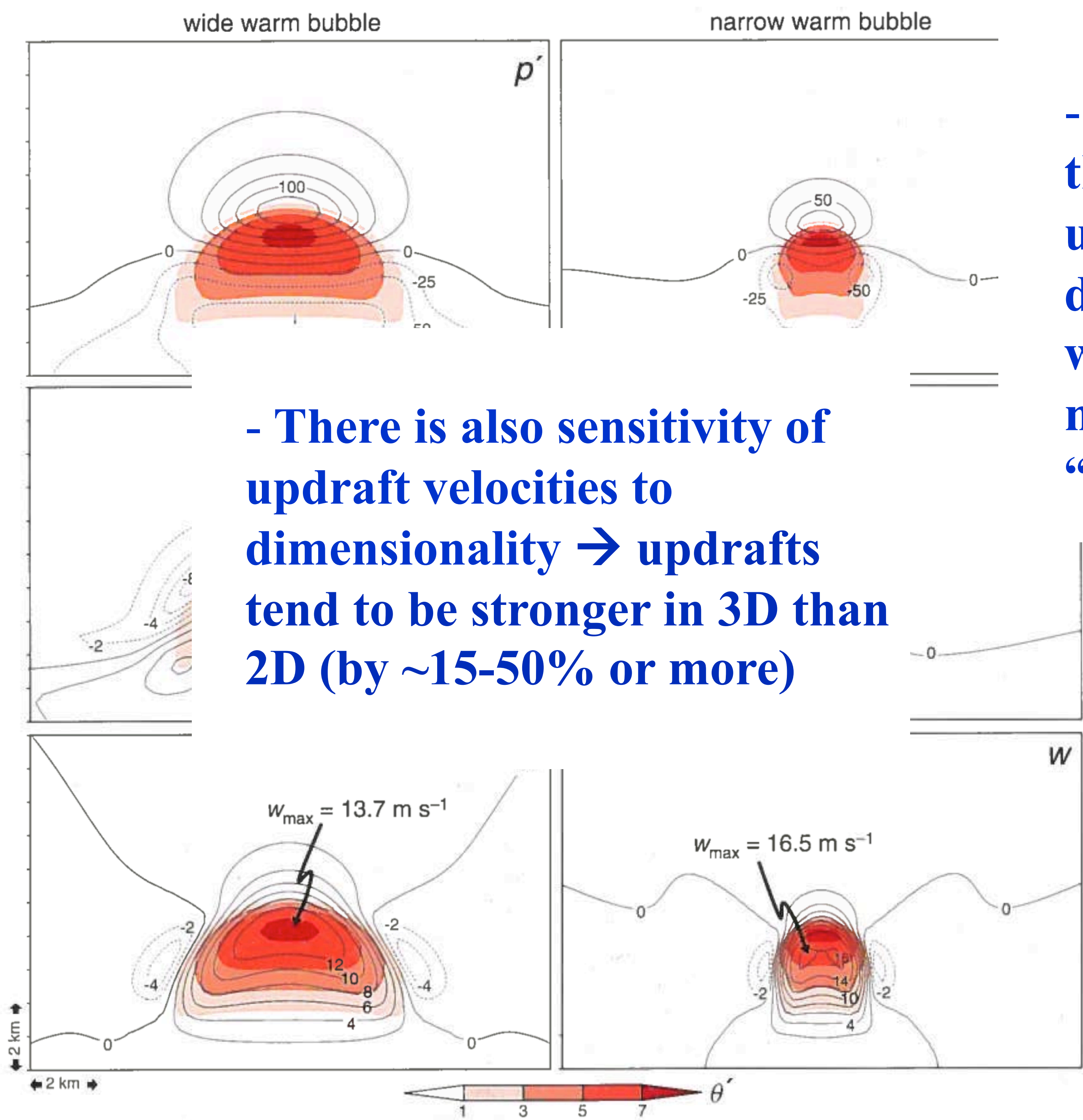
Impacts of updraft size and dimensionality on cumulus dynamics

Hugh Morrison

National Center for Atmospheric Research*
(MMM Division, NESL)

*NCAR is sponsored by the
National Science Foundation

CMMAP Winter Meeting
January 14, 2015



- There is also sensitivity of updraft velocities to dimensionality → updrafts tend to be stronger in 3D than 2D (by ~15-50% or more)

- It is well known that convective updraft velocities depend on updraft width (and hence model Δ*x* in the "grey zone")

Idealized 3D simulations using CM1 model

Markowski and Richardson (2010)

- These sensitivities are well known and attributable to perturbation pressure effects, however… quantification and deeper understanding are lacking.

