Chapter 6 Wallace and Hobbs
6.2 Microstructures of Warm Clouds

The average droplet radius is significantly smaller. To describe this distinction in another way, droplets with a radius of about $20 \mu$m exist in concentrations of a few per cubic centimeter in the marine clouds, whereas in continental clouds the radius has to be lowered to $10 \mu$m before there are droplets in concentrations of a few per cubic centimeter. The generally smaller droplets in continental clouds result in the boundaries of these clouds being well defined because the droplets evaporate quickly in the nonsaturated ambient air. The absence of droplets much beyond the main boundary of continental cumulus clouds gives them a "harder" appearance than maritime clouds.

We will see in Section 6.4.2 that the larger droplets in marine clouds lead to the release of precipitation in shallow clouds, and with smaller updrafts, than in continental clouds.

Shown in Fig. 6.8 are retrievals from satellite measurements of cloud optical thickness ($\tau$) and cloud droplet effective radius ($r_e$) for low-level water clouds over the globe. It can be seen that the $r_e$ values are generally smaller over the land than over the oceans, in agreement with the aforementioned discussion.

Visual extent of cloud

Marine

Continental

![Figure 6.6](a) Vertical air velocity (with positive values indicating updrafts and negative values downdrafts), (b) liquid water content, and (c) droplet size spectra at points 1, 2, and 3 in (b), measured from an aircraft as it flew in a horizontal track across the width and about half-way between the cloud base and cloud top in a small, warm, nonraining cumulus cloud. The cloud was about 2 km deep. [Adapted from *J. Atmos. Sci.*, 26, 1053 (1969).]

![Figure 6.7](a) Percentage of marine cumulus clouds with indicated droplet concentrations. (b) Droplet size distributions in a marine cumulus cloud. (c) Percentage of continental cumulus clouds with indicated droplet concentrations. (d) Droplet size distributions in a continental cumulus cloud. Note change in ordinate from (b). [Adapted from P. Squires, "The microstructure and colloidal stability of warm clouds. Part I—The relation between structure and stability," *Tellus* 10, 258 (1958). Permission from Blackwell Publishing Ltd.]

The formation of ice particles in clouds also affects their appearance (see Section 6.5.3).
6.3 Cloud Liquid Water Content and Entrainment

Instruments that can reveal the fine structures of clouds (Figs. 6.10 and 6.11), indicate that adiabatic cores, if they exist at all, must be quite rare.

Air entrained at the top of a cloud is distributed to lower levels as follows. When cloud water is evaporated to saturate an entrained parcel of air, the parcel is cooled. If sufficient evaporation occurs before the parcel loses its identity by mixing, the parcel will sink, mixing with more cloudy air as it does so. The sinking parcel will descend until it runs out of negative buoyancy or loses its identity. Such parcels can descend several kilometers in a cloud, even in the presence of substantial updrafts, in which case they are referred to as penetrative downdrafts. This process is responsible in part for the "Swiss cheese" distribution of LWC in cumulus clouds (see Fig. 6.6). Patchiness in the distribution of LWC in a cloud will tend to broaden the droplet size distribution, since droplets will evaporate partially or completely in downdrafts and grow again when they enter updrafts.

Over large areas of the oceans stratocumulus clouds often form just below a strong temperature inversion at a height of \( H_{1.5} \) km, which marks the top of the marine boundary layer. The tops of the stratocumulus clouds are cooled by longwave radiation to space, and their bases are warmed by longwave radiation from the surface. This differential heating drives shallow convection in which cold cloudy air sinks and droplets within it tend to evaporate, while the warm cloudy air rises and the droplets within it tend to grow. These motions are responsible in part for the cellular appearance of stratocumulus clouds (Fig. 6.13).
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Over large areas of the oceans stratocumulus clouds often form just below a strong temperature inversion at a height of \( \frac{H_1}{10} \approx 0.5 - 1.5 \) km, which marks the top of the marine boundary layer. The tops of the stratocumulus clouds are cooled by longwave radiation to space, and their bases are warmed by longwave radiation from the surface. This differential heating drives shallow convection in which cold cloudy air sinks and droplets within it tend to evaporate, while the warm cloudy air rises and the droplets within it tend to grow. These motions are responsible in part for the cellular appearance of stratocumulus clouds (Fig. 6.13).

Figure 6.10: High-resolution liquid water content (LWC) measurements (black line) derived from a horizontal pass through a small cumulus cloud. Note that a small portion of the cumulus cloud had nearly an adiabatic LWC. This feature disappears when the data are smoothed (blue line) to mimic the much lower sampling rates that were prevalent in older measurements. [Adapted from Proc. 13th Intern. Conf. on Clouds and Precipitation, Reno, NV, 2000, p. 105.]

Figure 6.11: Blue dots are average liquid water contents (LWC) measured in traverses of 802 cumulus clouds. Squares are the largest measured LWC. Note that no adiabatic LWC was measured beyond \( \frac{H_1}{10} \approx 900 \) m above the cloud base. Cloud base temperatures varied little for all flights, which permitted this summary to be constructed with a cloud base normalized to a height of 0 m. [Adapted from Proc. 13th Intern. Conf. on Clouds and Precipitation, Reno, NV, 2000, p. 106.]

Figure 6.12: Schematic of entrainment of ambient air into a small cumulus cloud. The thermal (shaded violet region) has ascended from cloud base. [Adapted from J. Atmos. Sci. 45, 3957 (1988).]
Relative humidity (%) \[ \begin{align*}
\text{Super-} & \\
\text{saturation (\%)} & \\
\end{align*} \]

Droplet radius (\( \mu m \))

Ambient supersaturation

Several such curves, derived from (6.8), are shown in Fig. 6.3. Below a certain droplet radius, the variation of the relative humidity (or supersaturation adjacent to a solution droplet with a specified radius would grow along the red curve in Fig. 6.4. As it does, the droplet will grow over the peak in solution vapor pressure, and the Kelvin curvature effect becomes weaker, the Kelvin curvature effect becomes essentially the same as that adjacent to a pure water droplet of the same size.

