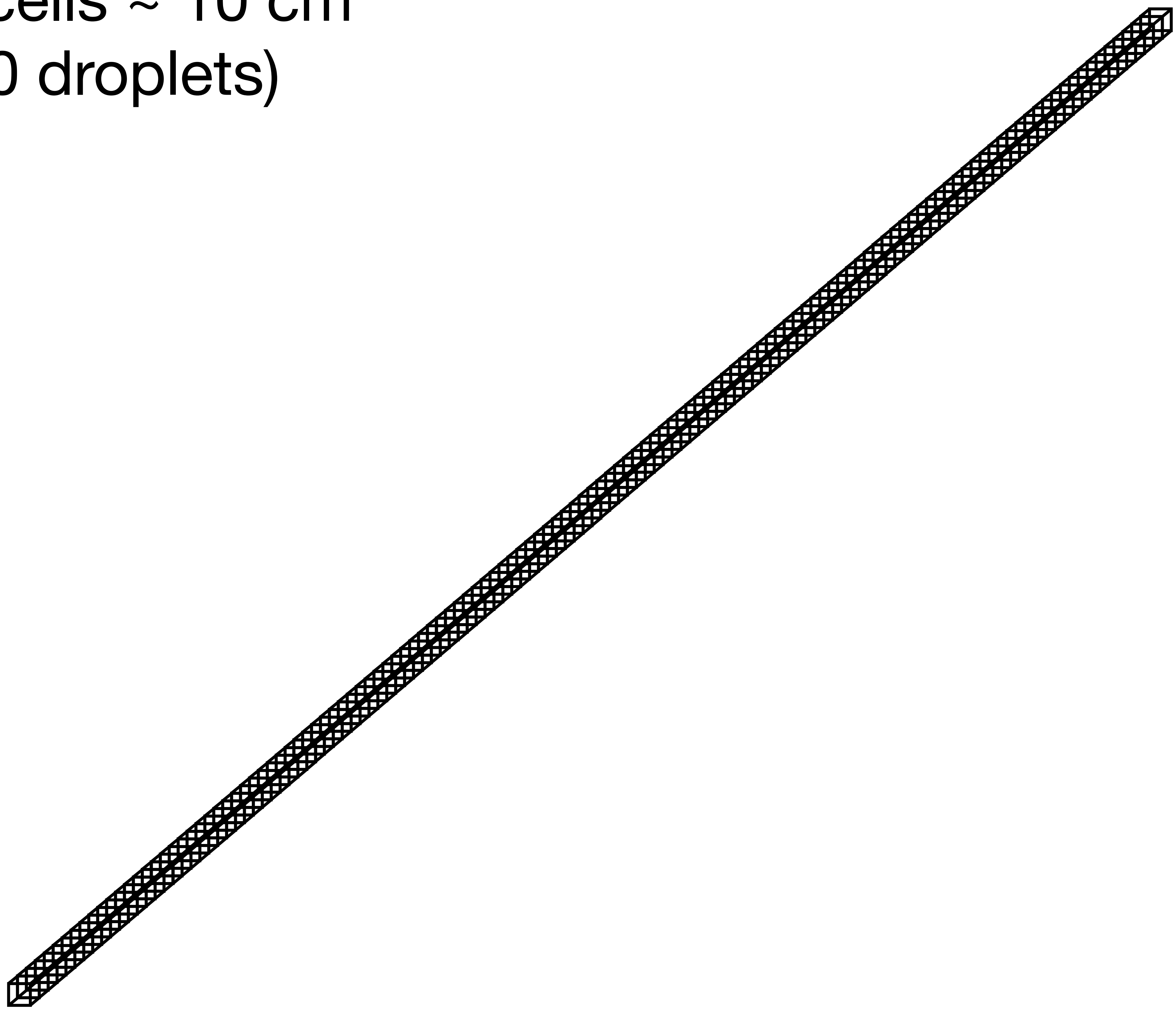
Each cell is ~ 1 mm^3



shown:
16 cells ~ 1.6 cm

Number of droplets per cell = concentration X cell volume = 100 cm^{-3} X 0.001 cm^3 = 0.1 (1 droplet per cm)

128 cells ~ 10 cm
(10 droplets)



1024 cells ~ 1 m
(100 droplets)

EMPM Required Inputs

- Required for a classical (instant mixing) parcel model calculation:

  Thermodynamic properties of cloud-base air

  Updraft speed

  Entrainment rate

  Thermodynamic properties of entrained air

  Aerosol properties

- In addition, the EMPM requires:

  Parcel size

  Entrained blob size, d

  Turbulence intensity (e.g., dissipation rate, ϵ)

Mixing Time Scale

$$\tau = \left(\frac{d^2}{\epsilon}\right)^{1/3},$$

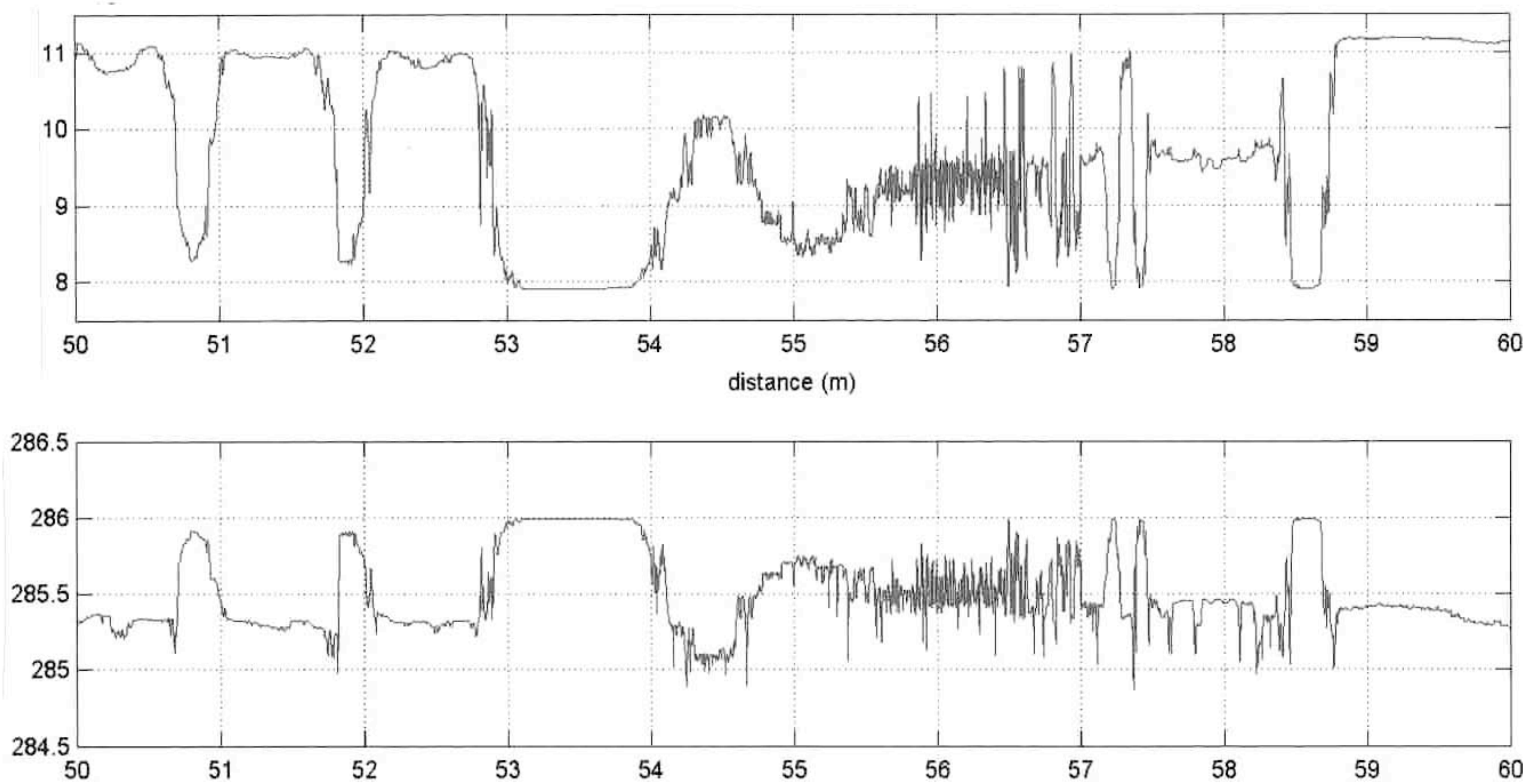
d is entrained blob size, ϵ is dissipation rate of turbulence kinetic energy.

For a **cumulus cloud,** $U \sim 2$ m/s, $L \sim 1000$ m, so $\epsilon \sim U^3/L = 10^{-2}$ m^2/s^3. For $d = 100$ m, $\tau \sim 100$ s.

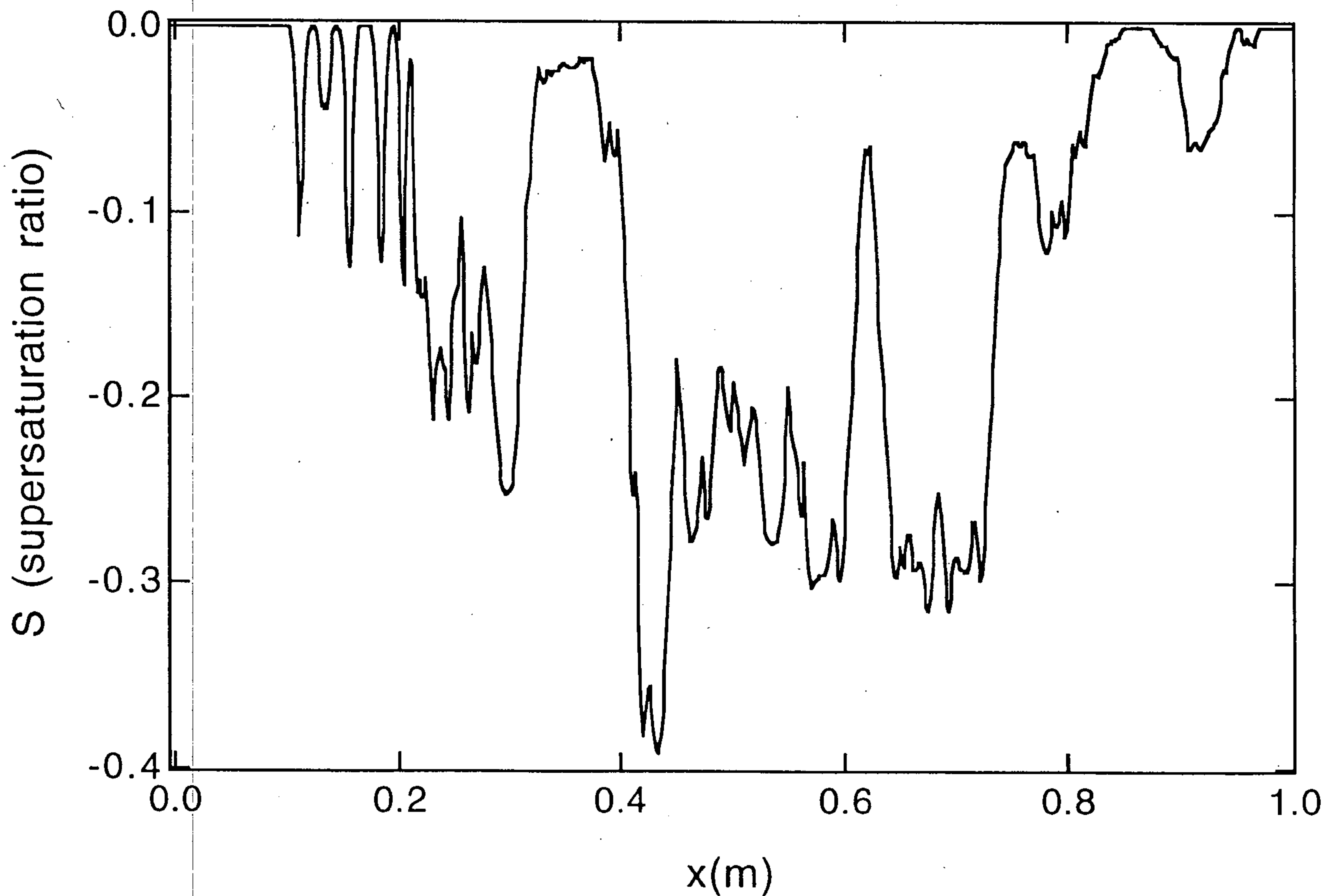
Classic (instant mixing) parcel model is recovered when

