Atmos 6220
Ekman Layer Spin-Down
(Holton Chapter 5)

Ekman Layer

Use flux-gradient model:

$$K_m \frac{\partial^2 u}{\partial z^2} + f(v - v_g) = 0$$

$$K_m \frac{\partial^2 v}{\partial z^2} - f(u - u_g) = 0.$$

Assume that these hold in the entire boundary layer. The boundary conditions are

$$u = 0,\ v = 0 \text{ at } z = 0,$$

$$u \longrightarrow u_g,\ v \longrightarrow v_g \text{ as } z \longrightarrow \infty.$$

Combine these into a single equation for $u+iv$, where $i=\sqrt{-1}$,

$$K_m \frac{\partial^2(u+iv)}{\partial z^2} - if(u+iv) = -if(u_g+iv_g).$$

Assume that $\mathbf{V}_g = (u_g, 0)$ is uniform. Then the solution is

$$u+iv = A\exp\left[(if/K_m)^{1/2}z\right] + B\exp\left[-(if/K_m)^{1/2}z\right] + u_g.$$

Use

$$\sqrt{i} = \frac{1+i}{\sqrt{2}}$$

and boundary conditions to get (for $f>0$)

$$A=0,\ \ B=-u_g$$

so

$$u + iv = -u_g \exp\left[-\gamma(1+i)z\right] + u_g,$$

where

$$\gamma \equiv \left(\frac{f}{2K_m}\right)^{1/2}.$$

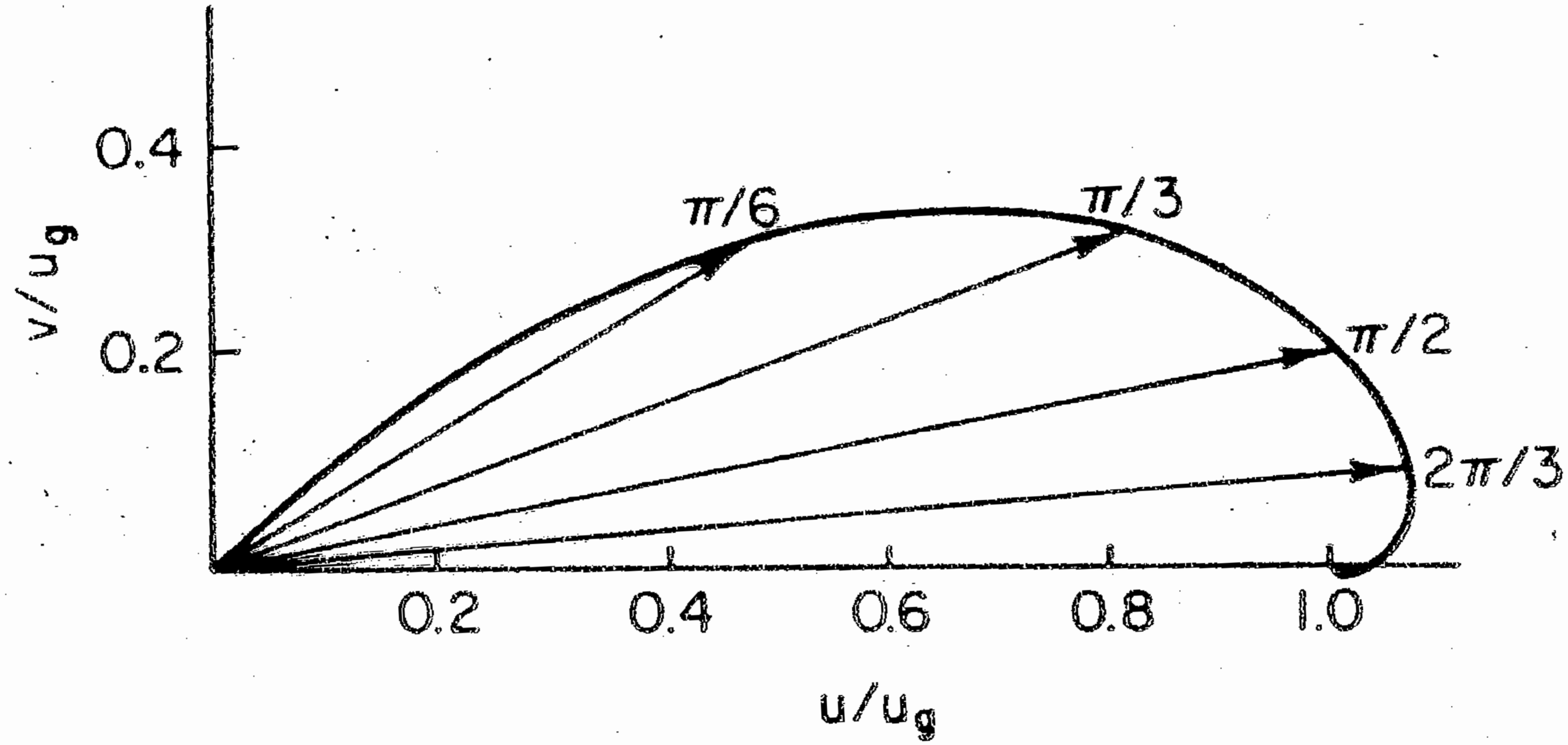
Use

$$\exp(-i\theta) = \cos\theta - i\sin\theta$$

to obtain

$$u = u_g\left(1 - e^{-\gamma z}\cos\gamma z\right)$$
$$v = u_g\left(e^{-\gamma z}\sin\gamma z\right).$$

The solution is plotted as a hodograph in Fig. 1. The boundary layer depth is π/γ.

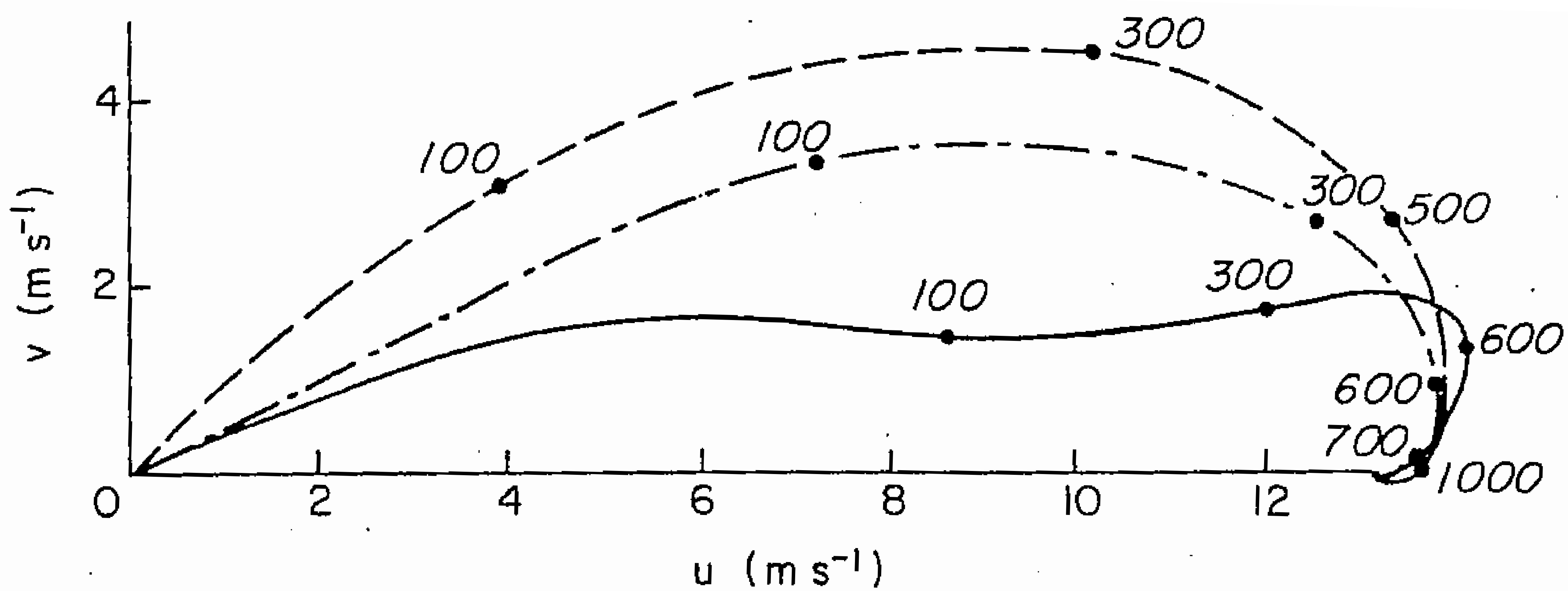


Hodograph of wind components in the Ekman spiral solution. Arrows show velocit for several levels in the Ekman layer, whereas the spiral curve traces out the velocity as a function of height. Points labeled on the spiral show the values of γz, w nondimensional measure of height.

