Potential Vorticity

Atmos 5110 Synoptic–Dynamic Meteorology I

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Suggested reading: Holton and Hakim (2013), section 4.5

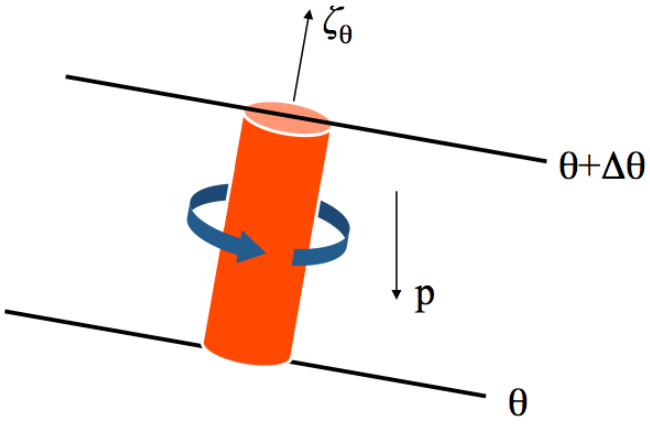
Another perspective on the development and decay of large-scale weather systems is obtained from the conservation of potential vorticity (PV), which is conserved for adiabatic and frictionless flow:

$$\text{P}= -g(\zeta_\theta + f)\frac{\partial \theta}{\partial p}; \quad \frac{DP}{Dt} = 0 \text{ for adiabatic, frictionless flow}$$

Components of Potential Vorticity

1. $(\zeta_\theta + f)$ = Absolute vorticity in isentropic coordinates

- Isentropic coordinates use potential temperature (θ) for horizontal surfaces
- Unless superadiabatic, θ increases with height
- $(\zeta_\theta + f)$ is the component of the absolute vorticity normal to the potential temperature surfaces



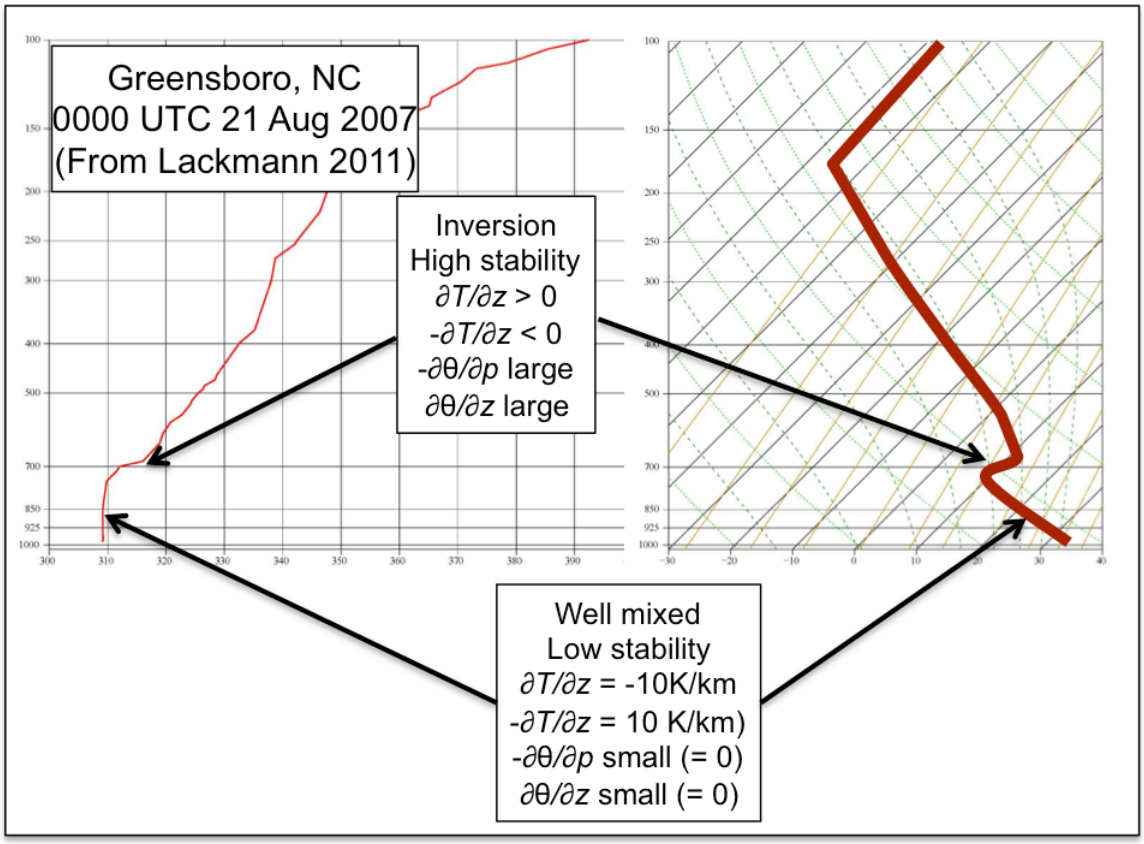
- Usually (but not always), $|\zeta_\theta + f| \cong |\zeta_z + f|$