To illustrate further the interpretation of the Köhler curves 2 and 5 from Fig. 6.3. Suppose that a particle of water supersaturation of 0.4% (indicated by the green curve from Fig. 6.3). Curve 2 is for a solution droplet containing 10^4 µm of (NH_4)_2SO_4, and curve 5 is 10^5 µm of (NH_4)_2SO_4. If the den-sity of the solution is 10 s, we reproduce in Fig. 6.4 the Köhler curves for a solution droplet of radius

\[ \text{relative humidity of the air adjacent to the droplet becomes weaker, the Kelvin curvature effect becomes essentially the same as that adjacent to a pure water droplet of the same size.} \]

Hilding Köhler (1882-1982) Swedish meteorologist. Former Chair of the Meteorology Department and Director of the Meteorological Observatory, University of Uppsala.
6.3 Cloud Liquid Water Content and Entrainment

Instruments that can reveal the fine structures of clouds (Figs. 6.10 and 6.11), indicate that adiabatic cores, if they exist at all, must be quite rare. Air entrained at the top of a cloud is distributed to lower levels as follows. When cloud water is evaporated to saturate an entrained parcel of air, the parcel is cooled. If sufficient evaporation occurs before the parcel loses its identity by mixing, the parcel will sink, mixing with more cloudy air as it does so. The sinking parcel will descend until it runs out of negative buoyancy or loses its identity. Such parcels can descend several kilometers in a cloud, even in the presence of substantial updrafts, in which case they are referred to as penetrative downdrafts. This process is responsible in part for the "Swiss cheese" distribution of LWC in cumulus clouds (see Fig. 6.6). Patchiness in the distribution of LWC in a cloud will tend to broaden the droplet size distribution, since droplets will evaporate partially or completely in downdrafts and grow again when they enter updrafts.

Over large areas of the oceans stratocumulus clouds often form just below a strong temperature inversion at a height of $H_{11011}^{0.5} - 1.5$ km, which marks the top of the marine boundary layer. The tops of the stratocumulus clouds are cooled by longwave radiation to space, and their bases are warmed by long-wave radiation from the surface. This differential heating drives shallow convection in which cold cloudy air sinks and droplets within it tend to evaporate, while the warm cloudy air rises and the droplets within it tend to grow. These motions are responsible in part for the cellular appearance of stratocumulus clouds (Fig. 6.13).

**Figure 6.10**
High-resolution liquid water content (LWC) measurements (black line) derived from a horizontal pass through a small cumulus cloud. Note that a small portion of the cumulus cloud had nearly an adiabatic LWC. This feature disappears when the data are smoothed (blue line) to mimic the much lower sampling rates that were prevalent in older measurements. [Adapted from Proc. 13th Intern. Conf. on Clouds and Precipitation, Reno, NV, 2000, p. 105.]

**Figure 6.11**
Blue dots are average liquid water contents (LWC) measured in traverses of 802 cumulus clouds. Squares are the largest measured LWC. Note that no adiabatic LWC was measured beyond $H_{11011}^{0.5}$ m above the cloud base. Cloud base temperatures varied little for all flights, which permitted this summary to be constructed with a cloud base normalized to a height of 0 m. [Adapted from Proc. 13th Intern. Conf. on Clouds and Precipitation, Reno, NV, 2000, p. 106.]

**Figure 6.12**
Schematic of entrainment of ambient air into a small cumulus cloud. The thermal (shaded violet region) has ascended from cloud base. [Adapted from J. Atmos. Sci. 45, 3957 (1988).]
Finally, using the ideal gas equation for the water vapor in the ambient atmosphere, we have:

\[ \frac{dM}{dt} = \frac{\partial}{\partial x} \left( v e \frac{dM}{dx} \right) \]

where:
- \( dM/dt \) is the rate of increase in the mass of the droplet,
- \( v \) is the velocity of the air well away from the droplet,
- \( e \) is the supersaturation of the ambient air, and
- \( \frac{dM}{dx} \) is the gradient in water vapor density.

Several assumptions have been made in the derivation of (6.20). For example, we have assumed that all of the water molecules that land on the droplet remain there and that the vapor adjacent to the droplet is at the same temperature as the environment. Due to the Kelvin curvature effect, the solute effect and the release of latent heat of condensation, the temperature at the surface of the droplet will, in fact, be somewhat higher than the temperature of the droplet.

In a cloud we are concerned with the growth of a large number of droplets in a rising parcel of air. As the parcel rises it expands, cools adiabatically, and increases with time, as shown schematically by curve (a) in Fig. 6.15. However, for droplets in excess of 1 µm or so in radius it can be seen from Fig. 6.3 that the solute effect and the Kelvin curvature effect are not very important so that (6.20) becomes

\[ \frac{dM}{dt} = \frac{\partial}{\partial x} \left( v e \frac{dM}{dx} \right) \]

(expressed as a fraction rather than a percentage).

Substituting, where

\[ \frac{dM}{dt} = \frac{\partial}{\partial x} \left( v e \frac{dM}{dx} \right) \]

follows from

\[ \frac{dM}{dt} = \frac{\partial}{\partial x} \left( v e \frac{dM}{dx} \right) \]

The collection of droplets (red curve).

Hence (6.20) becomes

\[ \frac{dM}{dt} = \frac{\partial}{\partial x} \left( v e \frac{dM}{dx} \right) \]

(6.21)

\[ \frac{dM}{dt} = \frac{\partial}{\partial x} \left( v e \frac{dM}{dx} \right) \]

where \( dM/dt \) is inversely proportional to the radius of the droplet. Consequently, the parcel rises it expands, cools adiabatically, and increases with time, as shown schematically by curve (a) in Fig. 6.15. However, for droplets in excess of 1 µm or so in radius it can be seen from Fig. 6.3 that the solute effect and the Kelvin curvature effect are not very important so that (6.20) becomes

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follows from

\[ \frac{dM}{dt} = \frac{\partial}{\partial x} \left( v e \frac{dM}{dx} \right) \]

The collection of droplets (red curve).
eventually reaches saturation with respect to liquid water. Further uplift produces supersaturations that initially increase at a rate proportional to the updraft velocity. As the supersaturation rises, CCN are activated, starting with the most efficient. When the rate at which water vapor in excess of saturation, made available by the adiabatic cooling, is equal to the rate at which water vapor condenses onto the CCN and droplets, the supersaturation in the cloud reaches a maximum value. The concentration of cloud droplets is determined at this stage (which generally occurs within 100 m or so of cloud base) and is equal to the concentration of CCN activated by the peak supersaturation that has been attained. Subsequently, the growing droplets consume water vapor faster than it is made available by the cooling of the air so the supersaturation begins to decrease. The haze droplets then begin to evaporate while the activated droplets continue to grow by condensation. Because the rate of growth of a droplet by condensation is inversely proportional to its radius \[\text{see (6.21)}\], the smaller activated droplets grow faster than the larger droplets. Consequently, in this simplified model, the sizes of the droplets in the cloud become increasingly uniform with time (i.e., the droplets approach a monodispersed distribution). This sequence of events is illustrated by the results of theoretical calculations shown in Fig. 6.16.