Figure 1: Holton Figure 5.4.

Modified Ekman Layer

The Ekman layer solution is not applicable to the surface layer. A better representation is obtained by combining a log wind profile for the surface layer with the Ekman spiral, as shown in Fig. 2. However, the observed boundary layer winds deviate substantially from the spiral pattern. Approximation to idealized Ekman layer conditions can occur in strong winds over mid-latitude oceans.

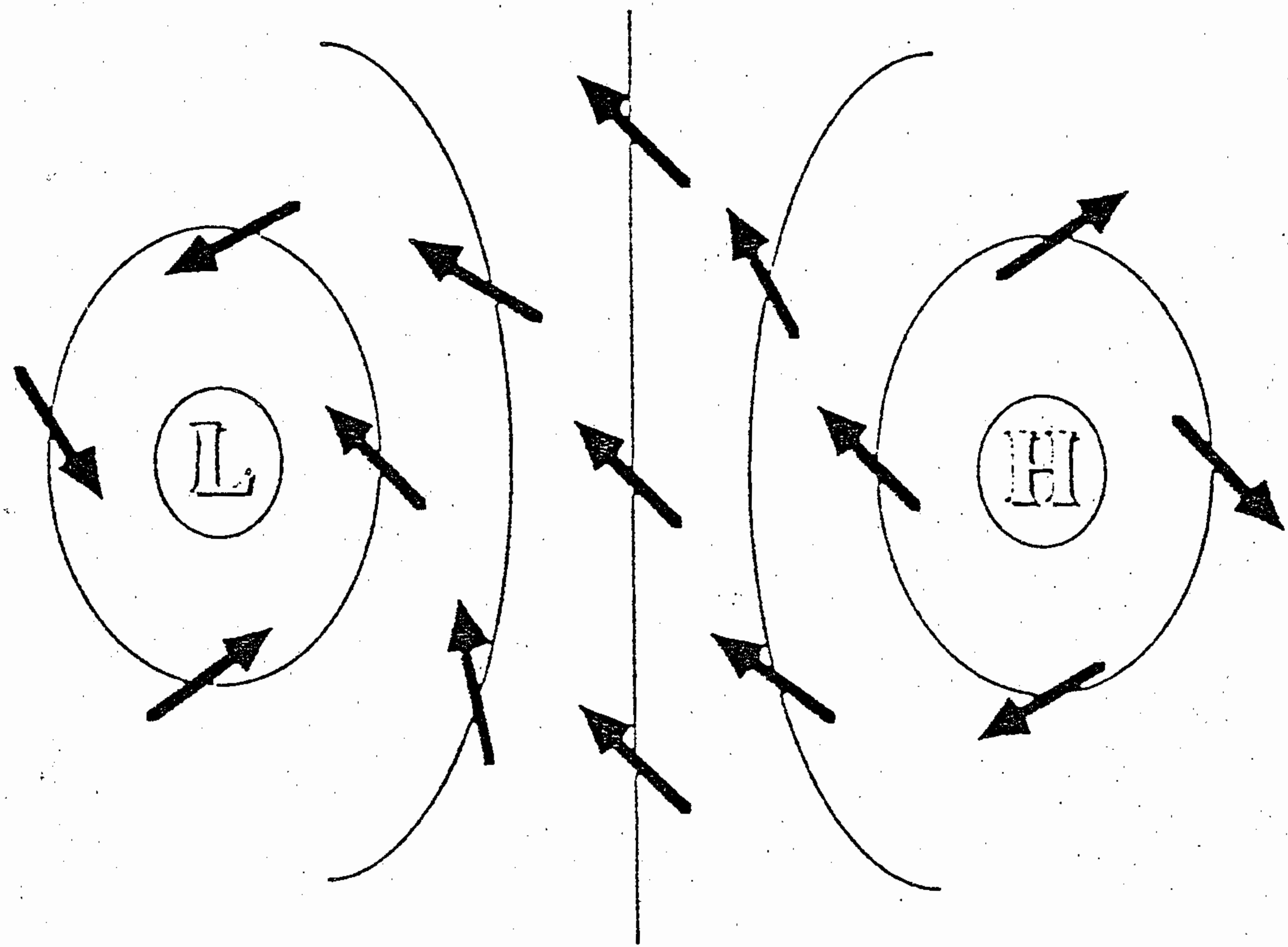


Mean wind hodograph for Jacksonville, Florida (≅30°N), 4 April 1968 (solid line) compared with the Ekman spiral (dashed line) and the modified Ekman spiral (dash-dot line) computed with $De \cong 1200$ m. Heights shown in meters. (Adapted from Brown, 1970. Reproduced with permission of the American Meteorological Society.)