- Entrained blob size, $d \to 0$
- Turbulence intensity, $\epsilon \to \infty$

EMPM water vapor and temperature fields



Snapshot of supersaturation ratio during mixing



Droplet radius histories during mixing

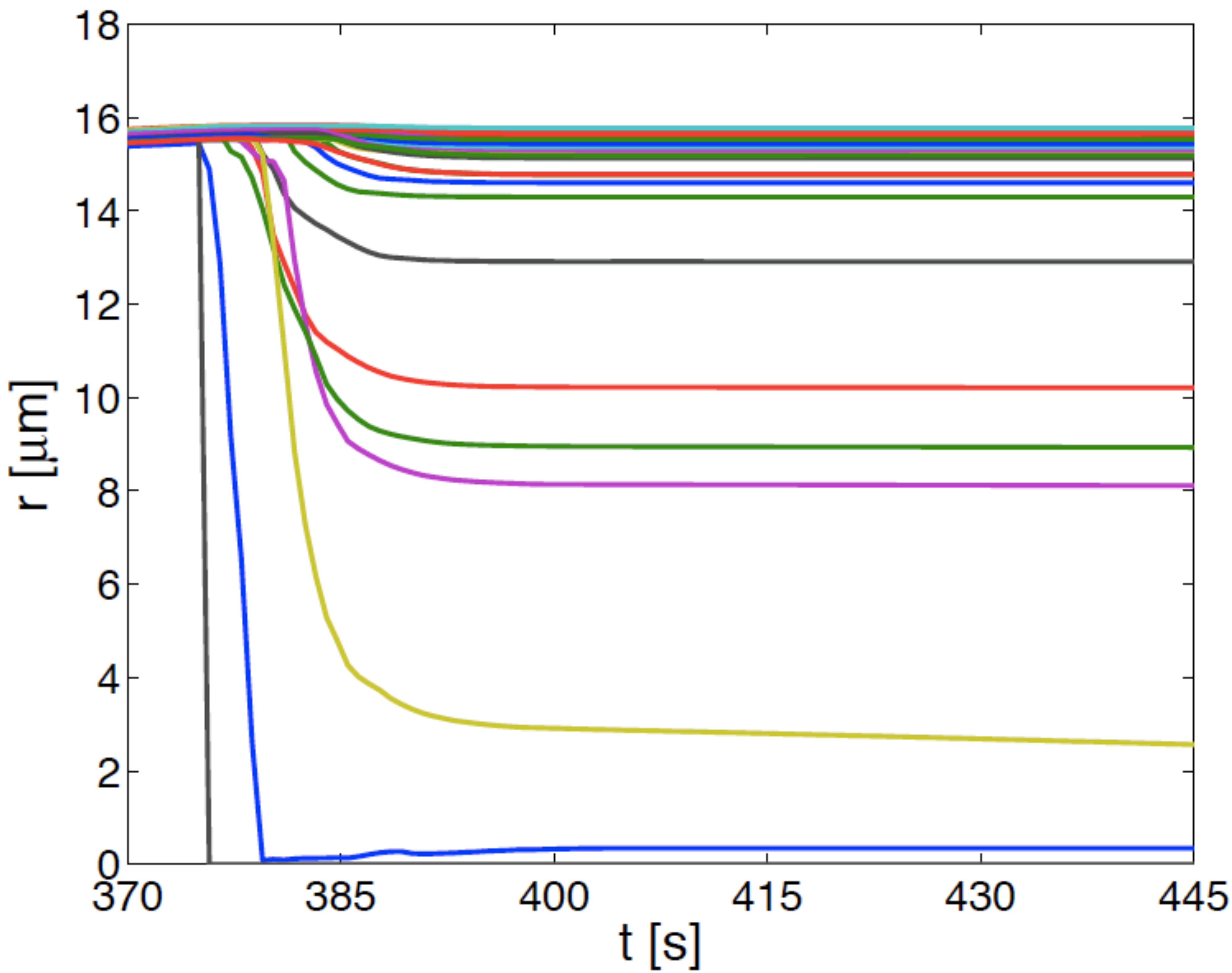
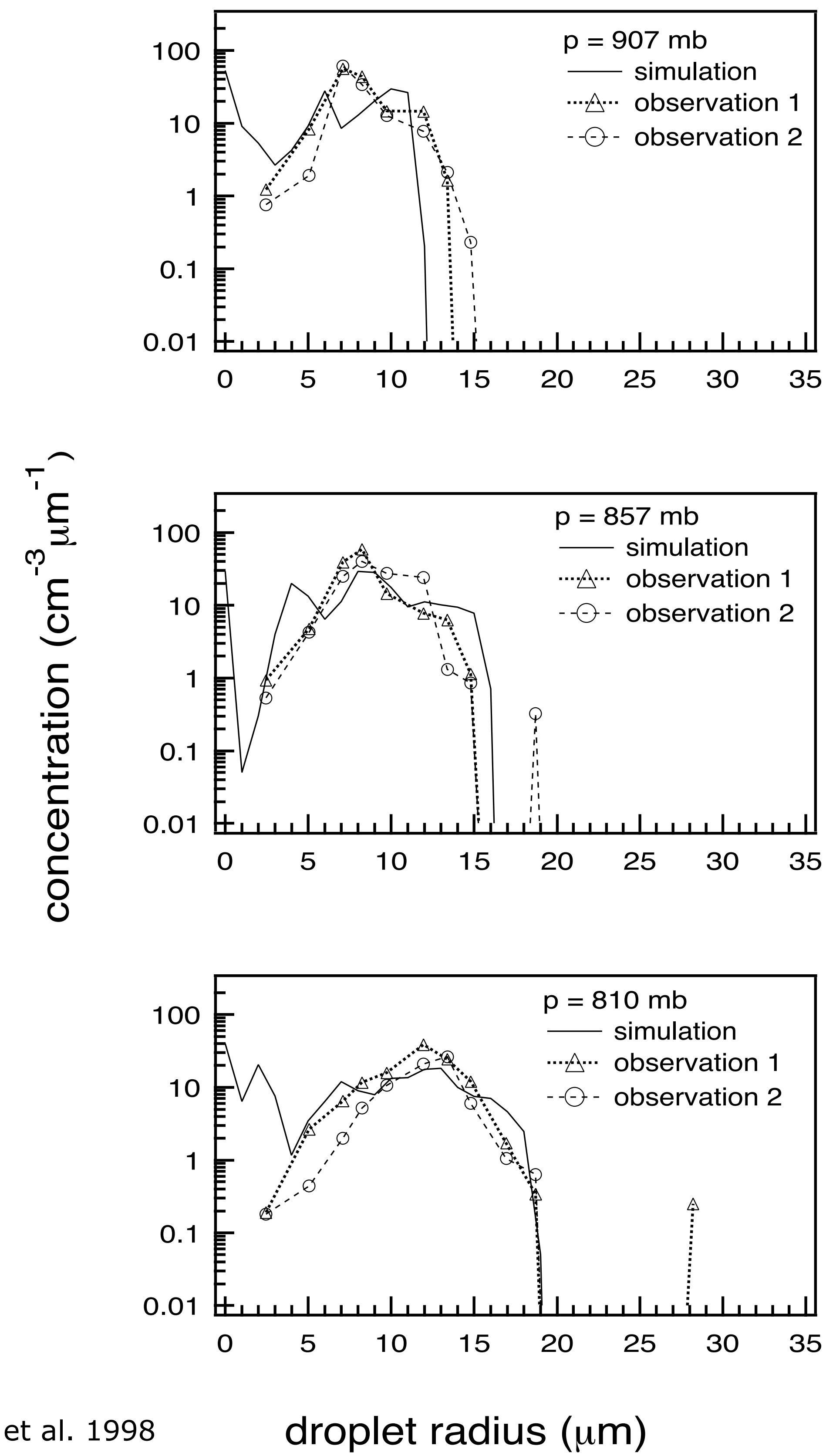


Figure 4.10: Radius histories of 30 droplets for $f = 0.1$ and $RH_e = 0.219$.