Comparisons of cloud droplet size distributions measured a few hundred meters above the bases of nonprecipitating warm cumulus clouds with droplet size distributions computed assuming growth by condensation for about 5 min show good agreement (Fig. 6.17). Note that the droplets produced by condensation during this time period extend up to only about \(10^{-9}\) m in radius. Moreover, as mentioned earlier, the rate of increase in the radius of a droplet growing by condensation decreases with time. It is clear, therefore, as first noted by Reynolds (1877), that growth by condensation alone in warm clouds is much too slow to produce raindrops with radii of several millimeters. Yet rain does form in warm clouds. The enormous increase in size required to transform cloud droplets into raindrops is illustrated by the scaled diagram shown in Fig. 6.18. For a cloud droplet \(10^{-9}\) m in radius to grow to a raindrop 1 mm in radius requires an increase in volume of one millionfold! However, only about one droplet in a million (about...
6.4 Growth of Cloud Droplets in Warm Clouds

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Conventional borderline between cloud droplets and raindrops

Typical cloud droplet

Large cloud droplet

Typical raindrop

CCN

Typical CCN

$224$
The presence of an electric field enhances coalescence. For example, in the experiment illustrated in Fig. 6.21a, droplets that bounce at a certain angle of incidence can be made to coalesce by applying an electric field of about 10 V/m. This behavior can be explained as follows. Whether the collector drop, but as the droplet and drop approach each other in size, deformation of the collector drop due to the impact, which, in turn, determines how much air is trapped between the drop and the droplet. The tendency for coalescence occurs depends on the relative magnitude of the impact energy to the surface energy of water. This energy ratio provides a measure of the possible charge.

The electric charge carried by a droplet is related to its radius and electrical potential by the equation $q = 4\pi \varepsilon_0 r_0 E$, where $\varepsilon_0$ is the permittivity of free space, $r_0$ is the radius of the droplet, and $E$ is the electric field. Measured charges on cloud drops are generally several orders of magnitude below the maximum electric charge that a water drop can carry, which is given by $q_{\text{max}} = 4\pi \varepsilon_0 r_1^2 l$, where $r_1$ is the radius of the drop.

For a drop 0.5 mm in radius, it is seen from Fig. 6.15 that for small cloud droplets, growth by collection until the collector drop has reached a radius of 200 μm is an accelerating process. This behavior is illustrated by the data in Fig. 6.22, which is within the range of measured values in clouds. Similarly, the coalescence efficiency is unity, so that growth by collection is an accelerating process. This so-called continuous collection model for the growth of a cloud drop by collisions and coalescence is given by $dM/M = −v_1 E^{′}$, which is illustrated in Fig. 6.23. The rate of increase in the growth of a drop by collection is initially dominant but, because coalescence is aided if the impacting droplet carries an electric charge in excess of about 0.03 pC, the impact efficiency reaches 100% for droplets of radius $r_1 < 200$ μm.

Let us now consider a collector drop of radius $r_2$. The maximum electric charge that a water drop can carry is $q_{\text{max}} = 4\pi \varepsilon_0 r_1^2 l$, where $r_1$ is the radius of the drop. Substituting into (6.26), where $E^{′}$ is the maximum electric field generated by the collector drop, which is given by $E^{′} = 0.1$, it follows that $E^{′}$ is with in $0.3$. The rate of increase in the growth of a drop by collection is given by $dM = −v_1 E^{′}$, which is illustrated in Fig. 6.23. The rate of increase in the growth of a drop by collection is initially dominant but, because coalescence is aided if the impacting droplet carries an electric charge in excess of about 0.03 pC, the impact efficiency reaches 100% for droplets of radius $r_1 < 200$ μm.

### Table 6.1: Coalescence Efficiencies

<table>
<thead>
<tr>
<th>$r_1$ (μm)</th>
<th>$E^{′}$</th>
<th>Coalescence Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>0.1</td>
<td>0.9</td>
</tr>
<tr>
<td>200</td>
<td>0.2</td>
<td>0.8</td>
</tr>
<tr>
<td>300</td>
<td>0.3</td>
<td>0.7</td>
</tr>
<tr>
<td>400</td>
<td>0.4</td>
<td>0.6</td>
</tr>
<tr>
<td>500</td>
<td>0.5</td>
<td>0.5</td>
</tr>
</tbody>
</table>

### Equation 6.27

$$dM = −v_1 E^{′}$$

### Equation 6.28

$$E^{′} = \frac{q_{\text{max}}}{4\pi \varepsilon_0 r_1^2}$$

### Equation 6.26

$$dM = −v_1 E^{′}$$
The presence of an electric field enhances coalescence. For example, in the experiment illustrated in Fig. 6.21a, droplets that bounce at a certain angle of incidence can be made to coalesce by applying an electric field of about 10 kV/m. For a drop 0.5 mm in radius, it is possible charge.

Measured charges on cloud drops are generally several orders of magnitude below the maximum charge that a water drop can carry. If the collector drop has a terminal fall speed equal to the speed of the largest cloud droplets, it is within the range of measured values in clouds. Similarly, the collector drop carries charges in excess of about 0.03 pC. The maximum electric charge that a water drop can carry occurs when the surface electrostatic stress equals the surface tension stress. For a droplet of radius $r$ and larger values of the size ratio, the impact energy the collector drop due to the impact, which, in turn, determines how much air is trapped and coalescence. At smaller size ratios of the droplet to the drop, the impact energy to the surface energy of this drop is relatively smaller and less able to prevent contact and larger values of the size ratio, the impact energy ratio provides a measure of the magnitude of the impact energy to the surface energy of the drop, which is illustrated in Fig. 6.23.