Figure 2: Holton Figure 5.5.

The Ekman layer wind profile is generally unstable for a neutrally stratified atmosphere. The results is boundary layer rolls, which cannot be parameterized by simple flux-gradient models. Such circulations transport momentum vertically and decrease the angle from the geostrophic wind, as shown in Fig. 2.

The profile differs from the Ekman spiral, but the vertically integrated horizontal mass transport is still cross-isobar, which is important for synoptic and large-scale motions.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Atmosphere, Ocean and Climate Dynamics

(MIT OpenCourseWare)

http://ocw.mit.edu/courses/earth-atmospheric-and-planetary-sciences/12-003-atmosphere-ocean-and-climate-dynamics-fall-2008/labs/

Ekman layers: frictionally-induced cross-isobaric flow

- We bring the cylindrical tank, filled to a depth of 10 cm or so with water at a uniform temperature, up to solid-body rotation at a speed of 5 rpm.
- We sprinkle a few small crystals of potassium permanganate in to the tank.
- Note the Taylor columns.

Ekman layers: frictionally-induced cross-isobaric flow

- Now we reduce the rotation rate to 3 rpm.
- The fluid continues in solid rotation like a cyclonic vortex with lower pressure in the center.
- The plumes of dye from the crystals on the bottom of the tank flow inward to the center of the tank at about 45 degrees relative to the geostrophic current.

Ekman layers: frictionally-induced cross-isobaric flow

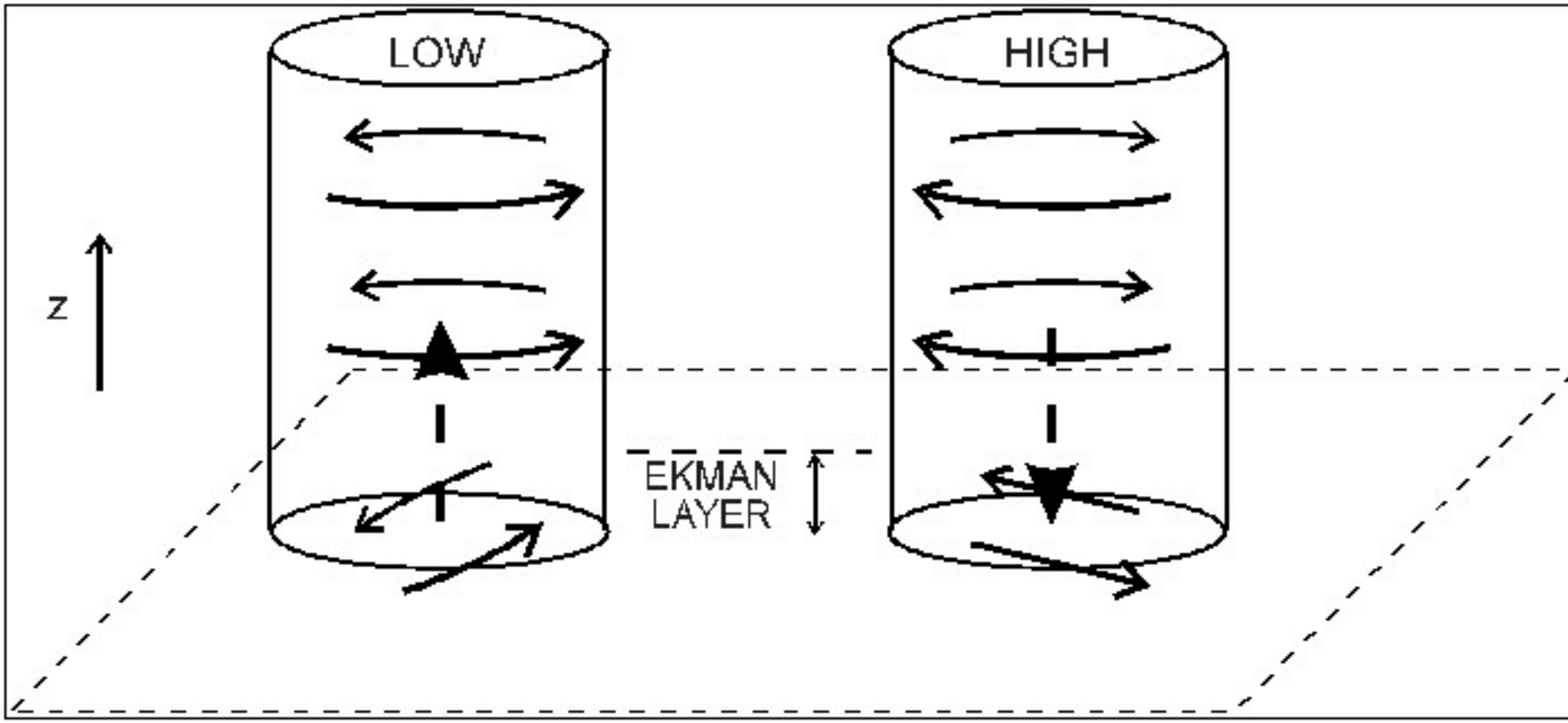
- Now we increase the rotation rate.
- The relative flow is now anticyclonic, with high pressure in the center.
- The plumes of dye sweep around to point outward.