- Relevance for models:

1) understanding sensitivity of "grey zone" modeling ($\Delta x \sim 1$ to 10 km) to grid resolution and dimensionality

2) representing perturbation pressure effects in convection parameterizations

Review of the key equations

Inviscid, nonhydrostatic momentum equation (anelastic):

$$\frac{\partial \vec{u}}{\partial t} + \vec{u} \bullet \nabla \vec{u} = -\frac{1}{\bar{\rho}(z)} \nabla p + B\hat{k}$$

$$B = -\frac{\rho}{\bar{\rho}(z)} g$$

Vertical component of the momentum equation (2D):

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + B$$

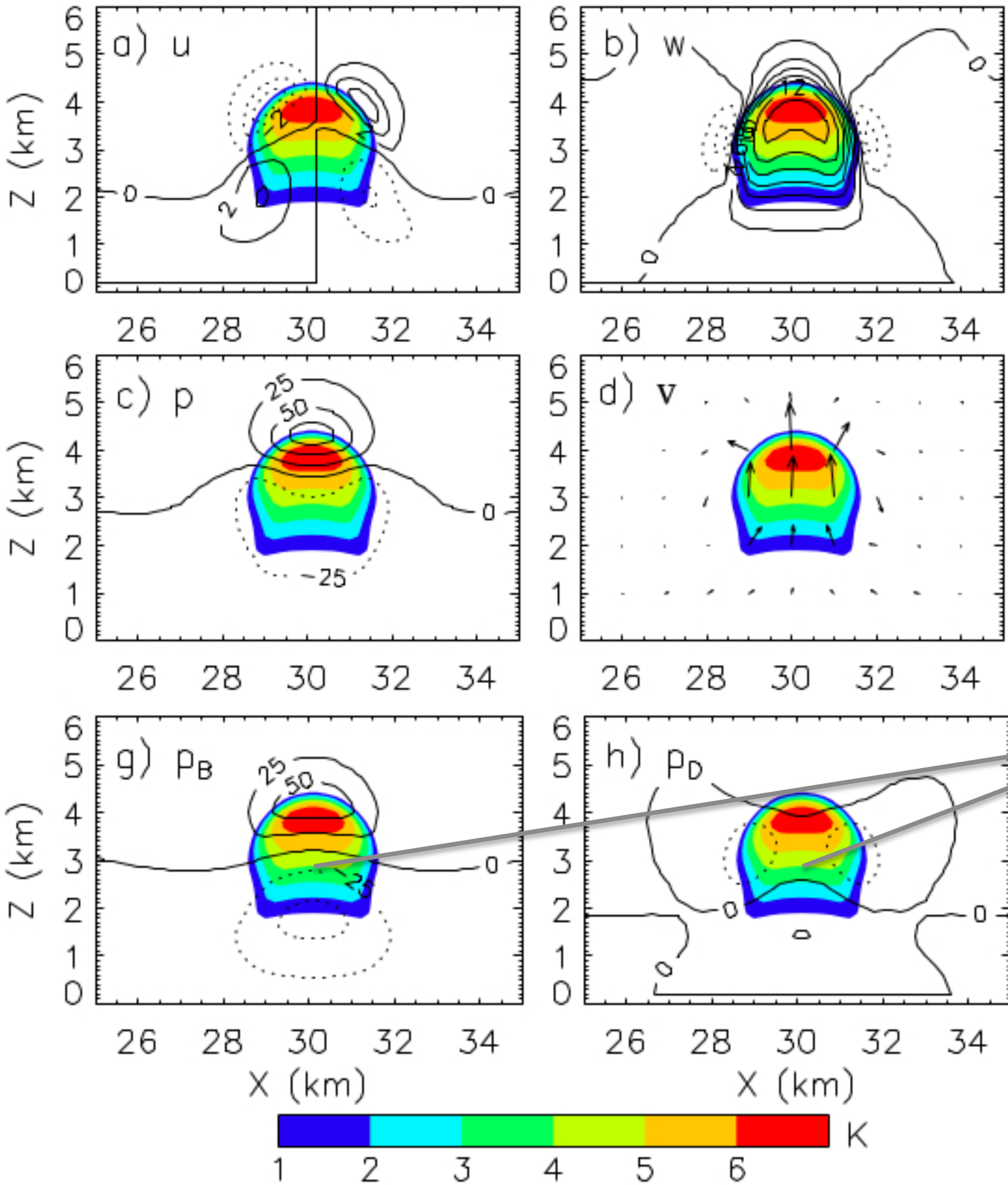
Thermodynamic maximum *w*:

$$w = \sqrt{2CAPE}$$

$$CAPE = \int_{LFC}^{LNB} B\,dz$$

Diagnostic perturbation pressure equation:

$$\nabla^2 p = \nabla^2 p_D + \nabla^2 p_B = -\nabla \bullet (\rho \vec{u} \bullet \nabla \vec{u}) + \frac{\partial(\rho B)}{\partial z}$$



Idealized 3D simulations using CM1 model
(*similar to Markowski and Richardson 2010)*

For a weakly sheared environment, at the updraft center:

$$\frac{\partial p_B}{\partial z} >> \frac{\partial p_D}{\partial z}$$

For the numerical solution we solve:

$$\nabla^2 p \approx \nabla^2 p_B = \frac{\partial(\rho B)}{\partial z}$$

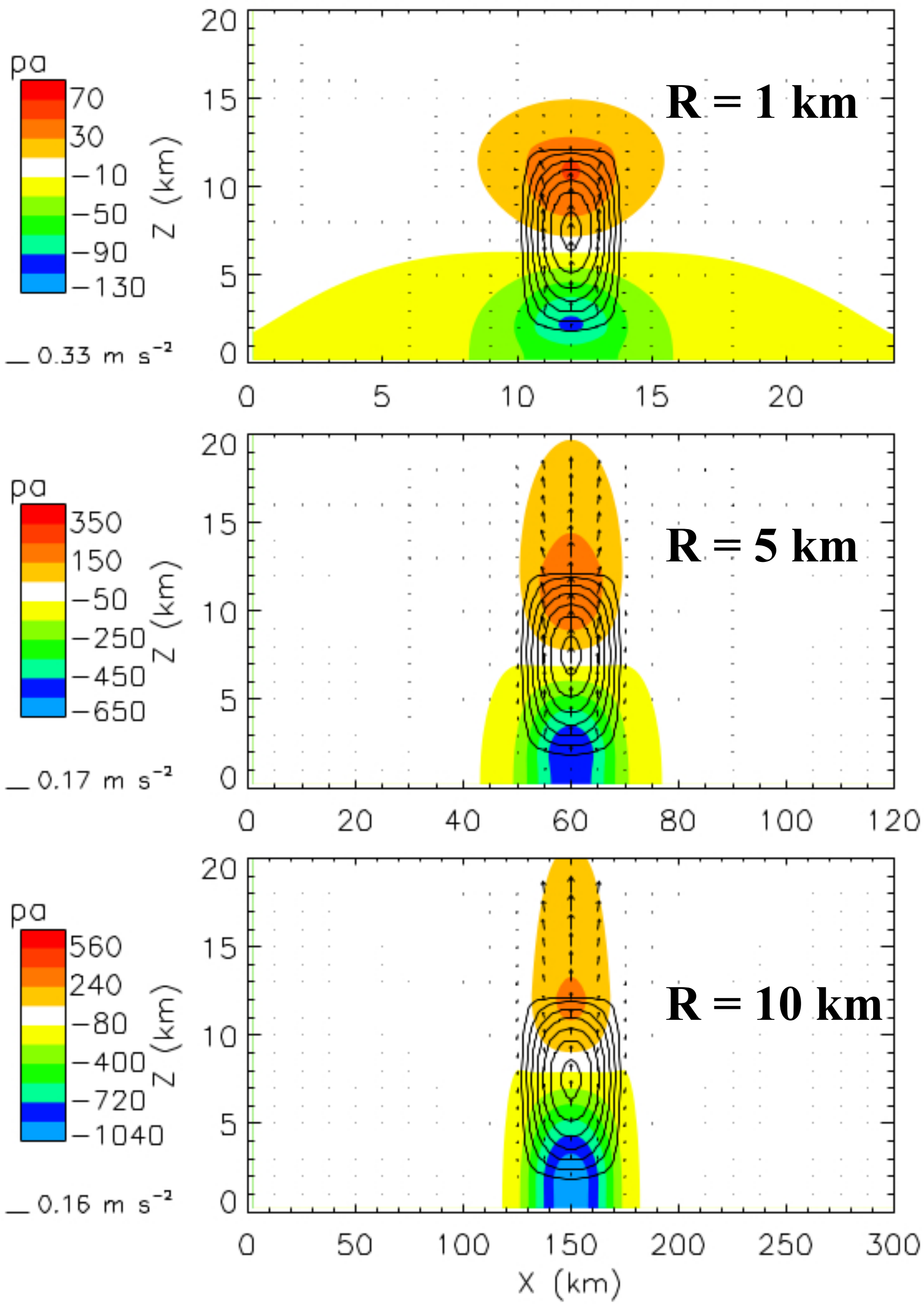
Methodology

1) Numerically solve the p_B Poisson equation and (steady state) vertical velocity in updraft center with specified *B* distributions

2) Derive theoretical scaling of *w* and perturbation pressure based on approximate analytic solutions to the governing momentum and continuity equations assuming steady state

- **2D Cartesian and axisymmetric cylindrical coordinates are used to compare 2D versus 3D updrafts**

- **Buoyancy profiles are from six real and idealized soundings, ranging from weak shallow convection to intense deep convection, with a range of horizontal buoyancy distributions tested for each sounding → *entrainment is not explicitly included***



Direct numerical solution

$$\nabla^2 p_B = \frac{\partial(\rho B)}{\partial z}$$

W-K idealized sounding
(*Weisman and Klemp 1982*)

Horizontal buoyancy distribution specified as cosine function from updraft center to edge.

Theoretical derivation

Approach: relate perturbation pressure at updraft edge to u^2 by 2 step horizontal integration of *u* momentum equation, combine with integrated continuity equation to relate *u* to *w*, then combine with *w* momentum equation with another 2 step integration (LFC to LMB and LMB to LNB).

Key assumptions:

1. **Impact of overshooting convection above LNB is neglected**
2. ***p* = 0 at the level of maximum buoyancy**
3. **Impact of downdrafts on updraft dynamics is neglected**
4. **vertical profile of *u*-wind is linear**
5. **Proportionality of *w* averaged across updraft to *w* at updraft center is equal to α, where α is given by ratio of *B* averaged across updraft to *B* at the updraft center**