Helena Schluter, Univ of Utah

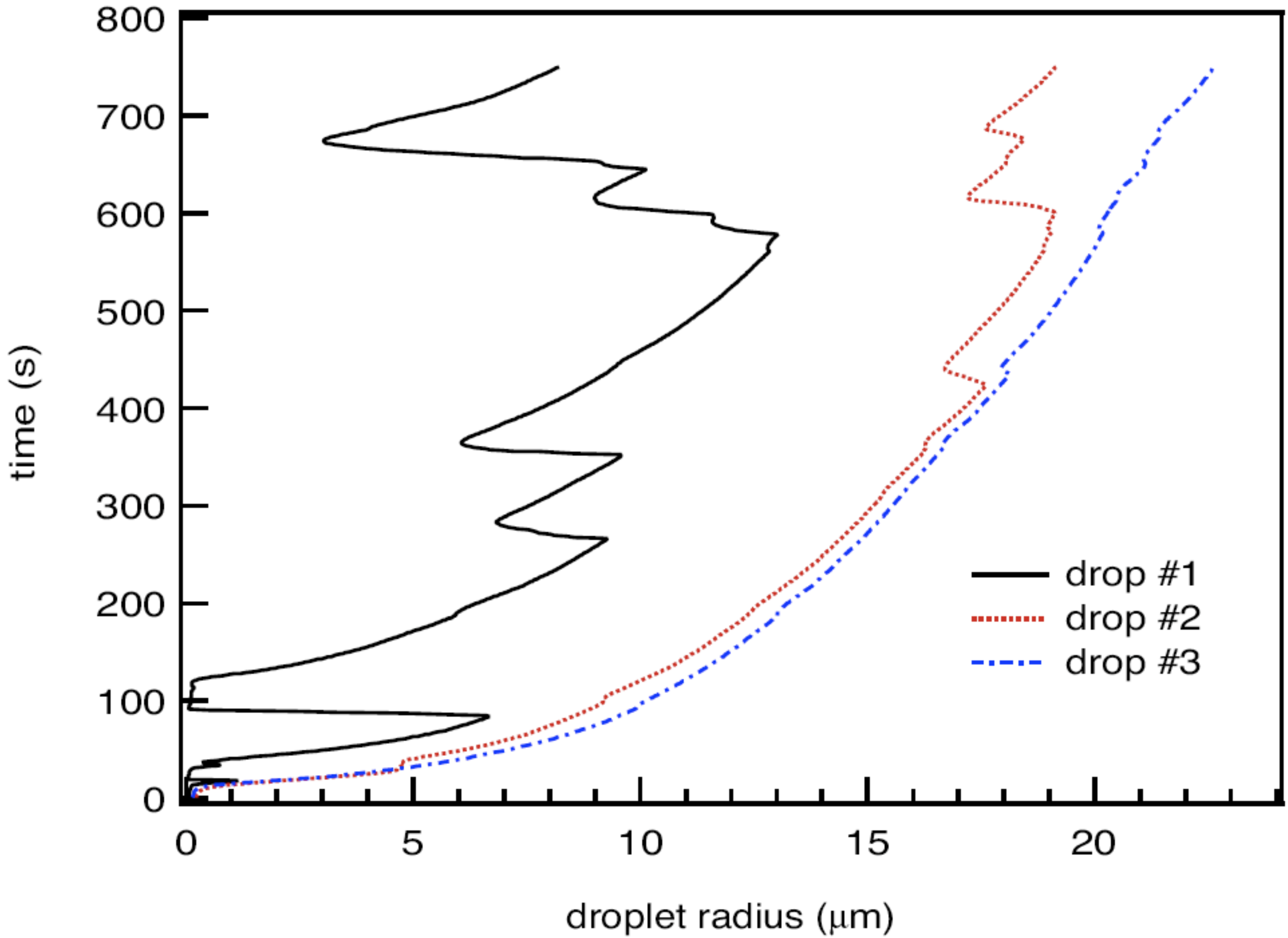
Applying the EMPM to Hawaiian Cumuli

The EMPM produced realistic, broad droplet size spectra that included super-adiabatic-sized droplets. The computed spectra agreed with those measured by aircraft.



Su et al. 1998

Large Droplet Production due to Entrainment and Mixing



800
700
600
500
400
300
200
100
0
time (s)
0
5
10
15
20
droplet radius (μm)
drop #1
drop #2
drop #3

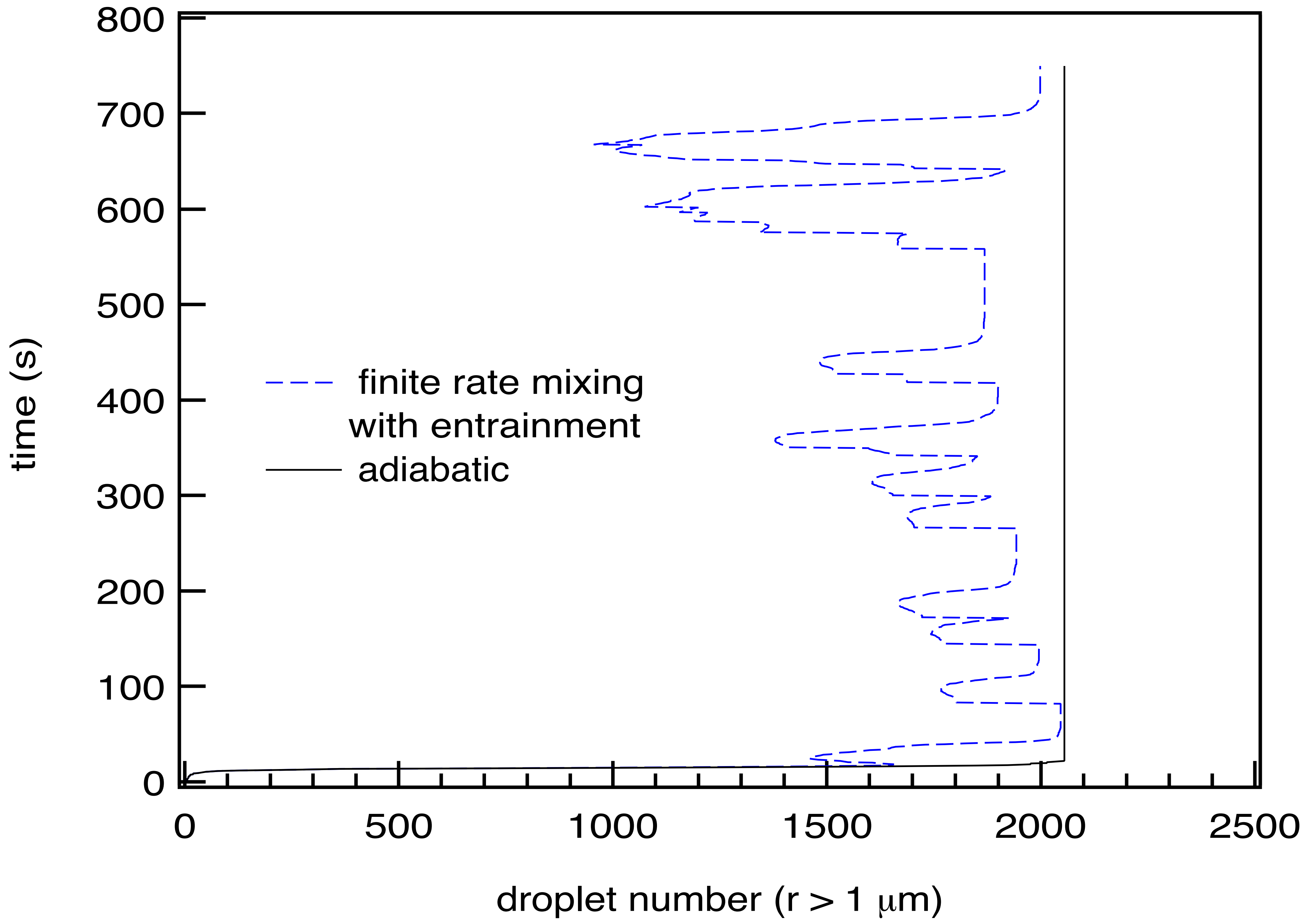
Some factors that affect large droplet production

- Turbulence intensity (dissipation rate)
- Entrained blob size
- Entrainment rate
- Relative humidity of entrained air

Entrainment of CCN affects droplet spectra in cumulus clouds

Droplet concentration profile with entrained CCN

Height = time * 2 (m)



(Su 1997)

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- **ClusColl (Clustering and Collision Model)**

- **ClusColl (Droplet Clustering and Collision Model)**
  - Inertial droplets move in response to Kolmogorov-scale turbulence and gravity.
  - Economically evolves 3D droplet positions and detects collisions.
  - Can be incorporated into EMPM.

- *Motivation:* To develop an economical model that represents the essential processes that contribute to the formation of rain drops by collision and coalescence of cloud droplets.

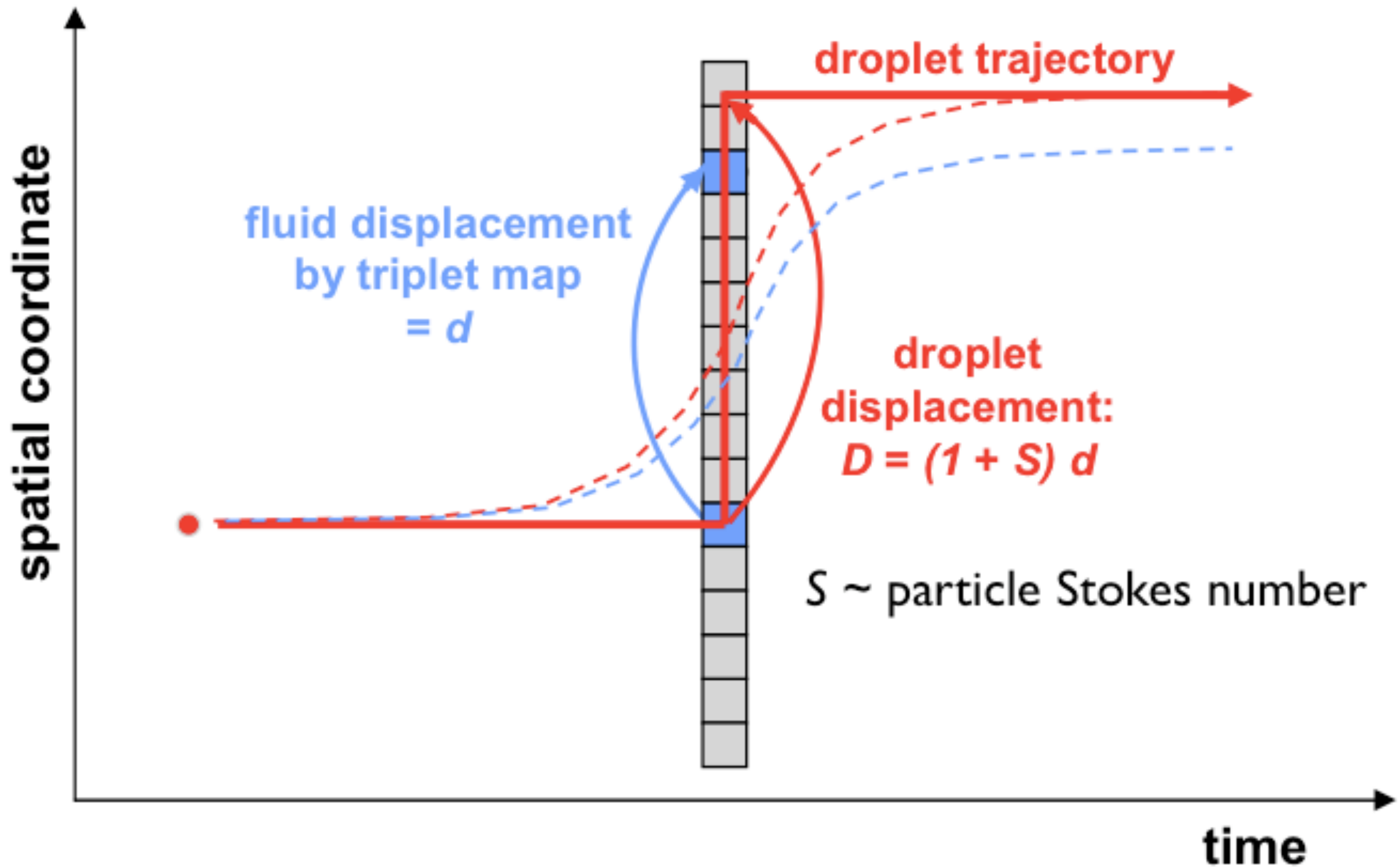
An Economical Simulation Method for Droplet Motions in Turbulent Flows

Each droplet has a radius and a 3-D position.