2. $-\frac{\partial\theta}{\partial p}$ = Static Stability

- I find it easier to think about static stability in height coordinates, i.e.,

$$-\frac{\partial\theta}{\partial p} \propto \frac{\partial\theta}{\partial z}$$

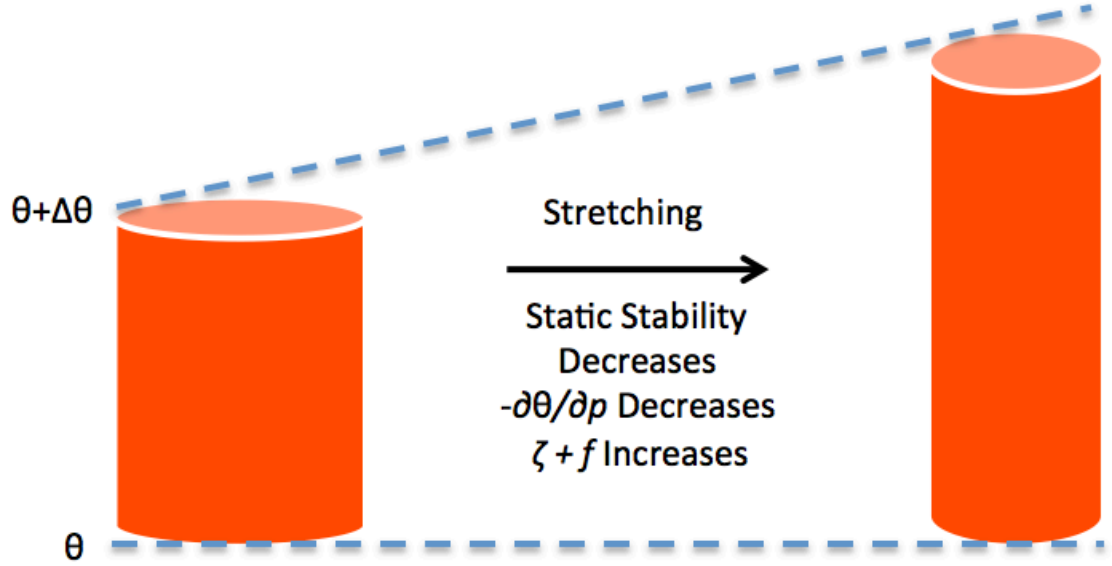
- Given that θ increases with height:
  - Large $\frac{\partial\theta}{\partial z}$ → Strong static stability
  - Small $\frac{\partial\theta}{\partial z}$ → Weak static stability