3D

$$\Delta p = \frac{2\rho_0 \alpha^2 R^2}{H^2}\left(1+\frac{2\alpha^2 R^2}{H^2}\right)^{-1} CAPE$$

$$w_{LMB} = \sqrt{2CAPE_1\left(1+\frac{\alpha^2 R^2}{H_1^2}\right)^{-1}}$$

$$w_{LNB} = \sqrt{2CAPE\left(1+\frac{2\alpha^2 R^2}{H^2}\right)^{-1}}$$

For *R*/*H* → 0:

$$w = \sqrt{2CAPE}$$

For *R*/*H* → infinity:

$$w = 0$$

2D

$$\Delta p = \frac{8\rho_0 \alpha^2 R^2}{H^2}\left(1+\frac{8\alpha^2 R^2}{H^2}\right)^{-1} CAPE$$

$$w_{LMB} = \sqrt{2CAPE_1\left(1+\frac{4\alpha^2 R^2}{H_1^2}\right)^{-1}}$$

$$w_{LNB} = \sqrt{2CAPE\left(1+\frac{8\alpha^2 R^2}{H^2}\right)^{-1}}$$

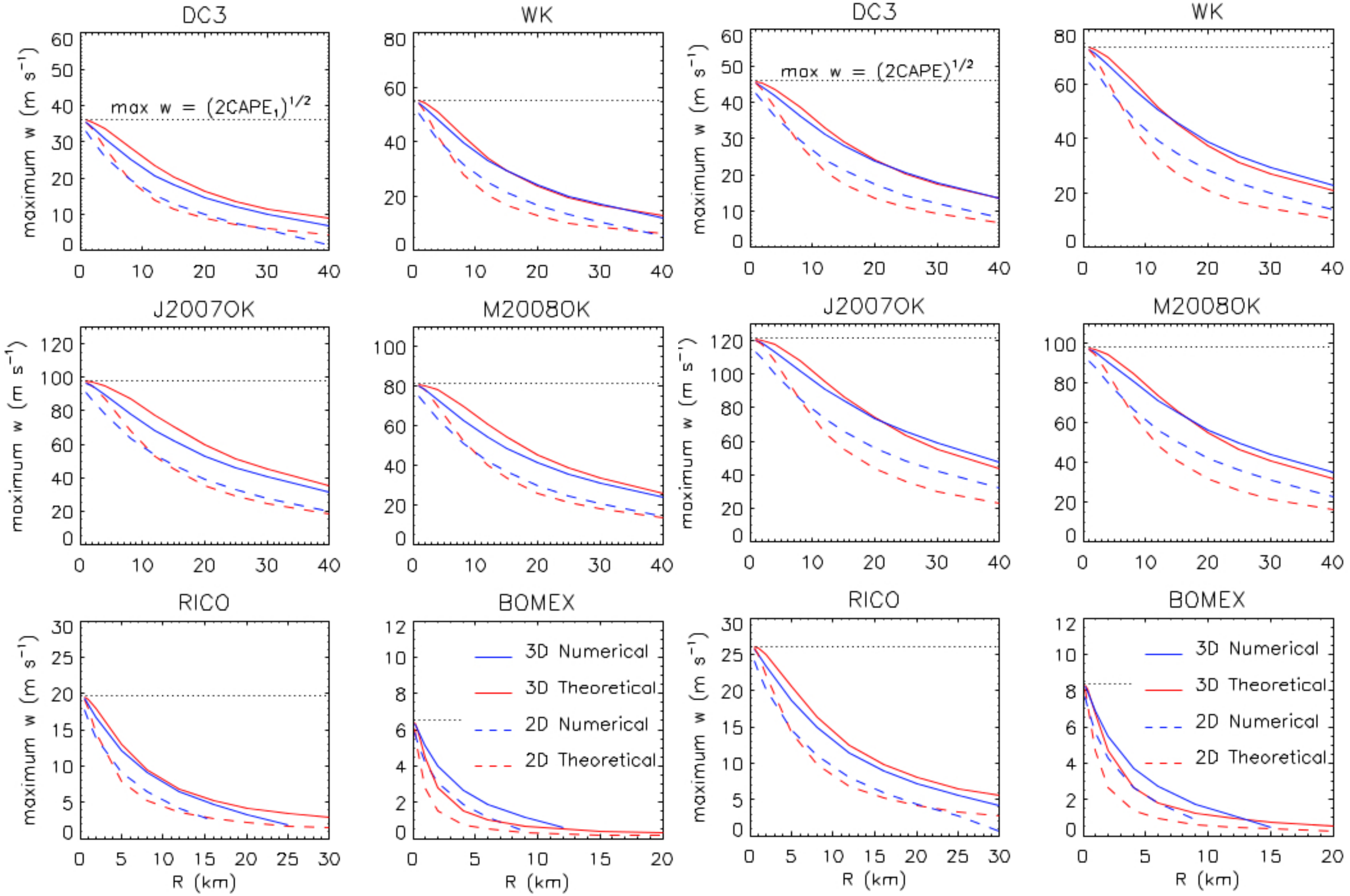
For *R*/*H* → 0:

$$w = \sqrt{2CAPE}$$

For *R*/*H* → infinity:

$$w = 0$$

w at LMB (COS) *w* at LNB (COS)