- *Radius* changes due to collision and coalescence.
- *Position* changes due to turbulence and sedimentation.
- *Map-based advection* is an efficient tool for capturing the physics that governs droplet motions and collisions in turbulence.

Turbulent Motion of *Droplets* can also be Represented by Applying 1-D Maps

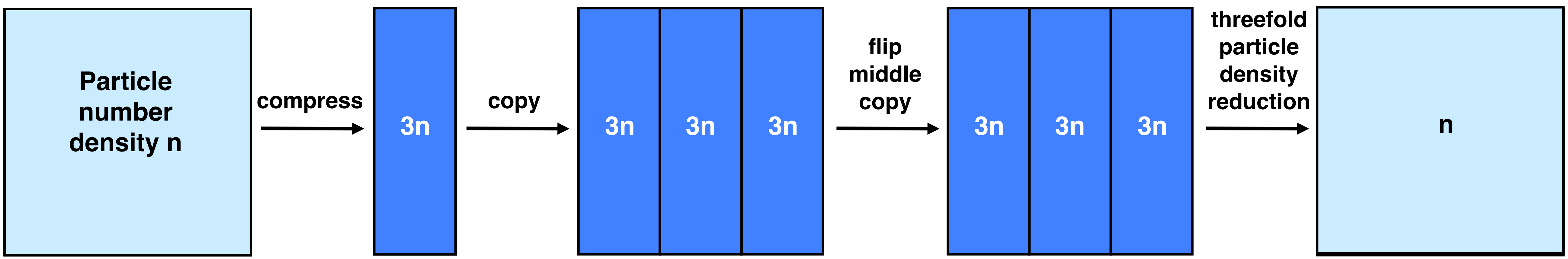
The droplet trajectory model idealizes droplet response to continuum flow (dashed curves: notional continuum fluid streamline and droplet trajectory)



Kerstein, A. R., and S. K. Krueger, 2006: Clustering of randomly advected low-inertia particles: A solvable model. *Phys. Rev. E*, **73,** 025302.

Continuum interpretation: slip induces fluctuations in an initially uniform particle-density field

Zero inertia: uniform multiplicative compression, compensated by number reduction



Non-zero inertia: non-uniform compression, inducing particle-density fluctuations

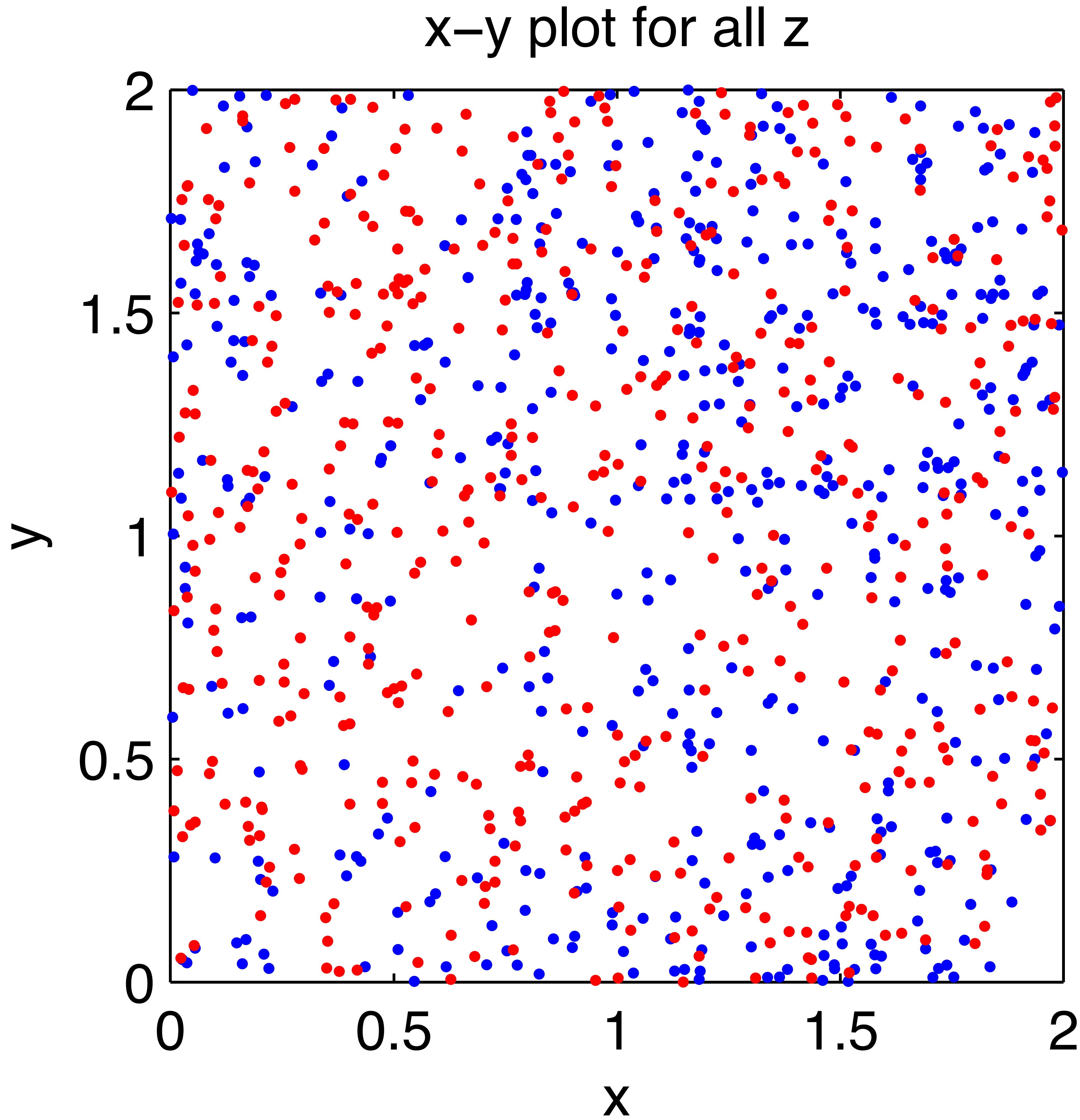
Alan Kerstein

Droplets are ejected from highly turbulent regions

Triplet Map for Droplets

Each triplet map has a
location, orientation, size, and time.

- *Location* is randomly chosen.
- *Orientation* is parallel to *x*-, *y*-, or *z*-coordinate and is randomly chosen.
- *Size* ~ Kolmogorov length scale.
- *Interval between maps* ~ Kolmogorov time scale.



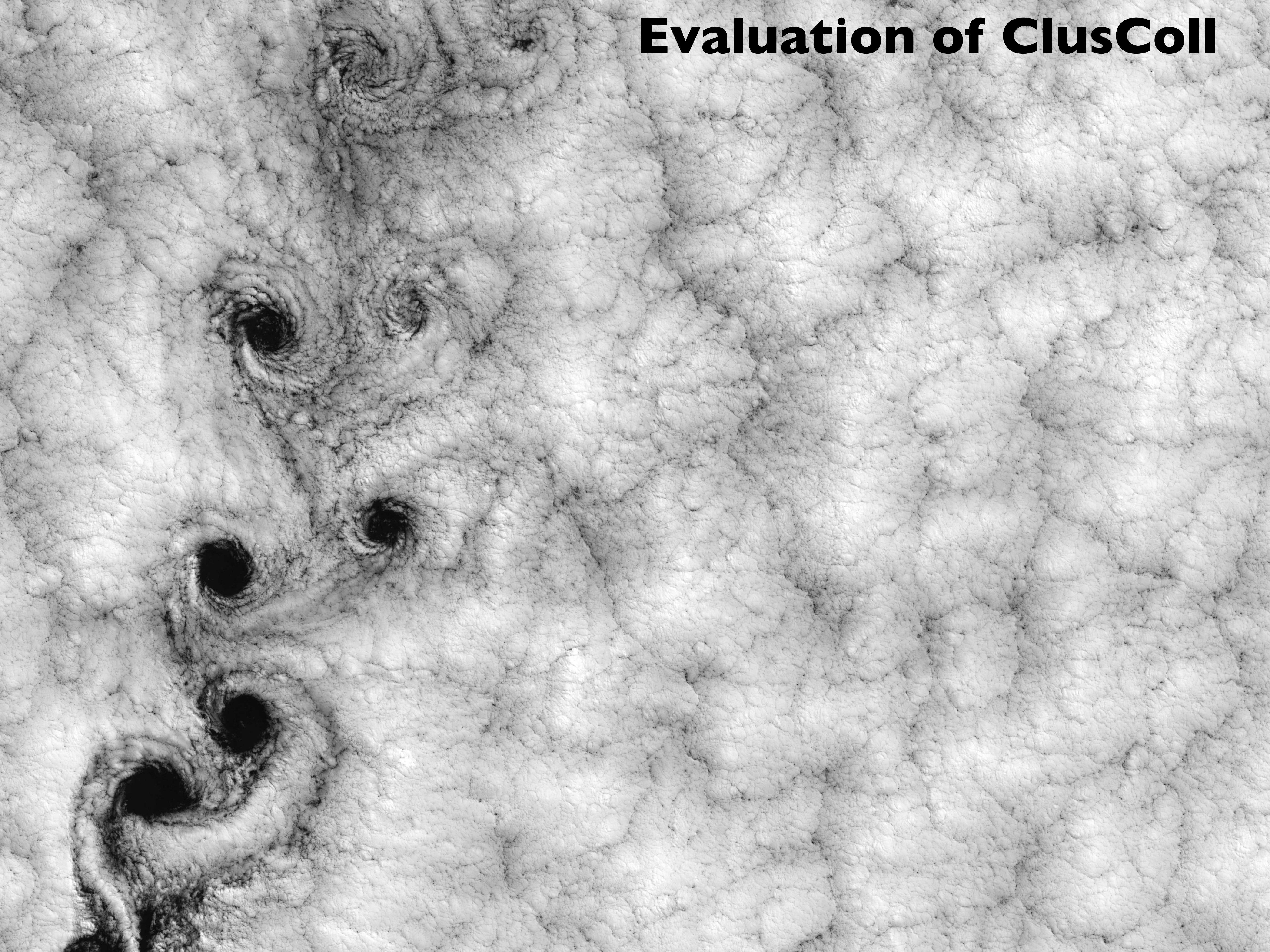
St = 0

St = 0.025

length unit is triplet map eddy size = 20 x Kolmogorov scale

- To use the triplet map to calculate droplet motions in turbulence we had to relate:
  - The *ratio of droplet displacement to fluid displacement* (S) for each map to the *particle Stokes number* (St).
  - The model's *map (eddy) size* to the *Kolmogorov length scale*.
  - The *map (eddy) interval* to the *Kolmogorov time scale*.

