Buoyant Plume: Line Source

Structure: Neutral Stratification

From Rouse, Yih, and Humphreys (1952), via section 2.3 of Emanuel's *Atmospheric Convection*, the vertical velocity in the plume is a function of x and z:

$$w(x,z) = 1.8\, F^{1/3} \exp\left(-32\frac{x^2}{z^2}\right),$$

where

$$F = \int_{-\infty}^{+\infty} wb\, dx,$$

is the integrated (or total) buoyancy flux of the plume and x is the coordinate normal to the line source.

The buoyancy in the plume is also a function of x and z:

$$b(x,z) = 2.6\, F^{2/3} z^{-1} \exp\left(-41\frac{x^2}{z^2}\right),$$

and the plume half-width is

$$R = 0.16z.$$

What is the vertical velocity at $x = R$?

$$\frac{w(R,z)}{w(0,z)} = \exp(-32(0.16)^2) \approx 0.44.$$

The mass flux of a plume per unit length along the line source (kg s^{-1} m^{-1}) is

$$M(F,z) = 1.8\, \rho(z)\, F^{1/3} z/4\, (\pi/2)^{1/2}.$$

Example: Stable Stratification

We will assume that the plume structure described above for neutral stratification is valid at levels below the maximum plume height, h.

1. The total buoyancy flux at the source is that for a fire with a 5-m flame length. Byram's (1959) fire intensity equation is

   $$I = 258 \times 10^3\, L_f^{2.17},$$

   where I is the fire intensity in W/m and L_f is the flame length in m. For $L_f = 5$ m, $I = 8.48$ MW/m. The corresponding total buoyancy flux is

   $$F = \frac{g}{\rho c_p \theta_0} I = 230 \text{ m}^3/\text{s}^3,$$

   for gravity $g = 9.81$ m/s^2, air density $\rho = 1.2$ kg/m^3, specific heat capacity at constant pressure $c_p = 1004$ J/kg/K, and reference potential temperature $\theta_0 = 300$ K.

2. A plume can penetrate to a particular height h that depends on F and the stability N:

$$h = 2.85\, F^{1/3} N^{-1}.$$

For F as specified above and $N = 0.01$ s^{-1}, a typical mid-latitude value, $h = 1747$ m.

3. The maximum (i.e., centerline) plume updraft speed at any level below h is $1.8\, F^{1/3}$. For F as specified, this speed is 11.0 m/s.

4. To construct top hat instead of Gaussian profiles, we must define the plume edge. If we define the edge by a plume threshold updraft speed, w_e, then the distance of the edge from the plume centerline is

$$R = z \left(\log(1.8\, F^{1/3}/w_e)/32\right)^{1/2}.$$

For $w_e = 0.5$ m s^{-1}, $z = 1000$ m, and F as specified, $R = 311$ m, so the plume is 622 m wide.

5. Once the plume edge is specified, average values of vertical velocity and buoyancy within the plume can be calculated using

$$w_p = \frac{\sqrt{\pi}\, 1.8\, F^{1/3} z}{2\sqrt{32}\, R} \mathrm{erf}\left(\sqrt{32}\frac{R}{z}\right)$$

and

$$b_p = \frac{\sqrt{\pi}\, 2.6\, F^{2/3}}{2\sqrt{41}\, R} \mathrm{erf}\left(\sqrt{41}\frac{R}{z}\right).$$

For F, z, and R as specified, $w_p = 5.5$ m s^{-1} and $b_p = 0.043$ m s^{-2}.