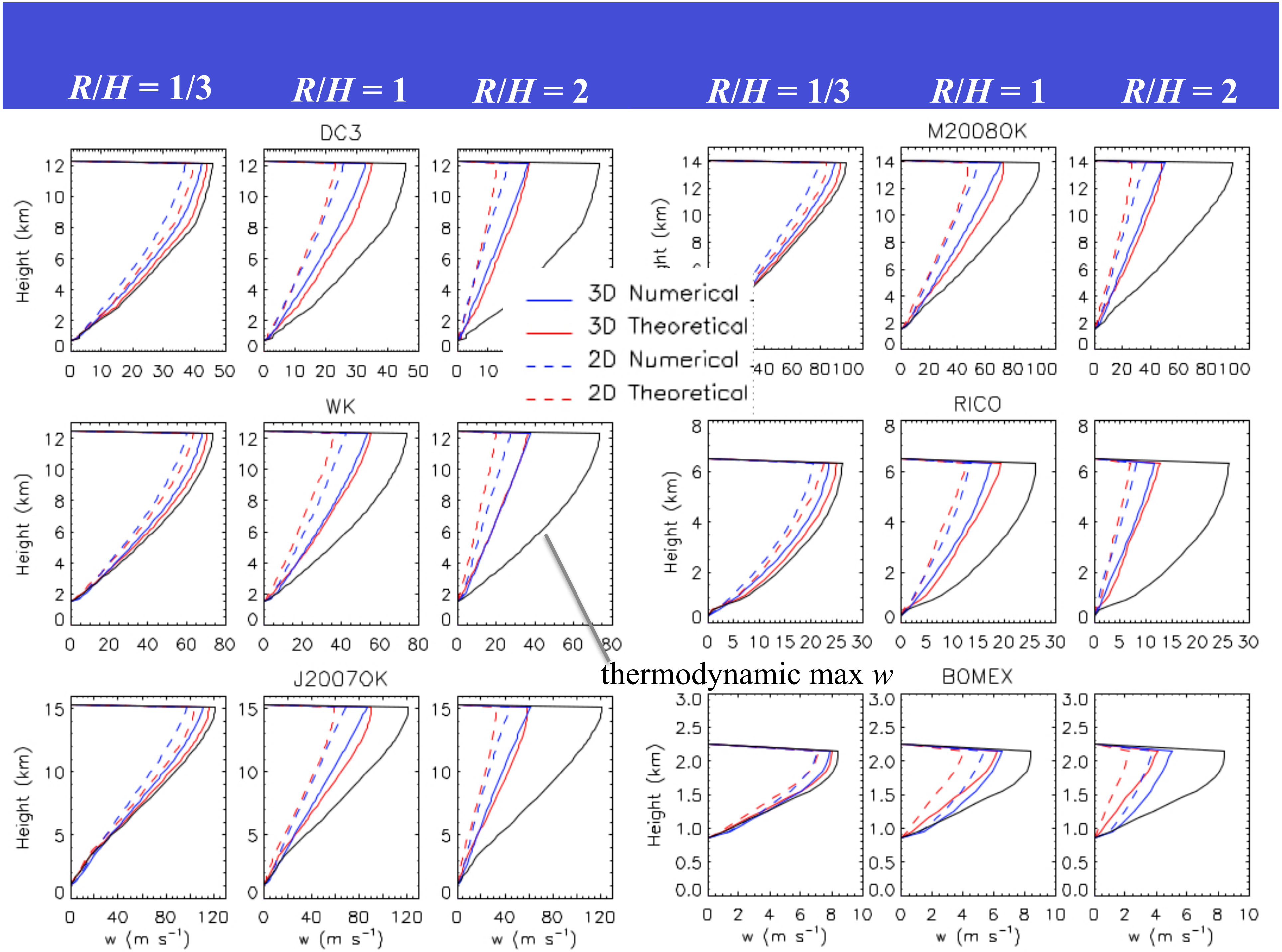


Vertical profiles of *w*

Theoretical profiles are derived by assuming a linear profile of the pressure scaling of *w* between the LMB and LNB (reasonable given smoothness of *p* field):