Ekman layers: frictionally-induced cross-isobaric flow

- In each case the rough bottom of the tank slows the currents down there, and induces a cross-isobaric flow from high to low pressure.

- Above the frictional layer, however, the flow remains close to geostrophic.

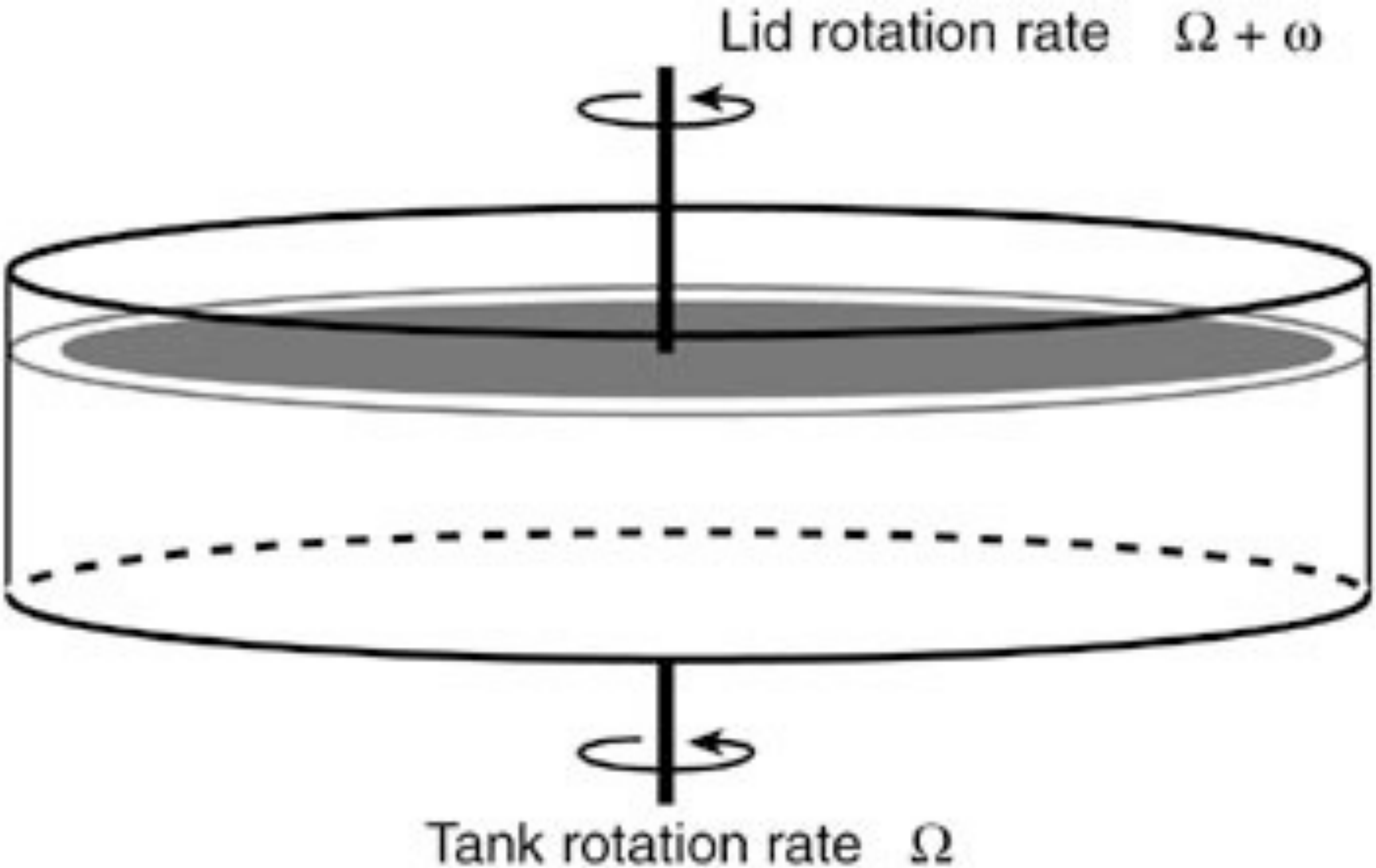
Ekman Pumping

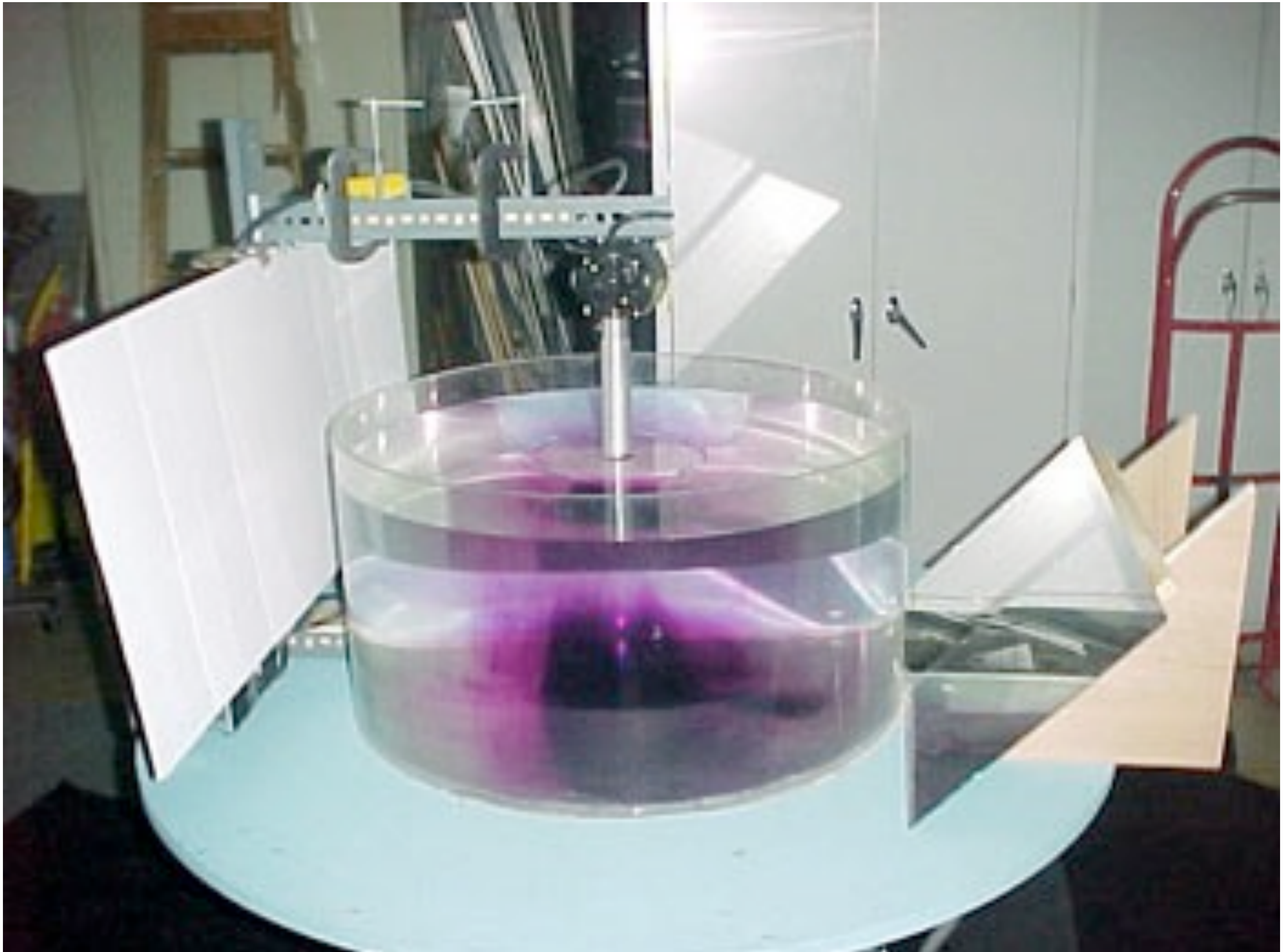