Evaluation of ClusColl

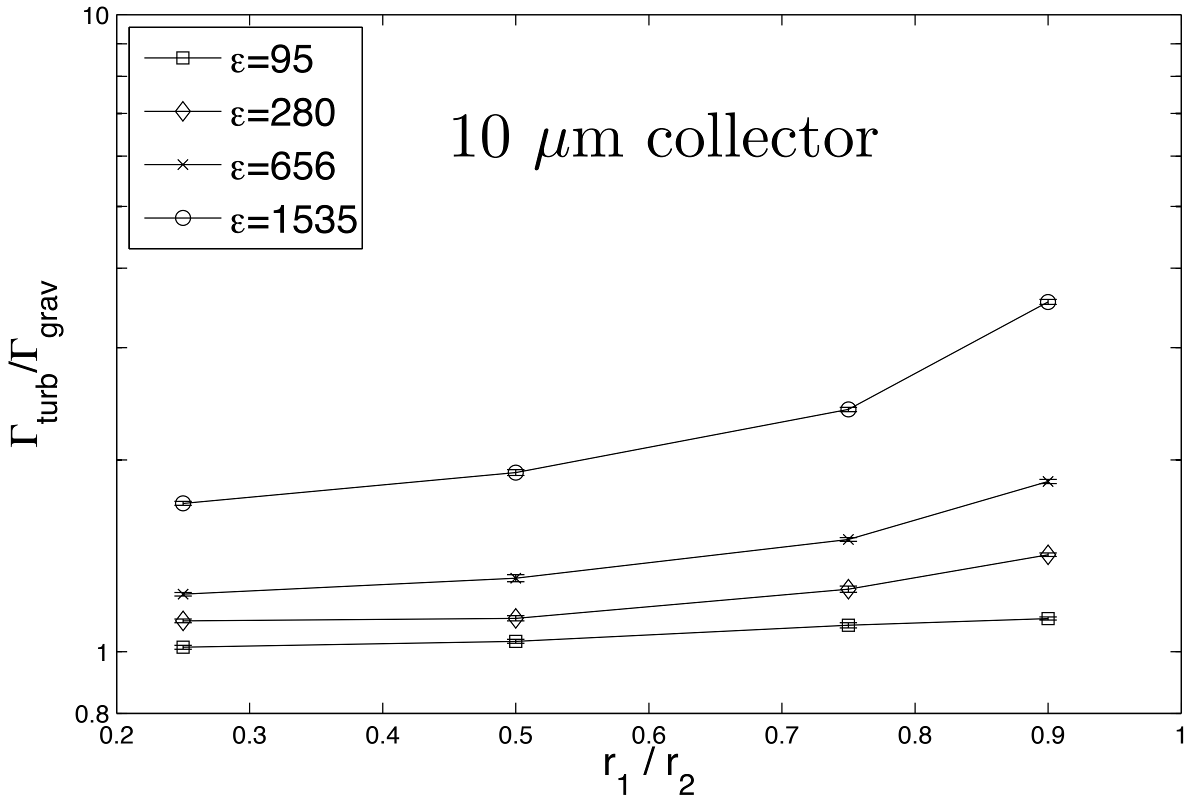


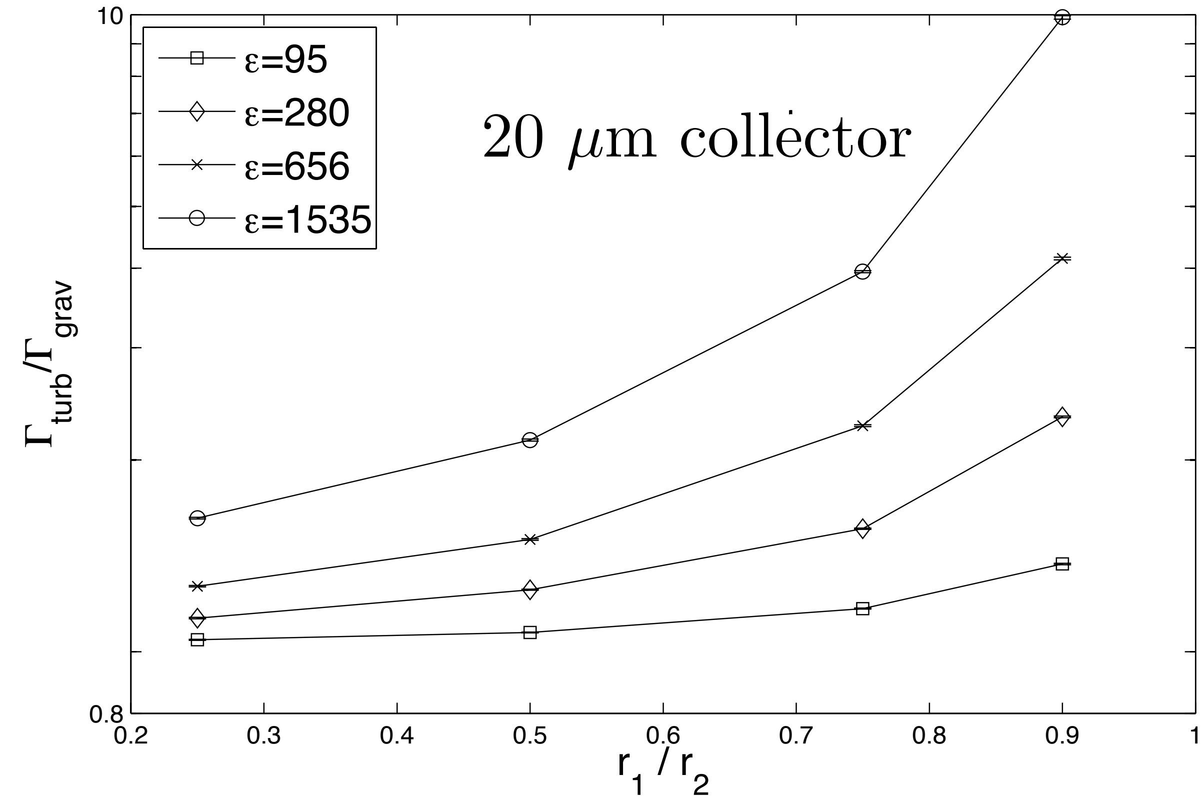
Collision Kernels

We implemented an efficient collision detection code and compared our *collision kernels* of

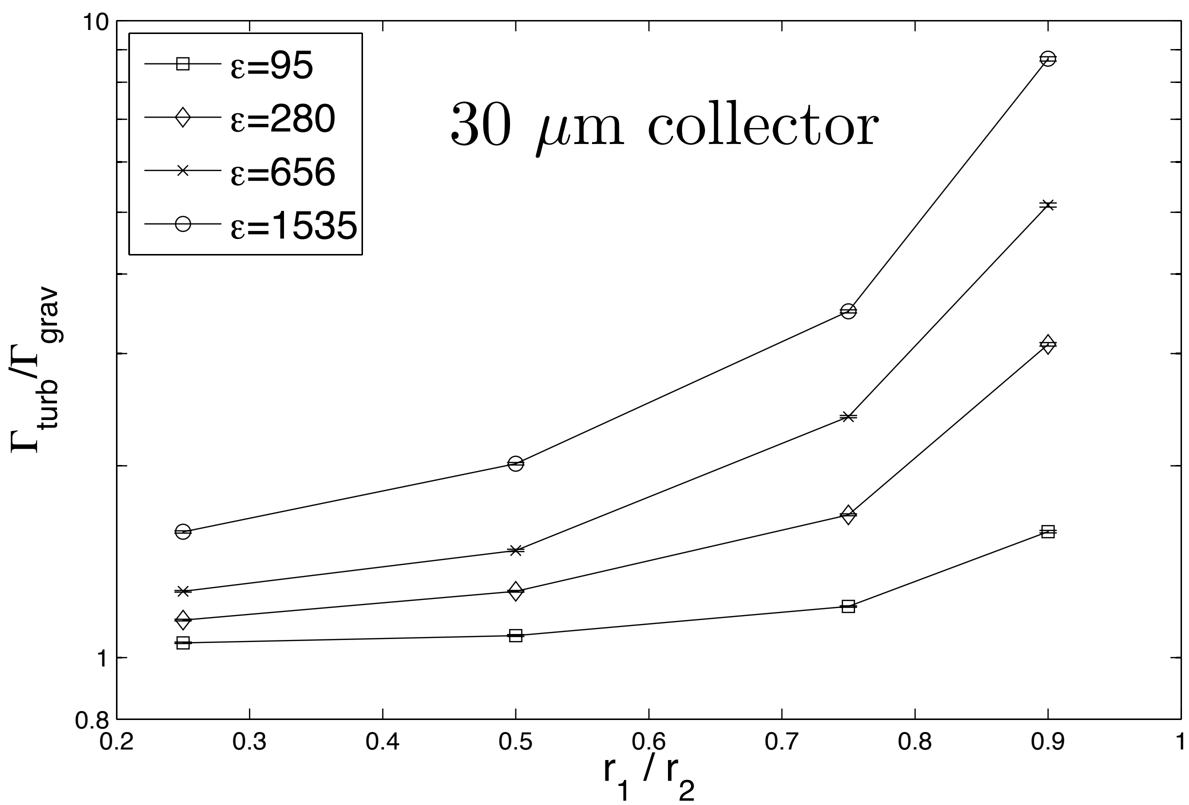
- bidispersions with inertia and gravity with those from DNS by Franklin et al. (2005).
- monodispersions with inertia and gravity with those from DNS by Ayala et al. (2008).

