5 Parcel Vertical Velocity

Equation (4) for the parcel vertical velocity neglects the vertical perturbation pressure gradient acceleration, which can be significant. We will include and then parameterize this acceleration. The complete equation for the parcel's vertical acceleration that was presented in section 1 is

$$\frac{dW}{dt} = -c_p \bar{\theta}_v \frac{\partial \pi_1}{\partial z} + g\left(\frac{\theta - \bar{\theta}}{\bar{\theta}} + 0.61(w - \bar{w}) - l - r\right) + D_W,$$

or in abbreviated form,

$$\frac{dW}{dt} = P + B + D_W. \tag{30}$$

The rate of change of W due to entrainment is

$$D_W \equiv \left(\frac{dW}{dt}\right)_{\text{entrainment}} = -\frac{1}{m}\frac{dm}{dt}W$$

if the entrained air has zero vertical velocity.

It is desirable to convert the time derivatives to height derivatives using $W = dz/dt$. We obtain

$$\frac{dW}{dt} = \frac{W\,dW}{W\,dt} = W\frac{dW}{dz} = \frac{1}{2}\frac{dW^2}{dz}$$

and

$$\frac{dm}{dt} = \frac{W\,dm}{W\,dt} = W\frac{dm}{dz},$$

so that

$$D_W = -\frac{1}{m}\frac{dm}{dt}W = -\frac{1}{m}\frac{dm}{dz}W^2 = -\lambda W^2,$$

where

$$\lambda \equiv \frac{1}{m}\frac{dm}{dz}.$$

Equation (30) becomes

$$\frac{1}{2}\frac{dW^2}{dz} = P + B - \lambda W^2. \tag{31}$$

To apply (31) to a parcel, we need to parameterize P, the vertical perturbation pressure gradient acceleration. P is primarily produced by the parcel's motion relative to the air around it, and acts to decelerate the parcel's motion relative to the fluid, and is therefore often called *form drag.* Form drag arises because of the form of the object. The general size and shape of the body is the most important factor in form drag: bodies with a larger apparent cross-section will have a higher drag than thinner bodies. Form drag increases with the square of the parcel's speed.[2] Therefore, form drag is often parameterized by increasing the entrainment term, which also increases with W^2. A vertical perturbation pressure gradient acceleration is also produced directly by buoyancy (that is, even without any parcel motion) that opposes the buoyancy acceleration, B. This contribution to P is often parameterized by reducing B. If we include both of these parameterizations in (31), the result is

$$\frac{1}{2}\frac{dW^2}{dz} = aB - b\,\lambda W^2. \tag{32}$$

The values used for a range from 1/3 to 1, while for b, 2 has been widely used.

[2] `http://en.wikipedia.org/wiki/Parasitic_drag#Form_drag`