We will assume that the droplets are uniformly distributed in space and that they are collected uniformly at the same rate by all collector drops of a given size. This so-called continuous collection model for the growth of a cloud drop by collisions and coalescence is aided if the impacting droplet carries a positive charge. The impact energy released at coalescence is given by:

$$\frac{dM}{dt} = \frac{1}{2}m v_1^2,$$

where $m$ is the density of liquid water, $v_1$ is the terminal fall speed of a cloud droplet of size $r_1$, and $v_2$ is the velocity with which this drop is falling in still air through a cloud of equal liquid water content. If $v_2 = v_1$, the growth of a drop by collection is an accelerated process. This behavior is illustrated by the red curve in Fig. 6.23, which indicates negligible growth of a 100 µm cloud droplet.

Because the rate of increase in the weight of the collector drop increases sharply, it is the growth of a drop by collection that is given by (6.27), which is illustrated in Fig. 6.23. The rate of increase in the weight of the collector drop due to collisions is:

$$\frac{dM}{dt} = M \frac{d^2r}{dt^2},$$

Substituting into (6.26), we get:

$$\frac{d^2r}{dt^2} = \frac{1}{l} \left( \frac{r}{c} \right)^2 \frac{d^2r}{dt^2} - \frac{1}{c} \frac{dr}{dt} \frac{d^2r}{dt^2} - \frac{1}{c} \frac{dr}{dt} \frac{d^2r}{dt^2} - \frac{1}{c} \frac{dr}{dt} \frac{d^2r}{dt^2},$$

where $l$ is the density of liquid water, $c$ is the LWC (in kg m$^{-3}$), $r$ is the cloud droplet radius, and $c$ is the LWC (in kg m$^{-3}$). Substituting into (6.27), we get:

$$\frac{dM}{dt} = \frac{1}{2}m v_1^2.$$
droplet sizes by statistical collisions. [Adapted from Fig. 6.24]

The growth of drops by collection is also accelerated if they pass through pockets of higher than average LWC, a further broadening of the droplet size spectrum. Consequently, the continuous collision model predicts that collector drops of this size have to be incorporated into computer models and numerical experiments to allow for the fact that collisions are individual events, distributed statistically in time and space. Consider, for example, 100 small cloud droplets that are distributed uniformly in space. Initially, these droplets have the same size as shown on line 1 in Fig. 6.24. After a certain interval of time, some of these droplets (let us say 10) will have collided with one another, making them larger. The second collisions are similarly statistically distributed, giving a further broadening of the droplet size spectrum used as input data to the two clouds were different. CCN concentrations on the development of larger drops. CCN concentrations based on measurements, with continental air having much higher concentrations of CCN than marine air. Much of what is presently known about both types of clouds reveals such pockets of high LWC (e.g., J. Atmos. Sci. 23, 689 (1967)).

In the continuous collision model, it is assumed that the collector drop exists for only a few minutes and occupies only a few percent of the cloud volume, they can produce significant concentrations of large drops when averaged over the entire cloud volume. Measurements have revealed pockets of high LWC much higher concentrations of CCN than marine air, whereas the continental cloud does not. These differences to the higher concentrations of CCN present in continental air (Fig. 6.5). Figure 6.25 illustrates the effects of these differences in cloud microstructure. We have attributed these differences in droplet size to the fact that continental cumulus cloud is more likely to rain than a continental cumulus cloud with similar updraft velocity, whereas the continental cloud does not. These droplet size distributions are significantly larger, and the droplet size spectra are much broader, in marine than in continental cumulus clouds (Fig. 6.7). We have attributed these differences to the fact that the marine cloud contains a small number of droplets near the middle of (a) a warm marine cumulus cloud and (b) a warm continental cloud after 67 min of growth. The collector drops near the middle of a warm marine cumulus cloud are markedly larger than those in continental air (Fig. 6.10). The numerical predictions of the mass spectrum of raindrops that are large enough to grow by collection, allow for the fact that collisions are individual events, distributed statistically in time and space. It can be seen that after 67 min the cumulus cloud contains a small number of drops near the middle of (a) a warm marine cumulus cloud and (b) a warm continental cloud after 67 min of growth.
X-rays  Ultraviolet  Visible  Infrared  Microwave radiation

FINE  COARSE

Small (Aitken)  Large  Giant

Gases  Smoke  Sea Salt  Dust  Sand

Dry Aerosol (CN)  Haze (CCN)  Cloud  Drizzle  Rain  Hail

$10^{-9}$ (1 nm)  $10^{-8}$ (0.1 μm)  $10^{-7}$ (1 μm)  $10^{-6}$ (10 μm)  $10^{-5}$ (100 μm)  $10^{-4}$ (1 mm)  $10^{-3}$ (1 cm)  $10^{-2}$ (1 m)  $10^{-1}$

Particle diameter, m
Fig. 6.24. After a certain interval of time, some of the droplets in a cloud can grow much faster than average by statistically distributed collisions. The growth of drops by collection is also accelerated if they pass through pockets of higher than average LWC, and depth.