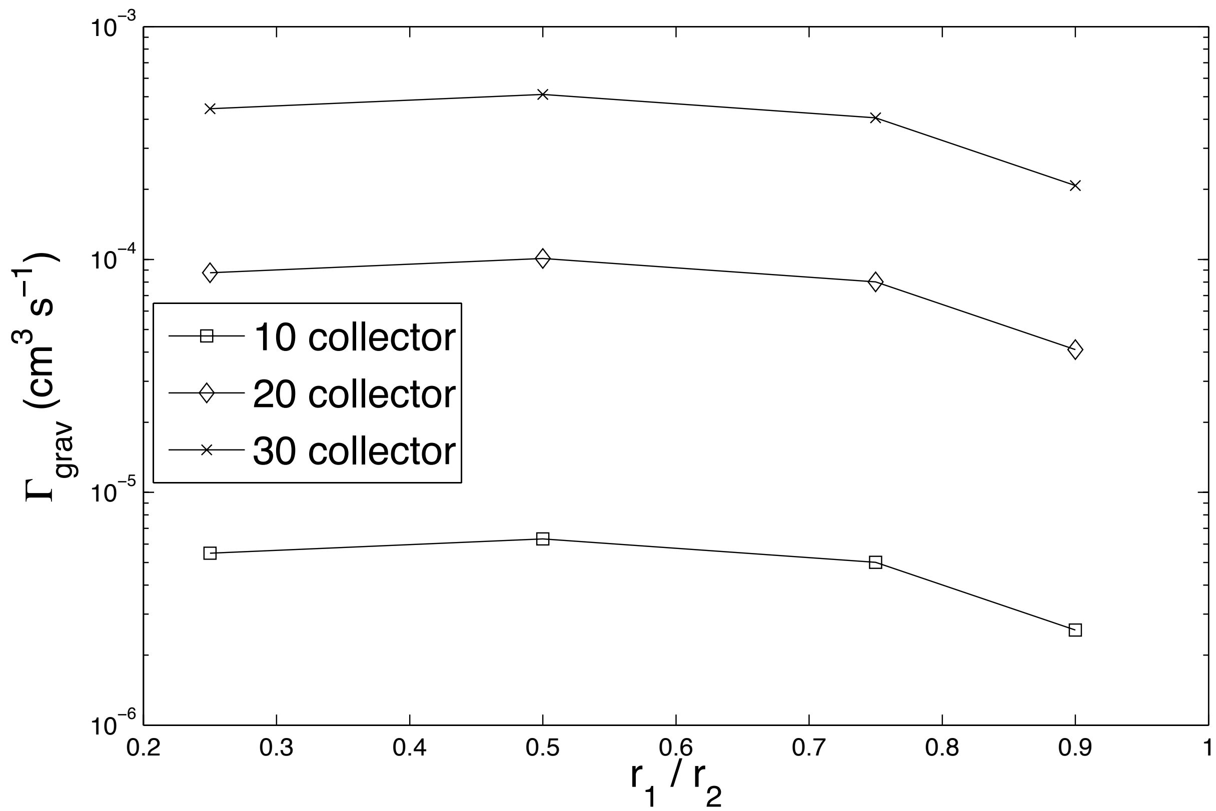
Normalized Collision Kernels



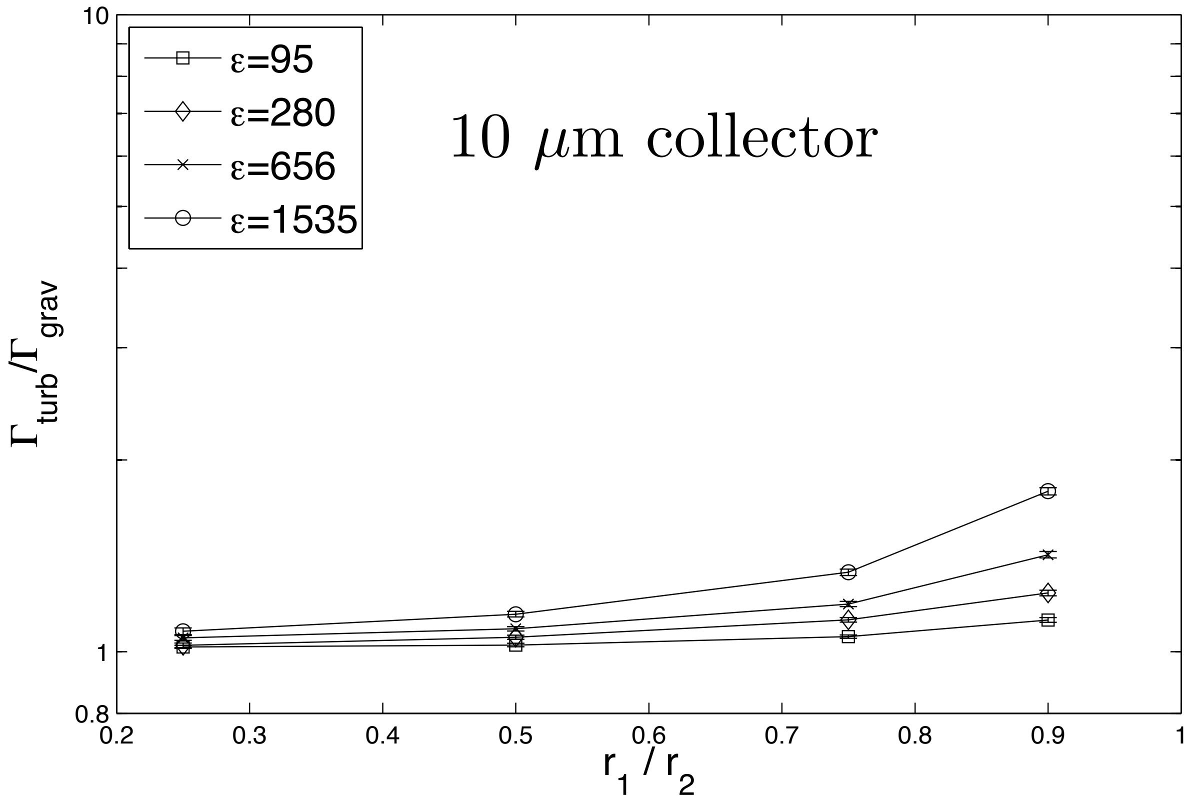


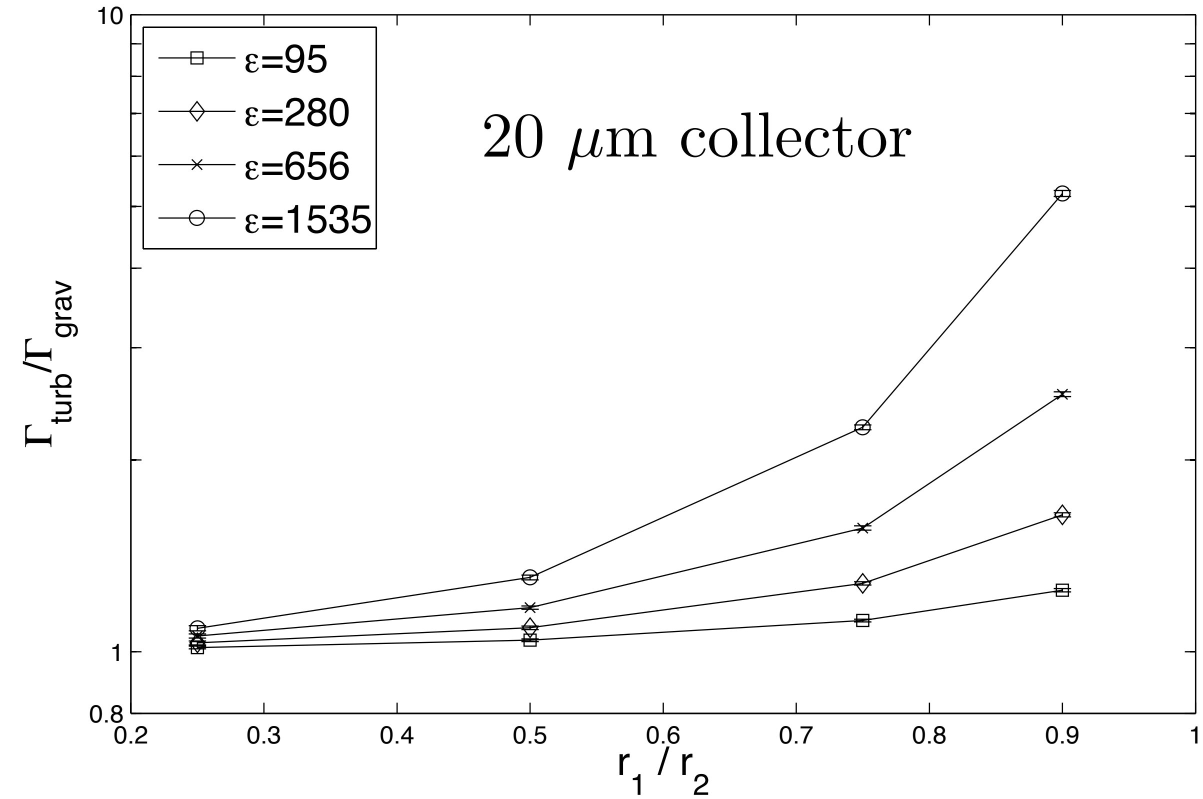
DNS by Franklin et al. (2005)