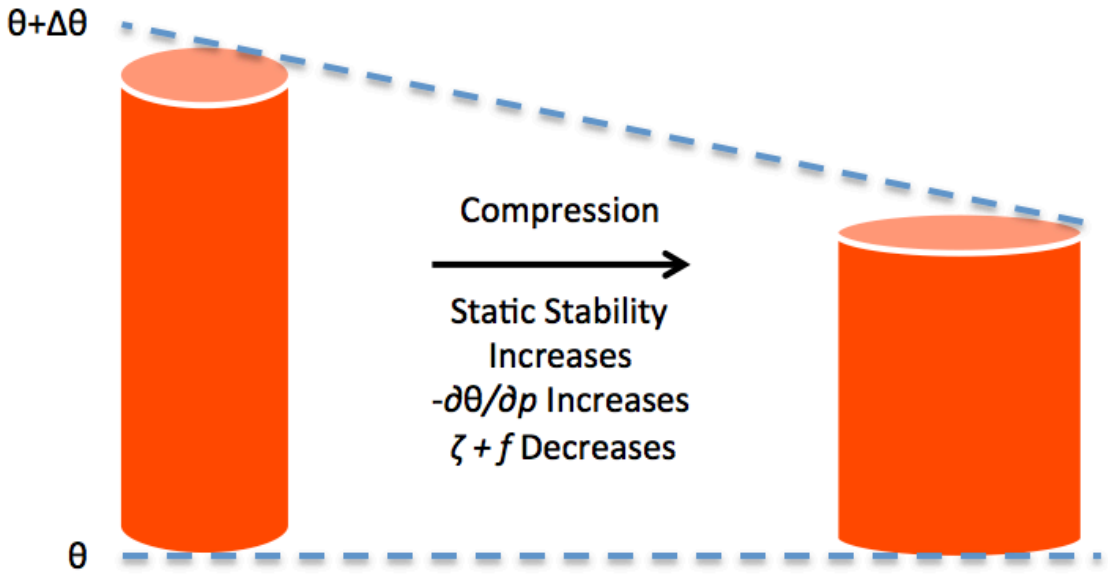
Class Activity: Using the 5110->PV-StaticStability bundle, explore the profiles and cross sections of temperature and potential temperature to better understand static stability and the relationship to potential vorticity.

Physical Interpretation

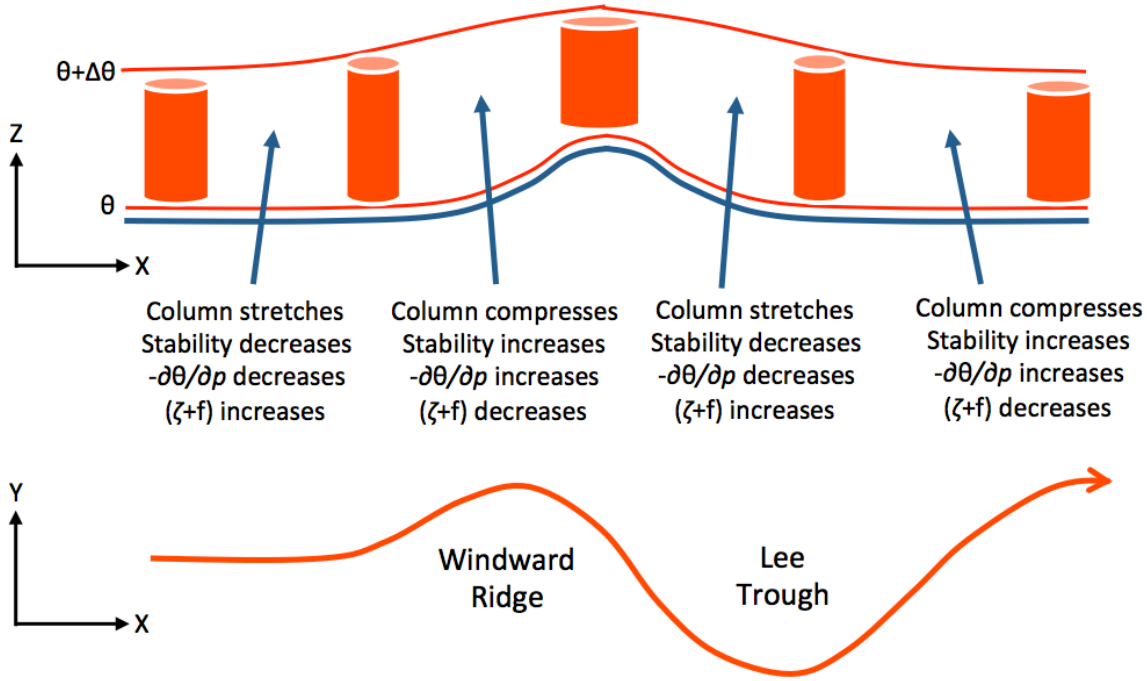
- If PV is conserved, than an increase in $(\zeta_\theta + f)$ must be accompanied by a decrease in $-\frac{\partial\theta}{\partial p}$ (or vice versa)
- For instance, if a fluid column is stretched, then the $\zeta_\theta + f$ must increase



- If a fluid column is compressed, then $\zeta_\theta + f$ must decrease



Example: Zonal flow with now initial relative vorticity impinging on a mountain barrier