**Schematic diagram to illustrate broadening of droplet size spectrum**

Line 1
- Consider the growth of drops, by condensation and collection. Numerical predictions of the mass spectrum of drops that are large enough to grow by collection, as depicted in line 2 of Fig. 6.24. Because of their more favored position for making further collisions, these 10 larger droplets are now in a much broader size, these 10 larger droplets are now in a much broader size spectrum, as shown in line 3 of Fig. 6.24 (where it has been assumed that in this time step, nine of the smaller droplets and one of the larger droplets collides in a continuous and uniform fashion with other droplets so that the distribution will now be statistically distributed, giving a further broadening of the droplet size spectra from the fairly uniform distributions used as input data to the two clouds were based on measurements, with continental air having much higher concentrations of CCN than marine air (about 200 versus 45 cm$^{-3}$). Measurements reveal how a small fraction of the droplets in a typical marine and continental air masses. As much of what is presently known about both typical marine and continental clouds reveals such pockets of high LWC (e.g., clouds with large drops near the middle of (a) a warm marine cumulus cloud is more likely to rain than a continental cloud with similar updraft velocity, whereas the continental cloud does not. These differences on the development of larger drops. CCN concentrations on line 2 each had a collision). Hence, by allowing stochastic collisions, in warm cumulus clouds in continental air (Fig. 6.5). Figure 6.25 illustrates much broader, in marine than in continental cumulus clouds (Fig. 6.7). We have attributed these differences to the higher concentrations of CCN present in continental air (Fig. 6.5). Figure 6.25 illustrates much broader, in marine than in continental cumulus clouds (Fig. 6.7). We have attributed these differences to the higher concentrations of CCN present in continental air (Fig. 6.5). Figure 6.25 illustrates much broader, in marine than in continental cumulus clouds (Fig. 6.7). We have attributed these differences to the higher concentrations of CCN present in continental air (Fig. 6.5). Figure 6.25 illustrates much broader, in marine than in continental cumulus clouds (Fig. 6.7). We have attributed these differences to the higher concentrations of CCN present in continental air (Fig. 6.5). Figure 6.25 illustrates much broader, in marine than in continental cumulus clouds (Fig. 6.7). We have attributed these differences to the higher concentrations of CCN present in continental air (Fig. 6.5). Figure 6.25 illustrates much broader, in marine than in continental cumulus clouds (Fig. 6.7). We have attributed these differences to the higher concentrations of CCN present in continental air (Fig. 6.5). Figure 6.25 illustrates much broader, in marine than in continental cumulus clouds (Fig. 6.7). We have attributed these differences to the higher concentrations of CCN present in continental air (Fig. 6.5). Figure 6.25 illustrates much broader, in marine than in continental cumulus clouds (Fig. 6.7). We have attributed these differences to the higher concentrations of CCN present in continental air (Fig. 6.5).
Cloud Microphysics

6.5 Microphysics of Cold Clouds

If a cloud extends above the 0°C level it is called a cold cloud. Even though the temperature may be below 0°C, water droplets can still exist in clouds, in which case they are referred to as supercooled droplets.

Cold clouds may also contain ice particles. If a cold cloud contains both ice particles and supercooled droplets, it is said to be a mixed cloud; if it consists entirely of ice, it is said to be glaciated.

This section is concerned with the origins and concentrations of ice particles in clouds, the various ways in which ice particles can grow, and the formation of precipitation in cold clouds.

6.5.1 Nucleation of Ice Particles; Ice Nuclei

A supercooled droplet is in an unstable state. For freezing to occur, enough water molecules must come together within the droplet to form an embryo of ice large enough to survive and grow. The situation is analogous to the formation of a water droplet from the vapor phase discussed in Section 6.2.1. If an ice embryo within a droplet exceeds a certain critical size, its growth will produce a decrease in the energy of the system. However, any increase in the size of an ice embryo smaller than the critical size causes an increase in total energy. In the latter case, from an energetic point of view, it is preferable for the embryo to break up.

If a water droplet contains no foreign particles, it can freeze only by homogeneous nucleation. Because the numbers and sizes of the ice embryos that form by chance aggregations increase with decreasing temperature, below a certain temperature (which depends on the volume of water considered), freezing by homogeneous nucleation becomes a virtual certainty.

The results of laboratory experiments on the freezing of very pure water droplets, which were probably nucleated homogeneously, are shown by the blue

\[ \text{Continued} \]

Theoretical predictions of the probability of breakup as a function of the sizes of two coalescing drops are shown in Fig. 6.28. For a fixed size of the larger drop, the probability of breakup initially increases as the size of the smaller drop increases, but as the smaller drop approaches the size of the larger, the probability of breakup decreases due to the decrease in the kinetic energy of the impact.

Measurements of the size distributions of raindrops that reach the ground can often be fitted to an expression, known as the Marshall–Palmer distribution, which is of the form 

\[ N(D) = N_0 \exp \left( -\frac{D - D_0}{D_1} \right) \]

where \( N(D) \) is the number of drops per unit volume with diameters between \( D \) and \( D + dD \) and \( N_0 \) and \( D_1 \) are empirical fitting parameters. The value of \( N_0 \) tends to be constant, but \( D_1 \) varies with the rainfall rate.

![Fig. 6.28](image-url)

Empirical results for the probability of breakup (expressed as a fraction and shown on the contours) following the collision and initial coalescence of two drops. The shaded region is covered by Fig. 6.21, but note that Fig. 6.21 shows the probability of coalescence rather than breakup. [Based on J. Atmos. Sci. 39, 1600 (1982).]
Fig. 6.29 Median freezing temperatures of water samples as a function of their equivalent drop diameter. The different symbols are results from different workers. The red symbols and red line represent heterogeneous freezing, and the blue symbols and line represent homogeneous freezing. [Adapted from B. J. Mason, *The Physics of Clouds*, Oxford Univ. Press, Oxford, 1971, p. 160. By permission of Oxford University Press.]
...and kaolinite (green). [Adapted from...

Fig. 6.30 indicate that for a variety of materials, conditions for condensation-freezing and deposition shown in Fig. 6.30 are indicated. Ice nucleators. Decayed plant leaves contain copious ice crystals from upper level clouds that evaporate before reaching the ground may leave behind preactivation. Thus, about one particle in 10^5 acts as an ice nucleus. In urban air, the total concentration of aerosol is on the order of 10^5 cm^-3, and 1797 J. Atmos. Sci...

Temperature (°C)

Ice supersaturation (%)
The probability is equal to the volume of the drop (in meters) at temperature . Therefore, the number of active freezing nuclei containing by the number of active freezing nuclei per liter.

Given by the Poisson distribution for random events, probability contains as shown by the red line in Fig. 6.29. If a drop of diameter , a number of drops should vary with their diameter (6.33), show that the median freezing temperature of a drop that are active at temperature is now the median freezing temperature of ice nuclei, is frozen is given by

\[ n = \frac{V}{N} \exp \left( \frac{-T}{T_0} \right) \]

Where \( n \) is the number of active freezing nuclei, \( V \) is the volume of the drop, \( N \) is the number of active freezing nuclei per liter, \( T \) is the temperature at which it freezes in a given time interval, and \( T_0 \) is the temperature at which it freezes in a given time interval. Therefore, the number of active freezing nuclei per liter.