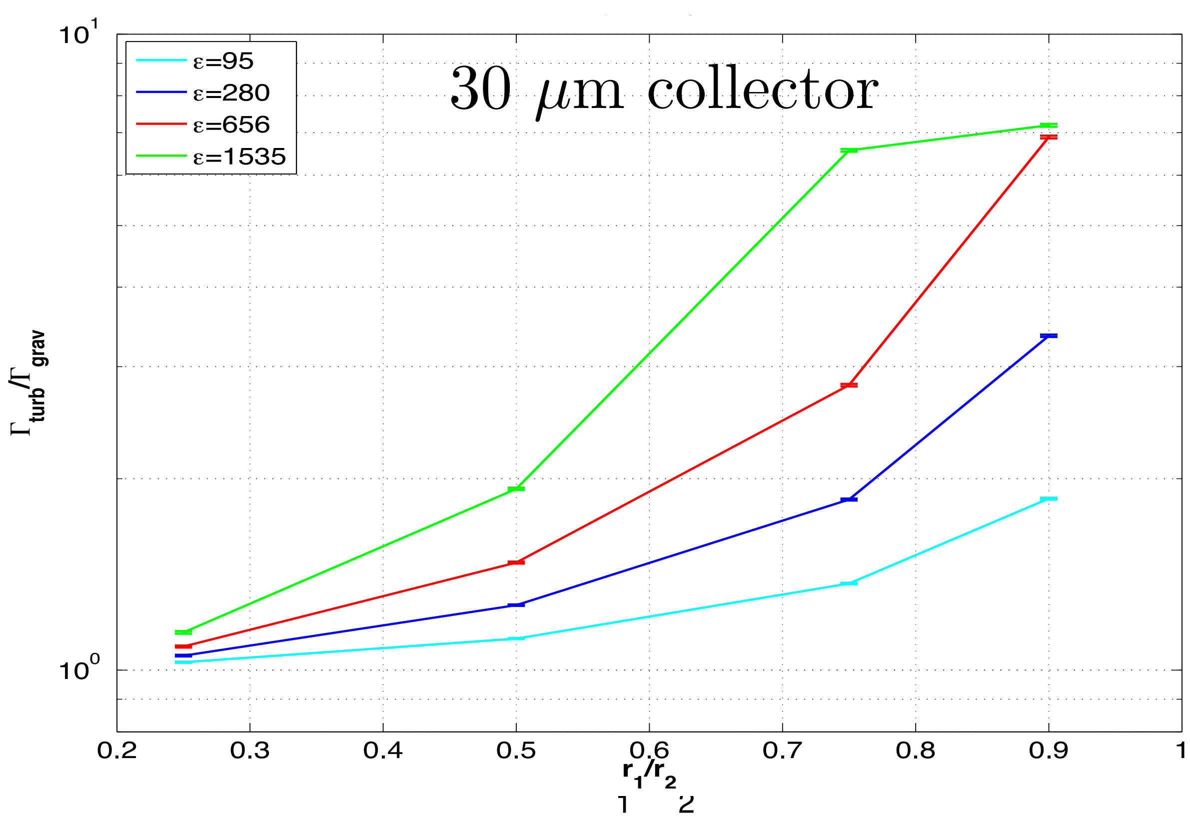


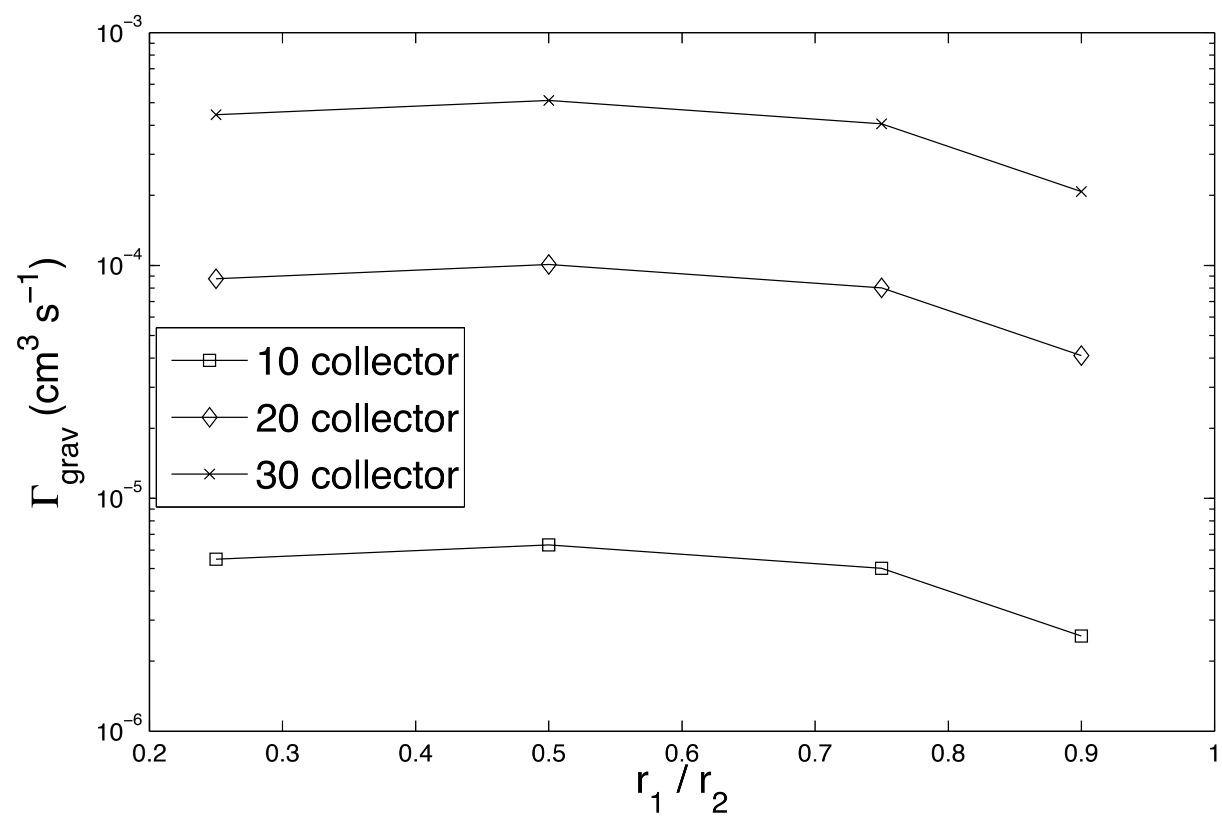
Normalized Collision Kernels





Triplet Map for Droplets





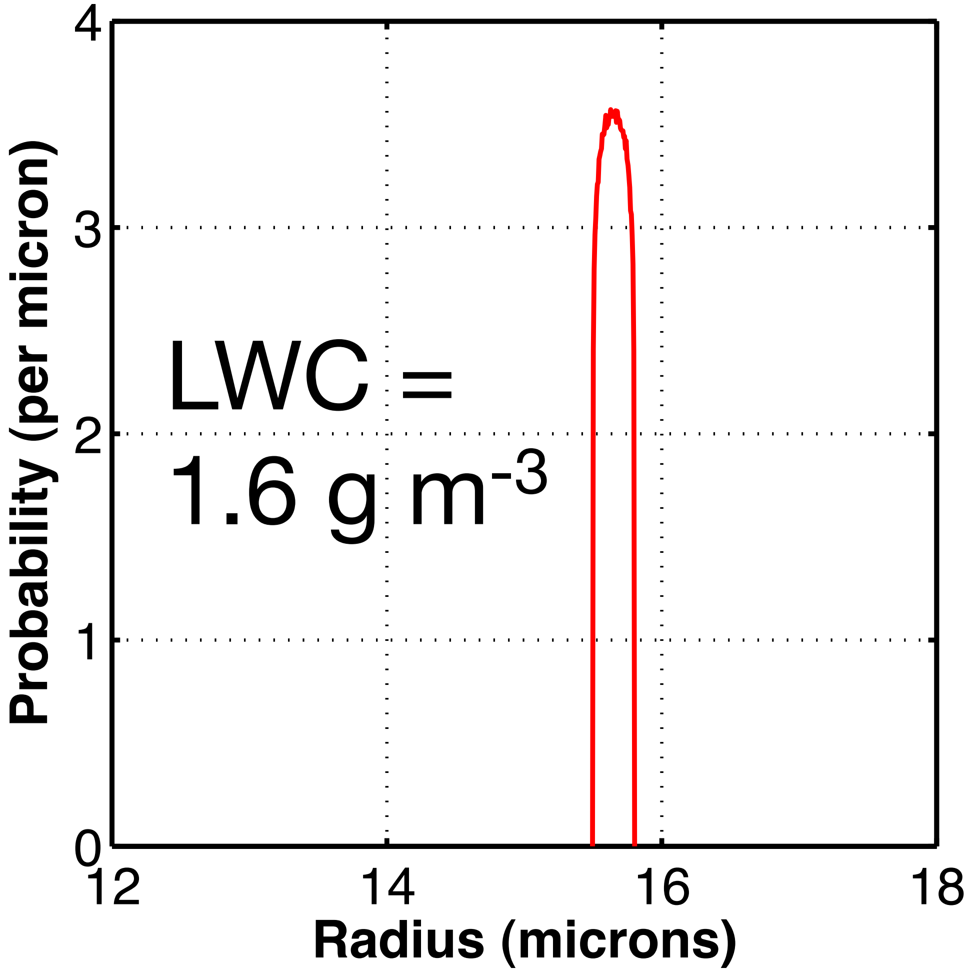
Collision and Coalescence Calculations

- *Case 1:* Narrow DSD from 15.5 to 15.8 microns. LWC = 1.6 g m^{-3}.

- *Case 2:* Wide DSD from 12 to 16.5 microns. LWC = 1.4 g m^{-3}.