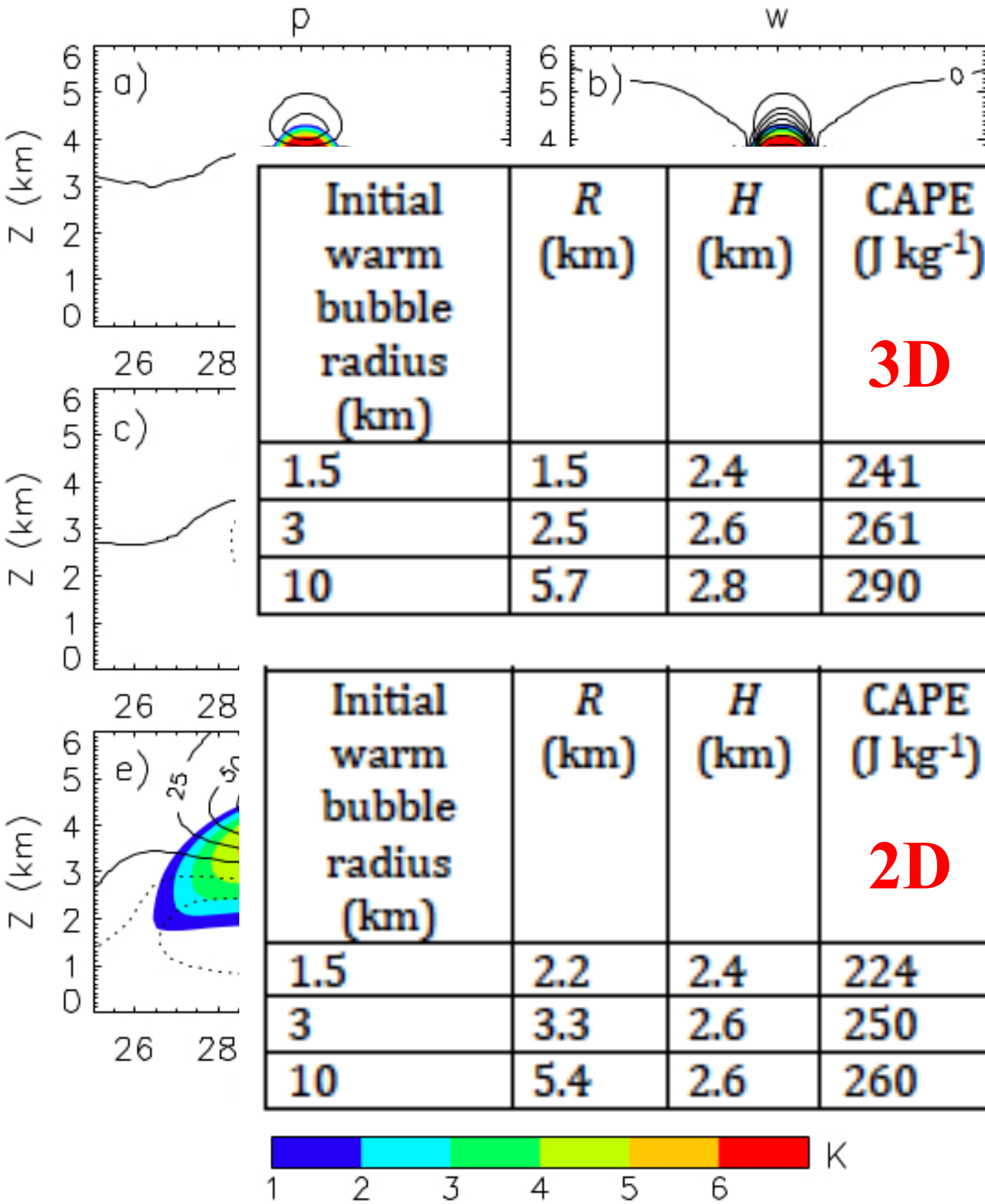
$$w(z) = \sqrt{2\left(1+\frac{\alpha^2 R^2}{H_1^2}\right)^{-1} \int_{z_{LFC}}^{z} B\,dz} \qquad z \le z_{LMB}$$

$$w(z) = \sqrt{2\left[\left(1+\frac{\alpha^2 R^2}{H_1^2}\right)+\frac{(z-z_{LMB})}{H_2}\left(\frac{\alpha^2 R^2}{H^2}-\frac{\alpha^2 R^2}{H_1^2}\right)\right]^{-1} \int_{z_{LFC}}^{z} B\,dz} \qquad z > z_{LMB}$$



R/H = 1/3
R/H = 1
R/H = 2
R/H = 1/3
R/H = 1
R/H = 2
DC3
M2008OK
WK
RICO
J2007OK
BOMEX
Height (km)
w (m s⁻¹)
3D Numerical
3D Theoretical
2D Numerical
2D Theoretical
thermodynamic max w

Comparison with 2D and 3D fully dynamical updraft simulations



3D

Initial warm bubble radius (km)	*R* (km)	*H* (km)	CAPE (J kg^{-1})	*α*	Δ*p* (hPa)		Max *w* (m s^{-1})	
					SIM	TH	SIM	TH
1.5	1.5	2.4	241	0.78	76	66	18.1	18.8
3	2.5	2.6	261	0.82	123	125	14.4	15.7
10	5.7	2.8	290	0.77	179	207	11.0	9.8

2D

Initial warm bubble radius (km)	*R* (km)	*H* (km)	CAPE (J kg^{-1})	*α*	Δ*p* (hPa)		Max *w* (m s^{-1})	
					SIM	TH	SIM	TH
1.5	2.2	2.4	224	0.47	112	118	16.4	13.3
3	3.3	2.6	250	0.48	153	162	11.7	11.2
10	5.4	2.6	260	0.57	184	205	9.0	6.6

Implications for convection schemes

Convection parameterizations often include simplified "plume" representations of *w*: they typically ignore perturbation pressure or represent it by a constant scaling of *B* ("virtual mass coefficient" → parameter "*a*").

$$\frac{1}{2}\frac{\partial\left(w^2\right)}{\partial z}=aB-b\varepsilon w^2$$

Most schemes set "*a*" ~ 1/3 to 1, but w/o physical justification.

The theoretical solutions provide a physical interpretation of the virtual mass coefficient as a function of *R* and *H* and can improve treatment of perturbation pressure effects for almost no computational cost.

Summary and conclusions

- **A simple, generalized theoretical scaling of perturbation pressure effects on *w* is proposed (for weakly-sheared environments).**
- **The theoretical scaling compares well with direct numerical solutions for a wide range of regimes from shallow to deep convection and fully dynamical updraft simulations.**
- **Different geometries in 2D and 3D lead to fundamental differences in scaling of perturbation pressure effects consistent with results from fully dynamical models → provides a concise explanation for weaker 2D convection (directly related to differences in mass continuity)**

- **Results suggest perturbation pressure effects may be a key for grid resolution sensitivity of convective strength in "grey zone" models because of updrafts that are too wide**

- **Perturbation pressure effects in convection schemes can be improved and made consistent with other aspects that scale with *R* and/or *H* (e.g., entrainment) for little computational cost**