Position	$-\partial\theta/\partial p$	f	ζ	Comments
1	$=-(\partial\theta/\partial p)_0$	$=f_0$	=0	
1-2	Decreasing	Small Change	Increasing	Turns north
2	$<-(\partial\theta/\partial p)_0$	$>f_0$	>0	
2-3	Increasing	Inc than dec	Decreasing	Turns south
3	$>-(\partial\theta/\partial p)_0$	$=f_0$	<0	
3-4	Decreasing	Decreasing	Increasing	Turns cyclonically
4	$<-(\partial\theta/\partial p)_0$	$<f_0$	>0	
4-5	Increasing	Increasing	Decreasing	Overshoots f_0
5	$=-(\partial\theta/\partial p)_0$	Oscillates	Oscillates	Wave train

Result: Windward ridge and lee trough

Example: Suppose we have equatorward moving air with no initial relative vorticity, but we allow the relative vorticity to increase or decrease.

Since potential vorticity is conserved

$$P_i = P_f$$

$$-\left[g(\zeta + f)\frac{\partial\theta}{\partial p}\right]_i = -\left[g(\zeta + f)\frac{\partial\theta}{\partial p}\right]_f$$

g = constant and $\partial\theta$ doesn't change $\Rightarrow$

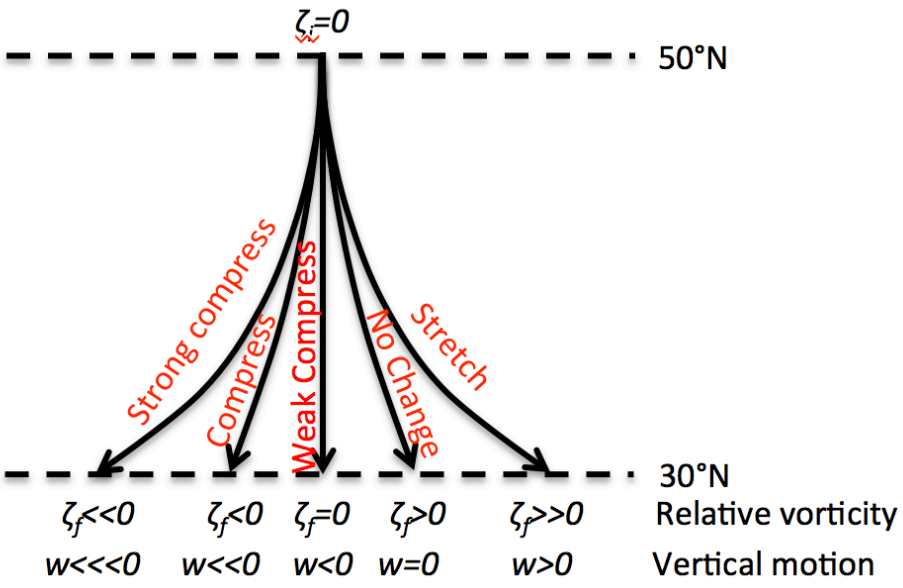
$$\frac{\zeta_i + f_i}{\Delta p_i} = \frac{\zeta_f + f_f}{\Delta p_f}$$

Setting $\zeta_i = 0$ (no initial relative vorticity) and rearranging yields

$$\frac{\Delta p_f}{\Delta p_i} = \frac{\zeta_f + f_f}{f_i}$$

Since the parcel is moving equatorward, $f_f < f_i$, so we have several possibilities:

1. $\zeta_f = 0 \Rightarrow \Delta p_f < \Delta p_i \Rightarrow$ Column compresses
2. $\zeta_f < 0 \Rightarrow \Delta p_f << \Delta p_i \Rightarrow$ Column compresses a lot
3. $\zeta_f > 0 \Rightarrow \Delta p_f \sim \Delta p_i \Rightarrow$ No major change
4. $\zeta_f >> 0 \Rightarrow \Delta p_f > \Delta p_i \Rightarrow$ Column stretches



Vertical motion assumes bottom of column remains at the ground

Synoptic Application

In the absence of other forcings:

- Northerly flow tends to be associated with subsidence
- Northerly flow with anticyclonic curvature tends to be associated with strong subsidence
- Northerly flow with cyclonic curvature tends to be associated with little vertical motion or weak ascent

Class Question Review

See classquestion.com