Solution: From (6.33) the number of active freezing nuclei, assume that the number of active freezing nuclei at temperature

\[ n = \frac{V}{N} \exp \left( \frac{-T}{T_0} \right) \]

Exercise 6.4

Measurements in many locations around the world. On measurements of ice nucleus concentrations based on Millipore filter measurements at close to water saturation in the northern and southern hemispheres. Southern hemisphere, expansion chamber (red); Antarctica, mixing chamber (black square); northern hemisphere, mixing chamber (brown). Vertical lines show the range and mean values (dots) of ice nucleus concentrations based on Millipore filter measurements at close to water saturation in the northern and southern hemispheres. Southern hemisphere, expansion chamber (red); Antarctica, mixing chamber (black square); northern hemisphere, mixing chamber (brown).
The probability is equal to the volume of the drop (in m^3) containing a number of active freezing nuclei, is frozen.

From (6.33) the number of active freezing nuclei, assume that the number of active freezing nuclei per m^3 contains

\[ p \exp (-n \ln(a + \frac{T - T_0}{D} \ln(D(\frac{T - T_0}{D}))) \exp (n \ln(D(\frac{T - T_0}{D})))]. \]

Therefore, the number of active freezing nuclei, is frozen is given by

\[ \text{median freezing temperature of the ice nucleus concentration}. \]

\[ \text{solution:} \]

\[ \text{Ice nucleus concentration measurements versus ice free-}

\text{zation or a deposition nucleus depends not only on temperature but also on the supersaturation of the ice nucleus concentrations} \]

\[ \text{is} \]

\[ \text{on measurements of ice nucleus concentrations is plotted against the median freezing temperature.} \]

\[ \text{As we have seen, the activity of a particle as a freezing or a deposition nucleus depends not only on temperature but also on the supersaturation of the ice nucleus concentrations.} \]
Fig. 6.33

Percentage of clouds containing ice particle concentrations greater than about 1 per liter as a function of temperature. Note that on the abscissa temperatures decrease to the right. Blue curve: continental cumuliform clouds; red curve: marine cumuliform clouds and clean arctic stratiform clouds; green curve: ice multiplication.

The probability of ice particles being present in a cloud increases as the temperature decreases below about 0°C containing drizzle or raindrops prior to the formation of ice. [Data from P. V. Hobbs, "Ice particles in stratiform clouds in the Arctic Ocean," Quart. J. Roy. Met. Soc 117, 15,066 (2001) Copyright 2001 American Geophysical Union. Reproduced by permission of the American Geophysical Union; Clouds; Ice Multiplication.]
6.6 Artificial Modification of Clouds and Precipitation

Fig. 6.48 Causality or chance coincidence? Explosive growth of cumulus cloud (a) 10 min; (b) 19 min; 29 min; and 48 min after it was seeded near the location of the arrow in (a). [Photos courtesy of J. Simpson.]

Many artificial ice nucleating materials are now known (e.g., lead iodide, cupric sulfide) and some organic materials (e.g., phloroglucinol, metaldehyde) are more effective as ice nuclei than silver iodide. However, silver iodide has been used in most cloud seeding experiments.

Since the first cloud seeding experiments in the 1940s, many more experiments have been carried out all over the world. It is now well established that the concentrations of ice crystals in clouds can be increased by seeding with artificial ice nuclei and that, under certain conditions, precipitation can be artificially initiated in some clouds. However, the important question is: under what conditions (if any) can seeding with artificial ice nuclei be employed to produce significant increases in precipitation on the ground in a predictable manner and over a large area? This question remains unanswered.

So far we have discussed the role of artificial ice nuclei in modifying the microstructures of cold clouds. However, when large volumes of a cloud are glaciated by overseeding, the resulting release of latent heat provides added buoyancy to the cloudy air. If, prior to seeding, the height of a cloud were restricted by a stable layer, the release of the latent heat of fusion caused by artificial seeding might provide enough buoyancy to push the cloud through the inversion and up to its level of free convection. The cloud top might then rise to much greater heights than it would have done naturally. Figure 6.48 shows the explosive growth of a cumulus cloud that may have been produced by overseeding.

Seeding experiments have been carried out in attempts to reduce the damage produced by hailstones. Seeding with artificial nuclei should tend to increase the number of small ice particles competing for the available supercooled droplets. Therefore, seeding should result in a reduction in the average size of the hailstones. It is also possible that, if a hailstorm is overseeded with extremely large numbers of ice nuclei, the majority of the supercooled droplets in the cloud will be nucleated, and the growth of hailstones by riming will be reduced significantly. Although these hypotheses are plausible, the results of experiments on hail suppression have not been encouraging.

Exploratory experiments have been carried out to investigate if orographic snowfall might be redistributed by overseeding. Rimed ice particles have relatively large terminal fall speeds \( H^{_{11011}} \text{ms}^{-1} \), therefore they follow fairly steep trajectories as they fall to the ground. If clouds on the windward side of a mountain are artificially overseeded, supercooled droplets can be virtually eliminated and growth by riming significantly reduced (Fig. 6.49). In the absence of riming, the ice
Cloud Microphysics generally require about 10 min before showing signs of plentiful ice particles. It also appears from measurements in clouds that high ice particle concentrations occur after the formation of drops with diameters of 11000\(\mu\)m and when rimed ice particles appear. These observations are consistent with the hypothesis that the high ice particle concentrations are due to the ejection of ice splinters during riming. However, calculations based on the results of laboratory experiments on ice splinter production during riming suggest that this process is too slow to explain the explosive formation of extremely high concentrations of ice particles observed in some clouds.

As indicated schematically in Fig. 6.35, an additional "super" ice enhancement mechanism may sometimes operate, but the exact nature of this mechanism remains a mystery.

6.5.3 Growth of Ice Particles in Clouds

(a) Growth from the vapor phase. In a mixed cloud dominated by supercooled droplets, the air is close to saturated with respect to liquid water and is therefore supersaturated with respect to ice. For example, air saturated with respect to liquid water at 11000°C is supersaturated with respect to ice by 10% and at 21000°C it is supersaturated by 21%. These values are much greater than the supersaturations of cloudy air with respect to liquid water, which rarely exceed 1%. Consequently, in mixed clouds dominated by supercooled water droplets, in which the cloudy air is close to water saturation, ice particles will grow from the vapor phase much more rapidly than droplets. In fact, if a growing ice particle lowers the vapor pressure in its vicinity below water saturation, adjacent droplets will evaporate (Fig. 6.36).