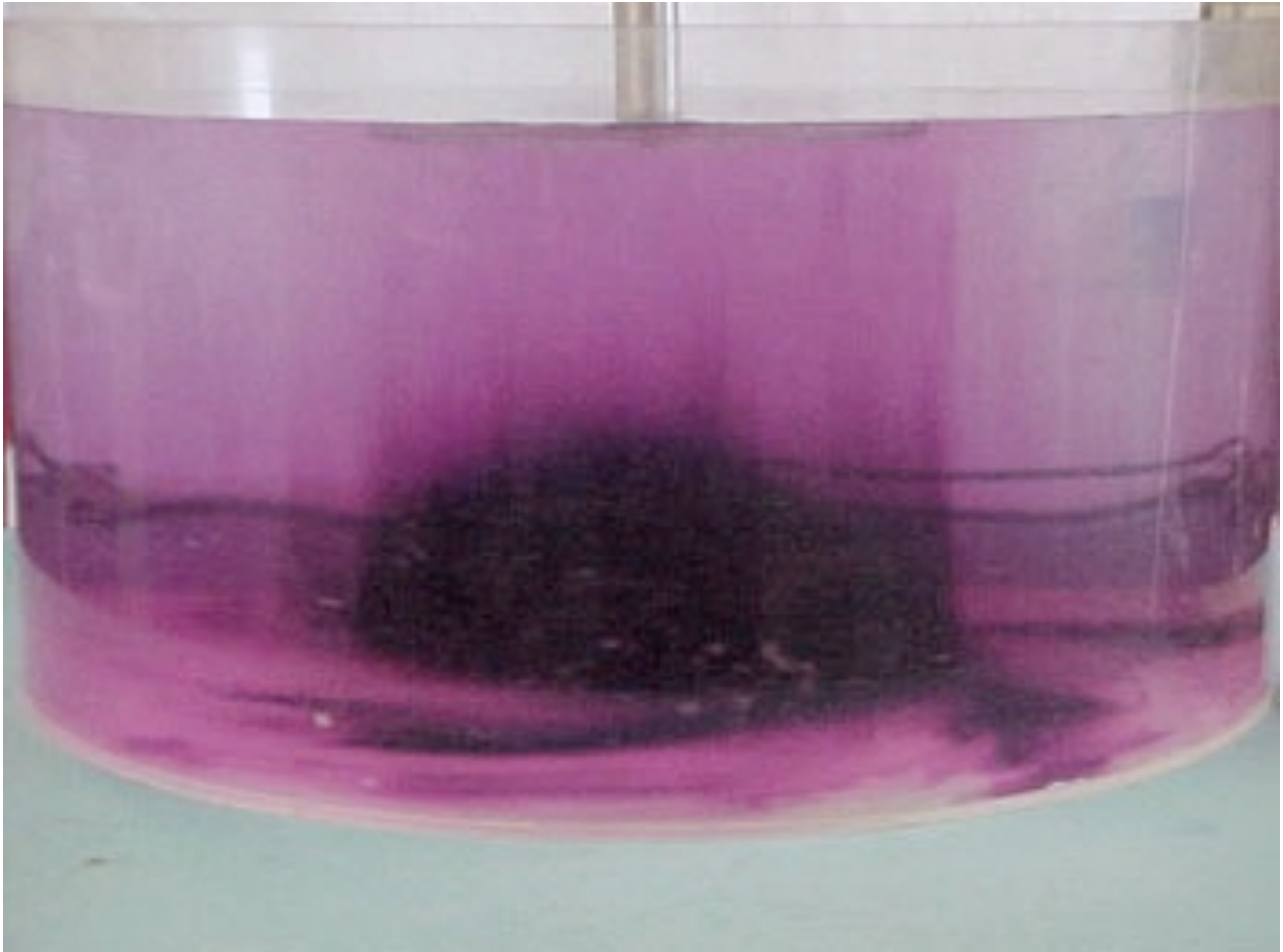
Ekman Pumping

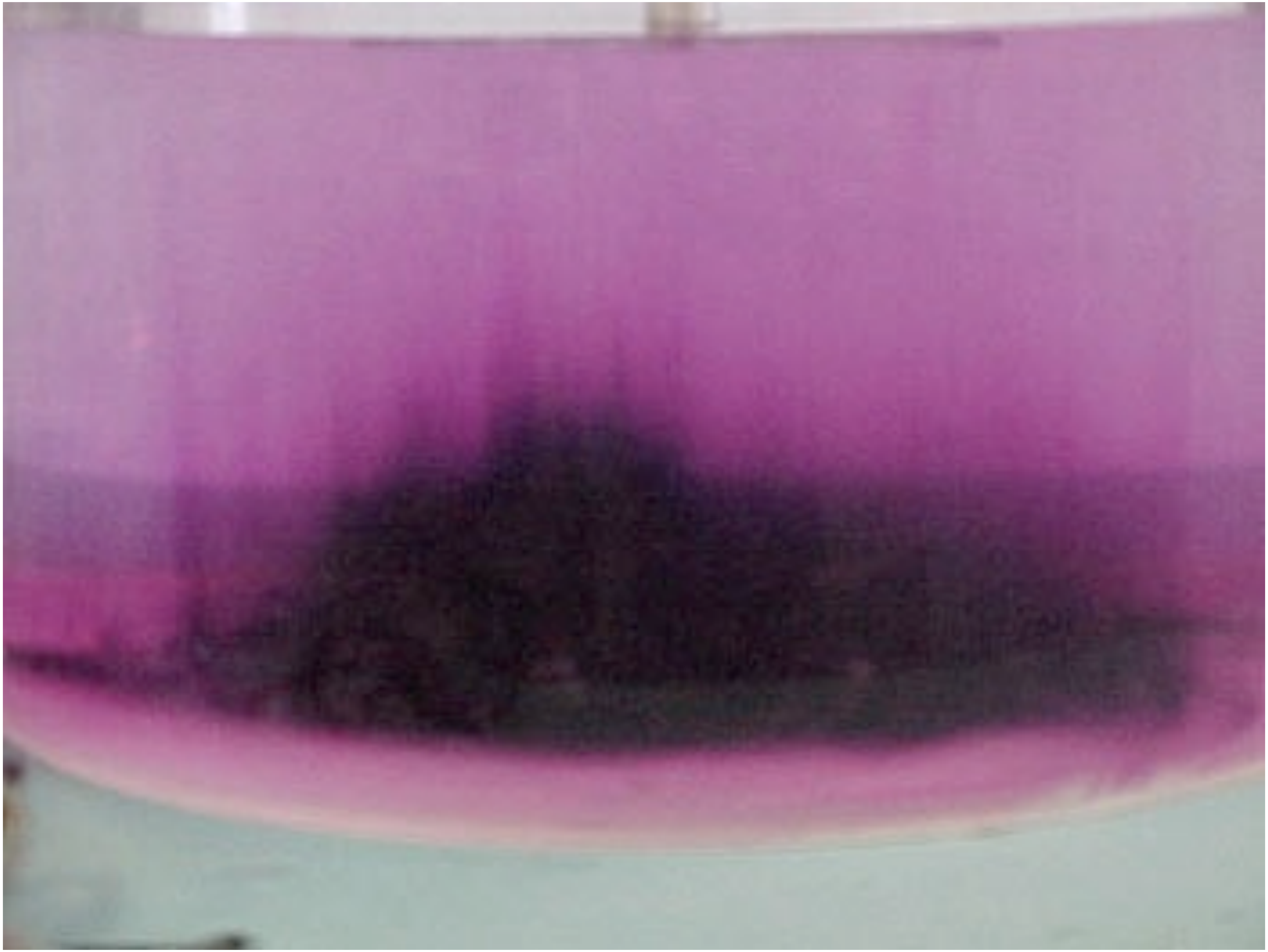
- Here we study the mechanism by which the wind stress drives ocean circulation.
- We induce circulation by rotating a disc at the surface of a tank of water which is itself rotating.

Ekman Pumping









Ekman Pumping

- In the interior (away from the bottom boundary) the horizontal flow is independent of height. Why? Since the water has uniform density, there is no "thermal wind" shear.

- Near the bottom boundary, there is inflow when ω has the same sign as Ω (cyclonic flow) and outflow when ω has the opposite sign (anticyclonic) - recall the Ekman layer experiment.

Ekman Pumping

- There is also an Ekman layer at the top (beneath the rotating lid), in which the radial component of the flow is opposite to that at the bottom boundary. (Can you figure out why? Can you picture the overall meridional (radial/vertical) circulation in the tank?)

- To help, have a look at the pictures above showing dyed water upwelling beneath a cyclonically rotating disc.