Meteorology 6150
A Numerical Model for Simulating Moist, Non-precipitating Convection

1. The dry model equations

The dry model is based on the quasi-compressible outflow model (QCOM) described in Droegemeier and Wilhelmson (1987). This model predicts the horizontal velocity (v), the vertical velocity (w), the potential temperature (θ), and the non-dimensional perturbation pressure (π_1). The compressible, non-rotating, adiabatic equations in Cartesian coordinates (y, z) are:

$$\frac{\partial v}{\partial t} = -v\frac{\partial v}{\partial y} - w\frac{\partial v}{\partial z} - c_p\theta_0\frac{\partial \pi_1}{\partial y} + D_v, \tag{1}$$

$$\frac{\partial w}{\partial t} = -v\frac{\partial w}{\partial y} - w\frac{\partial w}{\partial z} - c_p\theta_0\frac{\partial \pi_1}{\partial z} + g\left(\frac{\theta}{\theta_0} - 1\right) + D_w, \tag{2}$$

$$\frac{\partial \theta}{\partial t} = -v\frac{\partial \theta}{\partial y} - w\frac{\partial \theta}{\partial z} + D_\theta, \tag{3}$$

$$\frac{\partial \pi_1}{\partial t} = -\frac{c_s^2}{c_p\theta_0^2}\left[\frac{\partial}{\partial y}(\theta_0 v) + \frac{\partial}{\partial z}(\theta_0 w)\right]. \tag{4}$$

See Klemp and Wilhelmson (1978) for a derivation of (4). In our version, we neglect the height variation of the density. In the equations above, $\pi = (p/p_r)^{R/c_p}$, where $p_r = 1000$ mb, R is the gas constant for dry air, and c_p is the specific heat capacity at constant pressure for dry air, and c_s is the constant speed of sound. The terms D_v, D_w, and D_θ represent turbulent mixing. Variables with a subscript 0 refer to the basic state, which varies with height only. A subscript 1 indicates the departure from the basic state. The basic state is in hydrostatic balance:

$$\frac{d\pi_0}{dz} = -\frac{g}{c_p\theta_0}. \tag{5}$$

2. The model equations including water vapor and cloud water

Density now depends on water vapor mixing ratio, q_v, and cloud water mixing ratio, q_c, as well as temperature through the equation of state:

$$p = \rho RT(1 + 0.61q_v - q_c) \equiv \rho RT_v,$$

where T_v is the virtual temperature. As a consequence, the buoyancy acceleration becomes proportional to departures from the basic state of the virtual potential temperature, θ_v, so

$$\frac{g}{\theta_0}(\theta_v - \theta_{v0}) = g\left(\frac{\theta}{\theta_0} - 1 + 0.61(q_v - q_{v0}) - q_c\right)$$

where

$$\theta_v \equiv \theta + \theta_0(0.61q_v - q_c)$$

and

$$\theta_{v0} \equiv \theta_0(1 + 0.61q_{v0}).$$

The basic state hydrostatic balance now involves θ_{v0}, so

$$\frac{d\pi_0}{dz} = -\frac{g}{c_p\theta_{v0}}. \tag{6}$$

For consistency with the basic state hydrostatic balance, θ_0 in the pressure gradient accelerations must be replaced by θ_{v0}, so the v and w equations become

$$\frac{\partial v}{\partial t} = -v\frac{\partial v}{\partial y} - w\frac{\partial v}{\partial z} - c_p\theta_{v0}\frac{\partial \pi_1}{\partial y} + D_v, \tag{7}$$

$$\frac{\partial w}{\partial t} = -v\frac{\partial w}{\partial y} - w\frac{\partial w}{\partial z} - c_p\theta_{v0}\frac{\partial \pi_1}{\partial z} + g\left(\frac{\theta}{\theta_0} - 1 + 0.61(q_v - q_{v0}) - q_c\right) + D_w. \tag{8}$$

For consistency with the v and w equations, θ_0 in the π_1 equation must also be replaced by θ_{v0}, so it becomes

$$\frac{\partial \pi_1}{\partial t} = -\frac{c_s^2}{c_p\theta_0^2}\left[\frac{\partial}{\partial y}(\theta_{v0}v) + \frac{\partial}{\partial z}(\theta_{v0}w)\right]. \tag{9}$$

When there is no heating other than that due to condensation or evaporation, and there is no precipitation, liquid water potential temperature, defined as

$$\theta_l \equiv \theta - \frac{L}{c_p\pi_0}q_c,$$

where L is the latent heat of condensation, and total (suspended) water mixing ratio,

$$q_w \equiv q_v + q_c,$$

are conserved, so we replace the conservation equation for θ with analogous conservation equations for θ_l and q_w:

$$\frac{\partial \theta_l}{\partial t} = -v\frac{\partial \theta_l}{\partial y} - w\frac{\partial \theta_l}{\partial z} + D_{\theta_l}, \tag{10}$$

$$\frac{\partial q_w}{\partial t} = -v\frac{\partial q_w}{\partial y} - w\frac{\partial q_w}{\partial z} + D_{q_w}. \tag{11}$$

To close the set of equations, we must obtain θ, q_v, and q_c from θ_l and q_w. One way to do so is to assume that

$$q_c = (q_w - q_s)H(q_w - q_s)$$

where $q_s = q_s(T, p)$ is the saturation mixing ratio. Here $H(x) = 1$ when $x > 0$ and $H(x) = 0$ otherwise. This equation states that when the grid volume is saturated, $q_v = q_s$, and when unsaturated, $q_c = 0$. If $q_c = 0$, then $q_v = q_w$ and $\theta = \theta_l$. If $q_c > 0$, then

$$\theta = \theta_l + \frac{L}{c_p \pi_0}(q_w - q_s),$$

which cannot be solved for θ directly because q_s is a non-linear function of $T = \pi_0 \theta$. However, it can be solved iteratively using the saturation adjustment algorithm described by Krueger (2006).

3. Implementating saturation adjustment

Use subroutine `adjust.f` to implement saturation adjustment. The inputs for subroutine `adjust.f` are guesses for the adjusted (output) values of θ, q_v, and q_c that are consistent with the current values of θ_l and q_w. "Consistent" means that the guesses (θ^*, q_v^*, and q_c^*) satisfy

$$\theta_l = \theta^* - \frac{L}{c_p \pi_0} q_c^*$$

and

$$q_w = q_v^* + q_c^*.$$

The simplest guess is $q_c^* = 0$. Then

$$\theta^* = \theta_l$$

and

$$q_v^* = q_w.$$

Note that the model's predicted thermodynamic variables, θ_l and q_w, are both conserved during saturated adiabatic processes (including saturation adjustment).

4. Simulation: Isolated cumulus cloud

References

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