For simple periodic buoyancy forcing functions in 2D, e.g.,

$$\rho B = \rho_0 B_0 \cos\left(\frac{\pi x}{2R}\right)\sin\left(\frac{\pi z}{H}\right)$$

then a solution is:

$$p_B = -\frac{\rho_0 B_0}{\pi H}\left(\frac{1}{4R^2}+\frac{1}{H^2}\right)^{-1}\cos\left(\frac{\pi x}{2R}\right)\cos\left(\frac{\pi z}{H}\right)$$

This can be combined with the *w* momentum equation to give

$$w_{\max} = \sqrt{2CAPE\left(1+\frac{4R^2}{H^2}\right)^{-1}}$$

Analogous solutions can be derived for axisymmetric 3D updrafts by representing the buoyancy forcing using a Fourier-Bessel expansion.

$$p_B = -\frac{\rho_0 B_0}{\pi H}\left(\frac{1}{4R^2}+\frac{1}{H^2}\right)^{-1}\cos\left(\frac{\pi x}{2R}\right)\cos\left(\frac{\pi z}{H}\right). \qquad (6)$$

The difference in p_B from the LNB to LFC at the updraft center Δp is found by evaluating (6) at $x = 0$ and $z = H$ and subtracting (6) evaluated at $x = 0$ and $z = 0$. This can be expressed in terms of $\mathrm{CAPE} = \int_{\mathrm{LFC}}^{\mathrm{LNB}} B\,dz$ by integrating (5) from $z = 0$ to H, combining with the Δp evaluated from (6), and rearranging terms to yield

$$\Delta p = \rho_0 \mathrm{CAPE}\left(1+\frac{H^2}{4R^2}\right)^{-1} \qquad (7)$$

A similar expression for Δp can be derived for axisymmetric cylindrical quasi-3D updrafts using a Fourier–Bessel expansion. In this case, the single normal mode horizontal r component of the Laplacian in cylindrical coordinates is approximated as $-k_B^2 p$, where k_B is the first root of the Bessel function of the first kind, $J_0(k_B R) = 0$ (Holton 1973). This gives $k_B \sim 2.41/R$. Given that $J_0(0) = 1$, we can repeat the steps above to derive an expression for Δp in 3D analogous to (7):

$$\Delta p = \rho_0 \mathrm{CAPE} \left(1 + \frac{H^2}{2 c_0 R^2} \right)^{-1}, \tag{8}$$

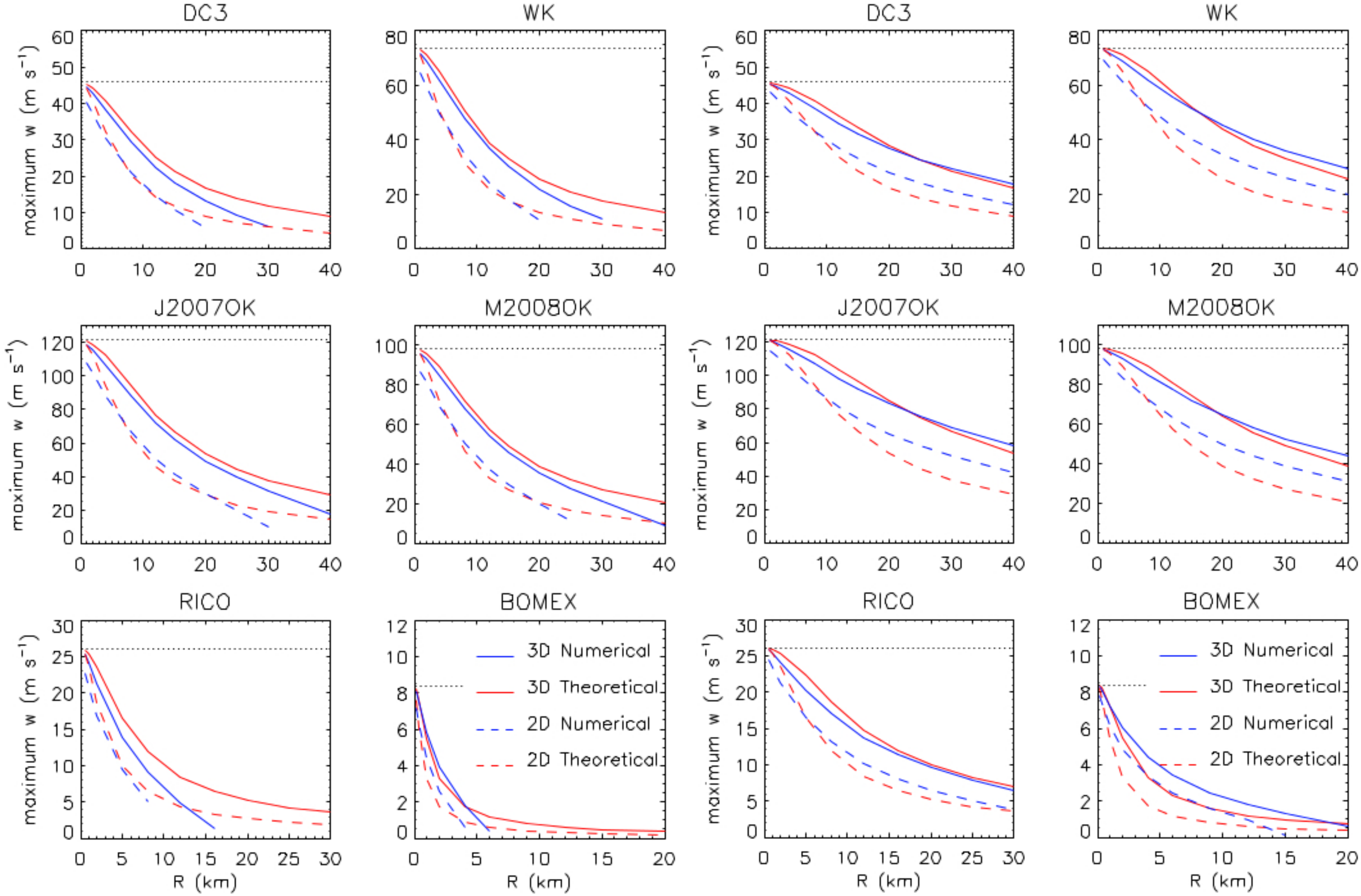
where $c_0 \approx \pi^2/(2 \times 2.41^2) \approx 0.849$.