Explosive formation of \(\sim 10^{-100}\) per liter of regular and irregular crystals. Liquid water content \(>0.5\) g m\(^{-3}\). Intense aggregation and fallout of ice particles. Concentrations of ice particles begin to decline and liquid water is depleted. Small aggregates and single ice crystals. No liquid water.

Particle size sorting produces filaments and virga. Large fast-falling ice particles.

Fig. 6.35 Schematic of ice development in small cumuliform clouds. [Adapted from Quart. J. Roy. Meteor. Soc. 117, 231 (1991). Reproduced by permission of The Royal Meteorological Society.]
Cloud Microphysics and generally require about 10 min before showing signs of plentiful ice particles. It also appears from measurements in clouds that high ice particle concentrations occur after the formation of drops with diameters \( \frac{D}{H} > 25 \) and when rimed ice particles appear. These observations are consistent with the hypothesis that the high ice particle concentrations are due to the ejection of ice splinters during riming. However, calculations based on the results of laboratory experiments on ice splinter production during riming suggest that this process is too slow to explain the explosive formation of extremely high concentrations of ice particles observed in some clouds.

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6.5.3 Growth of Ice Particles in Clouds

(a) Growth from the vapor phase. In a mixed cloud dominated by supercooled droplets, the air is close to saturated with respect to liquid water and is therefore supersaturated with respect to ice. For example, air saturated with respect to liquid water at \( \frac{T}{H} = 10 \) \( \frac{C}{H} \) is supersaturated with respect to ice by 10% and at \( \frac{T}{H} = 20 \) \( \frac{C}{H} \) it is supersaturated by 21%. These values are much greater than the supersaturations of cloudy air with respect to liquid water, which rarely exceed 1%. Consequently, in mixed clouds dominated by supercooled water droplets, in which the cloudy air is close to water saturation, ice particles will grow from the vapor phase much more rapidly than droplets. In fact, if a growing ice particle lowers the vapor pressure in its vicinity below water saturation, adjacent droplets will evaporate (Fig. 6.36).

Explosive formation of \(~10^3–10^4\)'s per liter of regular and irregular crystals. Liquid water content >0.5 g m\(^{-3}\). Intense aggregation and fallout of ice particles. Concentrations of ice particles begin to decline and liquid water is depleted.

Small aggregates and single ice crystals. No liquid water. Particle size sorting produces filaments and virga. Large fast-falling ice particles. Frozen drizzle drops and/or small graupel, isolated single and irregular ice crystals ~0.1–20 per liter.

Fig. 6.35 Schematic of ice development in small cumuliform clouds. [Adapted from Quart. J. Roy. Meteor. Soc. 117, 231 (1991). Reproduced by permission of The Royal Meteorological Society.]
Microphysics of Cold Clouds

Cumulus turrets containing relatively large ice particles often have ill-defined, fuzzy boundaries, whereas turrets containing only small droplets have well-defined, sharper boundaries, particularly if the cloud is growing (Fig. 6.37). Another factor that contributes to the difference in appearance of ice and water clouds is the lower equilibrium vapor pressure over ice than over water at the same temperature, which allows ice particles to migrate for greater distances than droplets into the nonsaturated air surrounding a cloud before they evaporate. For the same reason, ice particles that are large enough to fall out of a cloud can survive great distances before evaporating completely, even if the ambient air is subsaturated with respect to ice; ice particles will grow in air that is subsaturated with respect to water, provided that it is supersaturated with respect to ice. The trails of ice crystals so produced are called fallstreaks or virga (Fig. 6.38).

The factors that control the mass growth rate of an ice crystal by deposition from the vapor phase are similar to those that control the growth of a droplet by condensation (see Section 6.4.1). However, the problem is more complicated because ice crystals are not spherical and therefore points of equal vapor density do not lie on a sphere centered on the crystal (as they do for a droplet). For the special case of a spherical ice particle of radius $r$, we can write, by analogy with (6.19),

$$\frac{dM}{dt} = \frac{4}{3} \pi r^3 \rho_v \left[ \frac{P}{H} \right] v_c$$

where $\rho_v$ is the density of the vapor just adjacent to the surface of the crystal and the other symbols were defined in Section 6.4.1. We can derive an expression for the rate of increase in the mass of an ice crystal of arbitrary shape by exploiting the analogy between the vapor field around an ice crystal and the field of electrostatic potential around a charged conductor of the same size and shape.

This analogy was suggested by Harold Jeffreys. Harold Jeffreys (1891–1989) English mathematician and geophysicist. First to suggest that the core of the Earth is liquid. Studied earthquakes and the circulation of the atmosphere. Proposed models for the structure of the outer planets and the origin of the solar system.
6.5 Microphysics of Cold Clouds

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$$\frac{dM}{dt} = \frac{4}{3} \pi r^3 \rho v_c \left( \frac{a}{H} \right)$$

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Cloud Microphysics
determined by the size and shape of the conductor. For a sphere, where 
\[ \varepsilon = 8.85 \times 10^{-12} \text{ F/m} \]
Combining the last two expressions, the mass growth rate of a spherical ice crystal is given by 
\[ G_i S_i = \frac{dM}{dt} \]
Equation (6.36) is quite general and can be applied to an arbitrarily shaped crystal of capacity 
\[ C \]
Provided that the vapor pressure corresponding to 
\[ v = e_i \]
is not too much greater than the saturation vapor pressure \( e_s \) over a plane surface of ice, and the ice crystal is not too small, (6.36) can be written as 
\[ G_i S_i = \frac{dM}{dt} \]
where 
\[ S_i \]
is the supersaturation (as a fraction) with respect to ice, 
\[ e_i \]
\[ G_i S_i = \frac{dM}{dt} \]
The variation of \( G_i S_i \) with temperature for the case of an ice crystal growing in air saturated with water is shown in Fig. 6.39. The product \( G_i S_i \) attains a maximum value at about 
\[ T = -14 \degree C \]
which is due mainly to the fact that the difference between the saturated vapor pressures over water and ice is a maximum near this temperature. Consequently, ice crystals growing by vapor deposition in mixed clouds increase in mass most rapidly at temperatures around 
\[ T = -14 \degree C \]
The majority of ice particles in clouds are irregular in shape (sometimes referred to as "junk" ice). This may be due, in part, to ice enhancement. However, laboratory studies show that under appropriate conditions ice crystals that grow from the vapor phase can assume a variety of regular shapes (or habits) that are either plate-like or column-like. The simplest plate-like crystals are plane hexagonal plates (Fig. 6.40a), and the simplest column-like crystals. The simplest plate-like crystals are plane hexagonal plates (Fig. 6.40a), and the simplest column-like crystals.

\[ G_i S_i \]

\[ \text{Temperature} (\degree C) \]

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Cloud Microphysics
determined by the size and shape of the conductor.

