8 Simulation 2: Linear convection (Due March 28, 2013)

1. This case corresponds to Rayleigh convection in the linear convection regime. Use free-slip, conducting boundary conditions. **Code the finite-difference equations for v, w, θ and π_1.** These are fully described in sections 3-5. To obtain the linear solution, omit the non-linear advection terms for v and w, and linearize the advection terms for θ by replacing $\theta(y, z, t)$ with $\theta_0(z)$.

2. **Perform a numerical simulation.** Use $H = 500$ m, $L = 2^{3/2}H$, $K_v = K_w = K_\theta = 100$ m^2 s^{-1}, $\Delta\theta = 2.4$ K, and

   $$\theta_0(z) = 288. - z\frac{\Delta\theta}{H}.$$

   These parameters correspond to Ra=1000, which is slightly supercritical for free-slip boundary conditions, when K_v, K_w, and K_θ are interpreted as molecular diffusivities.

   Use jt=20 and kt=10. Try $c_s = 25$ m/s. Use $\Delta t = 0.5$ s. Run for 300 s.

   Start with $v = w = \pi_1 = 0$ everywhere and

   $$\theta(y, z) = \theta_0(z).$$

   To initiate convection, add $A\cos(ky)$ to the initial θ field at the two levels nearest to $z = H/2$, with $k = 2\pi/L$ and $A = 0.05\Delta\theta$.

3. **Make a contour plot of each perturbation field at 300 s** (i.e., $v, w, \theta_1 = \theta - \theta_0(z)$, and π_1). *Normalize (divide) each field by its maximum value.* The simplest way to make a contour plot is to print the field using the subroutine PRINT (`http://www.inscc.utah.edu/~krueger/6150/qcom.html`), then draw the contours by hand. Use a contour interval of 0.2. Instead of drawing contours by hand, you may use a suitable graphics program, such as MATLAB, NCAR Graphics, or IDL.

4. The simplest analytic solution is for horizontal rolls aligned in the x direction. The perturbation fields of w, v, $B \equiv g\theta_1/\theta_0$, and $p_1/\rho_0 = c_p\theta_0\pi_1$ for this solution are given on page 59 of *Atmospheric Convection*:

$$w = w_1(z)\cos(k_c y),$$

$$v = -\frac{1}{k_c}\frac{dw_1}{dz}\sin(k_c y),$$

$$B = \frac{\nu}{k_c^2}\left(\frac{d^2}{dz^2} - k_c^2\right)^2 w_1 \cos(k_c y),$$

$$\frac{p}{\rho_0} = \frac{\nu}{k_c^2}\left(\frac{d^2}{dz^2} - k_c^2\right)\frac{dw_1}{dz}\cos(k_c y),$$

where $\nu = K_w$. For free-slip boundaries, we found that the critical wave number is $k_c = \pi/\sqrt{2}$, and that

$$w_1(z) = \sum_{n=1}^{\infty} A_n \sin(n\pi z).$$

The modes with $n = 1$ are associated with the critical Rayleigh number. This applies to our case because the Rayleigh number is slightly supercritical, so we set $n = 1$:

$$w_1(z) = A_1 \sin(\pi z).$$

A_1 is the amplitude, or maximum value, of w_1, and therefore, of w.

Make contour plots of all four perturbation fields for $y = [0, 2\sqrt{2}]$ and $z = [0, 1]$. As in part 1, *divide each field by its maximum value* before plotting.