For less idealized forcing the first term in a Fourier/Fourier-Bessel expansion can be retained to give equivalent expressions.

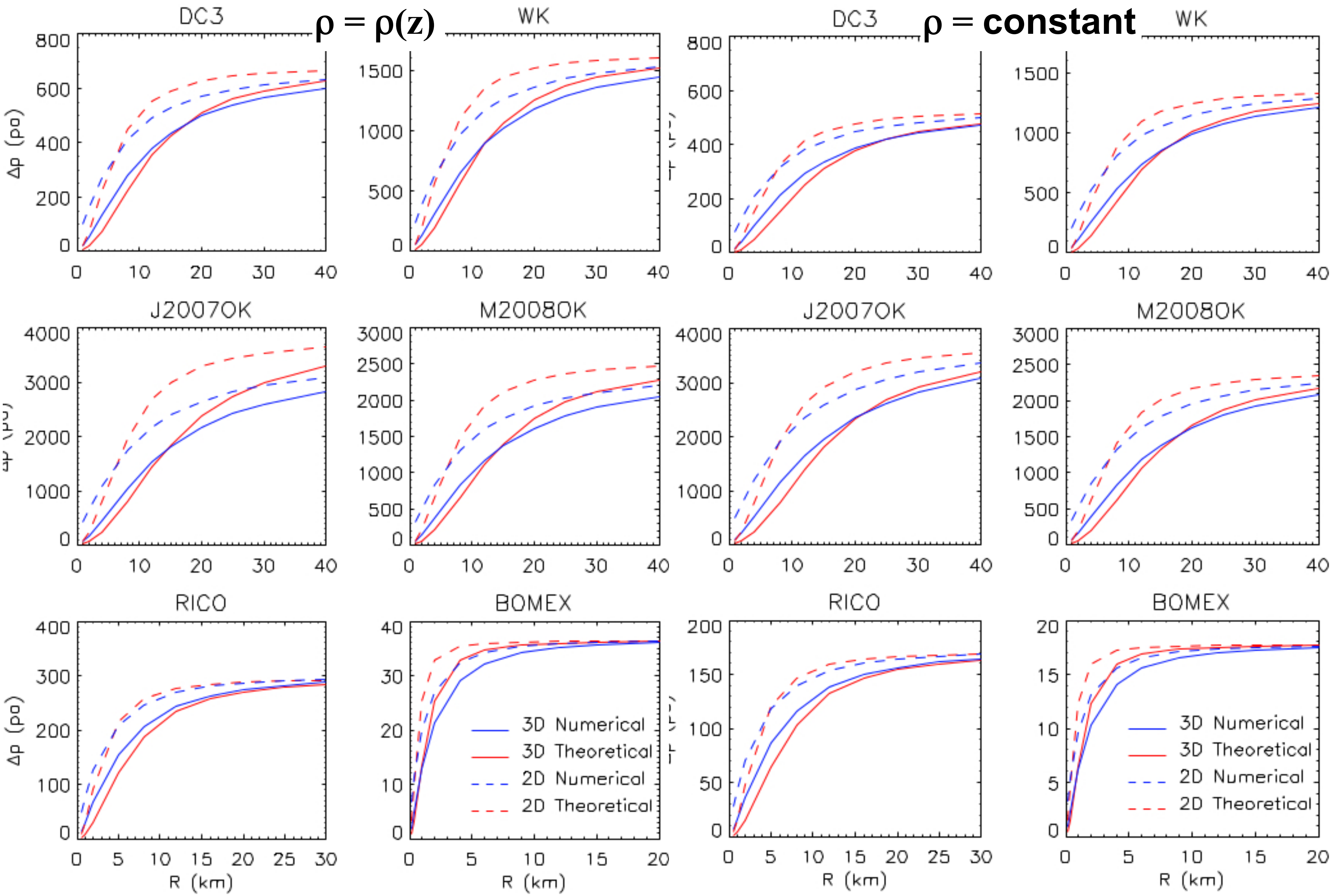
However, there are some conceptual issues:

- **The CAPE is that from single normal mode Fourier expansion of buoyancy forcing, which in general is different from the actual CAPE → can lead to large errors in integrated quantities (but one can assume scalings still apply)**
- **Underlying assumption of periodicity and symmetry in horizontal *and* vertical**

w at LNB (TOP-HAT)   w at LNB (COS^2)



Δp from LNB to LFC (COS)



But the effect on w is small!