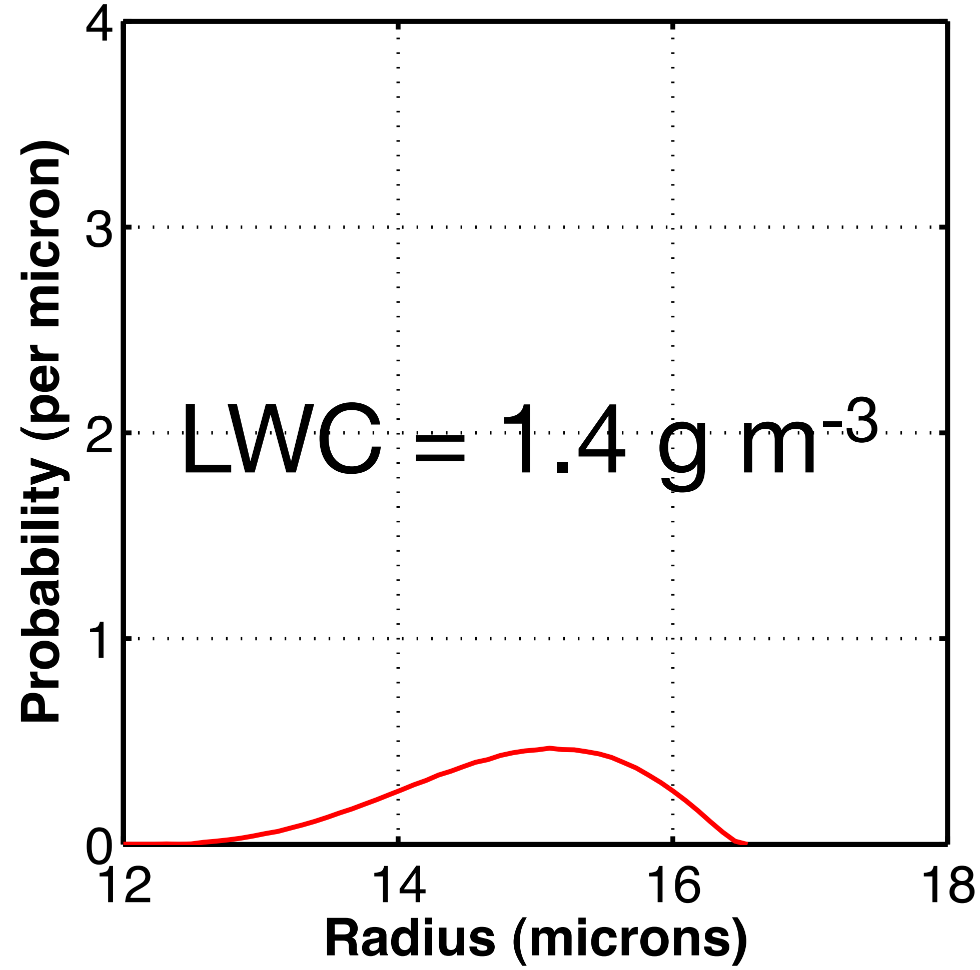
Case 1

adiabatic

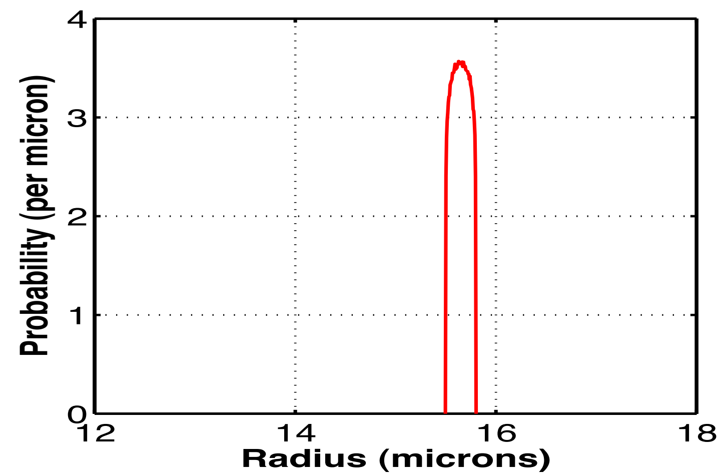


Case 2

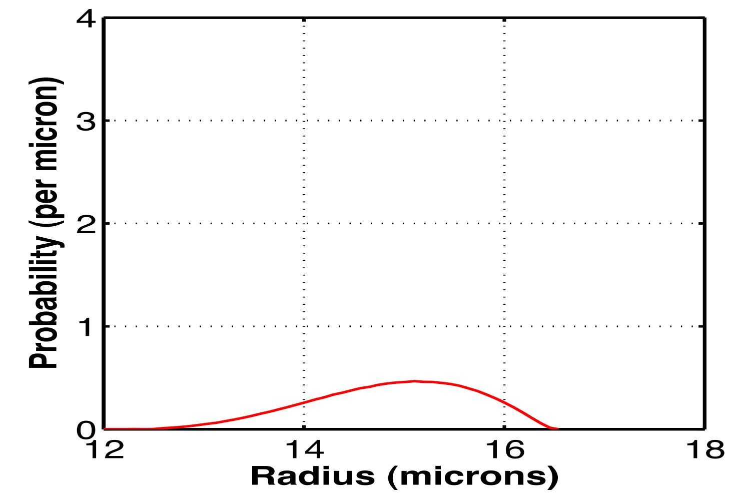
diluted

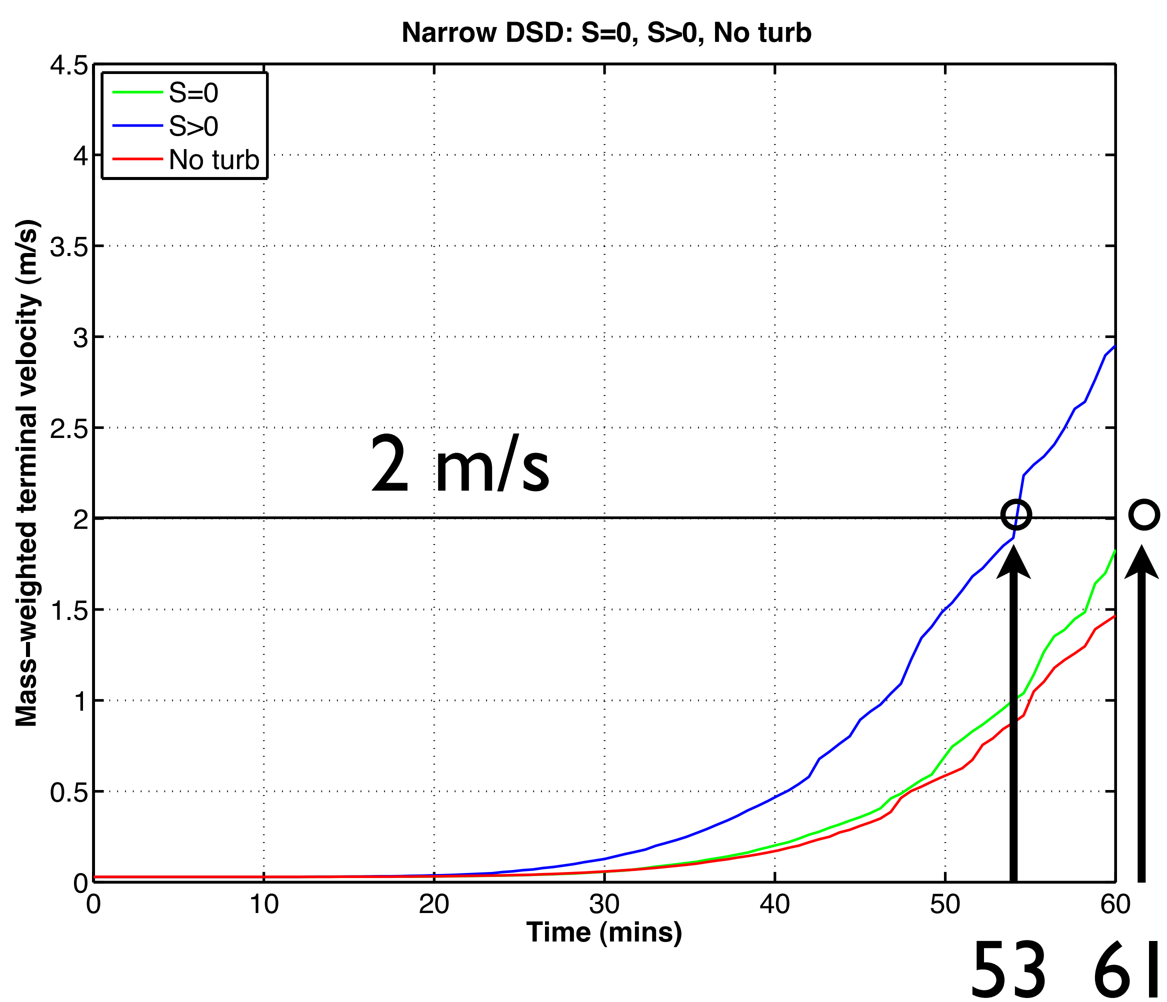


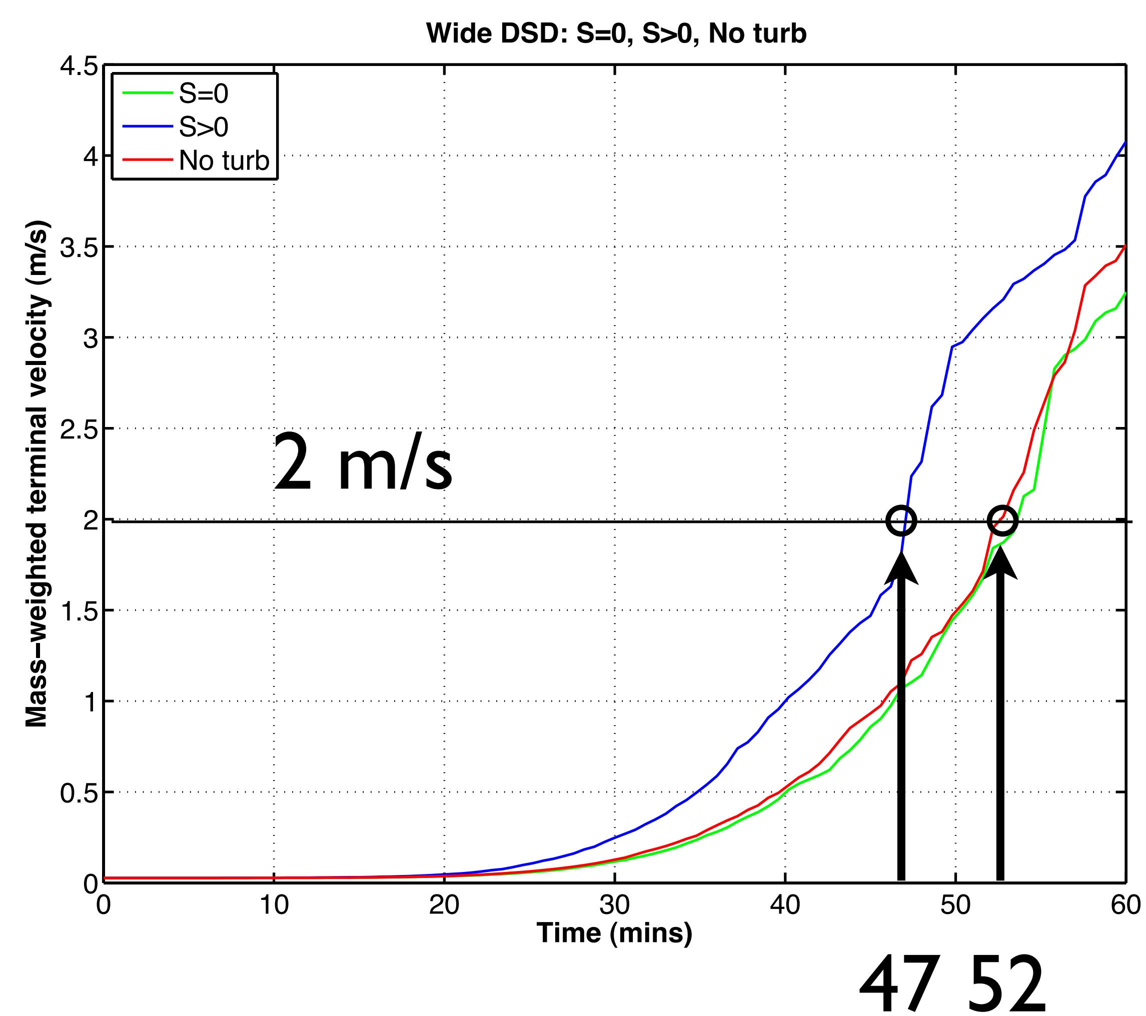
adiabatic



diluted







Collision and Coalescence Calculations

- Turbulence acting on zero-inertia droplets is similar to no turbulence.
- When turbulence acts on inertial droplets, rain forms 5 to 8 minutes sooner than with zero-inertia droplets or no turbulence.
- Under the same conditions, rain forms 6 to 9 minutes sooner with the broader DSD.

Summary

- An economical simulation method for droplet motions in turbulent flows has been developed.
- Collision kernels agree reasonably well with DNS results.
- Collision and coalescence calculations have been performed.
- These suggest that turbulence can accelerate rain formation by droplet clustering due to droplet inertia and by spectral broadening due to entrainment and mixing.

What's ahead...

- Combine EMPM and ClusColl into a single model.
- Extend the 1D approach to SGS modeling in LES of clouds.
- Use results of EMPM and ClusColl to improve conventional SGS models for LES of clouds.
- A difficult remaining problem is representing the effects of entrainment and mixing on DSDs in LES.

LES with 1D subgrid-scale model