For a sphere, where $\varepsilon_0$ is the permittivity of free space ($8.85 \times 10^{-12} \text{ F/m}$).

Combining the last two expressions, the mass growth rate of a spherical ice crystal is given by

$$G_i S_i$$

Equation (6.36) is quite general and can be applied to an arbitrarily shaped crystal of capacity $C$.

Provided that the vapor pressure corresponding to $\varepsilon_s$ is not too much greater than the saturation vapor pressure $\epsilon_{si}$ over a plane surface of ice, and the ice crystal is not too small, (6.36) can be written as

$$G_i S_i$$

where $S_i$ is the supersaturation (as a fraction) with respect to ice, $\frac{\varepsilon_s}{\varepsilon_{si}}$.

The variation of $G_i S_i$ with temperature for the case of an ice crystal growing in air saturated with water is shown in Fig. 6.39. The product $G_i S_i$ attains a maximum value at about $14\,\text{C}$, which is due mainly to the fact that the difference between $\frac{\varepsilon_i}{\varepsilon_s}$ and $\frac{\varepsilon_s}{\varepsilon_{si}}$ is a maximum near this temperature. Consequently, ice crystals growing by vapor deposition in mixed clouds increase in mass most rapidly at temperatures around $14\,\text{C}$.

The majority of ice particles in clouds are irregular in shape (sometimes referred to as "junk ice"). This may be due, in part, to ice enhancement. However, laboratory studies show that under appropriate conditions ice crystals that grow from the vapor phase can assume a variety of regular shapes (or habits) that are either plate-like or column-like.

The simplest plate-like crystals are plane hexagonal plates (Fig. 6.40a), and the simplest column-like $\varepsilon$.

**Fig. 6.39** Variation of $G_i S_i$ [see Eq. (6.37)] with temperature for an ice crystal growing in a water-saturated environment at a total pressure of 1000 hPa.

**Fig. 6.40** Ice crystals grown from the vapor phase: (a) hexagonal plates, (b) column, (c) dendrite, and (d) sector plate. [Photographs courtesy of Cloud and Aerosol Research Group, University of Washington.]

(e) Bullet rosette. [Photograph courtesy of A. Heymsfield.]
6.5 Microphysics of Cold Clouds

Crystals are solid columns that are hexagonal in cross-section (Fig. 6.40b).

Studies of the growth of ice crystals from the vapor phase under controlled conditions in the laboratory and observations in natural clouds have shown that the basic habit of an ice crystal is determined by the temperature at which it grows (Table 6.1). In the temperature range between 0 and \(-10^\circ C\), the basic habit changes three times. These changes occur near \(0^\circ C\), \(8^\circ C\), and \(40^\circ C\). When the air is saturated or supersaturated with respect to water, the basic habits become embellished. For example, at close to or in excess of water saturation, column-like crystals take the form of long thin needles between \(4^\circ C\) and \(6^\circ C\). Plate-like crystals appear like ferns, called dendrites (Fig. 6.40c), from \(9^\circ C\) to \(12^\circ C\) and \(16^\circ C\) to \(20^\circ C\). Sector plates grow (Fig. 6.40d), and below \(40^\circ C\) the column-like crystals take the form of column (often called bullet) rosettes (Fig. 6.40e). Because ice crystals are generally exposed to continually changing temperatures and supersaturations as they fall through clouds and to the ground, crystals can assume quite complex shapes.

Growth by riming; hailstones. In a mixed cloud, ice particles can increase in mass by colliding with supercooled droplets that then freeze onto them. This process, referred to as growth by riming, leads to the formation of various rimed structures; some examples are shown in Fig. 6.41. Figure 6.41a shows a needle that collected a few droplets on its leading edge as it fell through the air; Fig. 6.41b a uniformly, densely rimed column; Fig. 6.41c a rimed plate; Fig. 6.41d a rimed stellar; Fig. 6.41e a spherical graupel; and Fig. 6.41f a conical graupel. (Photographs courtesy of Cloud and Aerosol Research Group, University of Washington.)

### Table 6.1

<table>
<thead>
<tr>
<th>Supersaturation</th>
<th>Basic habit</th>
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<tbody>
<tr>
<td>Near to or greater than water saturation</td>
<td>Plate-like, Hexagonal plates, Dendrites, Hollow columns, Scrolls and sector plates, Dendrites, Sector plates</td>
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<td>Water saturation</td>
<td>Equiaxed, Equiaxed, Hollow column rosettes, Equiaxed, Equiaxed</td>
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<tr>
<td>Ice and Near to or greater than water saturation</td>
<td>Plate-like, Hexagonal plates, Dendrites, Hollow columns, Scrolls and sector plates, Dendrites, Sector plates</td>
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**a** From information provided by J. Hallett and M. Bailey.

**b** If the ice crystals are sufficiently large to have significant fall speeds, they will be ventilated by the airflow. Ventilation of an ice crystal has a similar effect on embellishing the crystal habit, as does increasing the supersaturation.

**c** At low supersaturations, crystal growth depends on the presence of molecular defects. As water saturation is approached, surface nucleation occurs near the crystal edges and layers of ice spread toward the crystal interior. Growth at the edges of a crystal is limited by vapor and heat transfer and in the interior of a crystal by kinetic processes at the ice–vapor interface.

**d** At lower supersaturations different crystal habits grow under identical ambient conditions depending on the defect structure inherited at nucleation.
viewed between crossed polarizing filters; see inset to Fig. 6.42) can also reveal whether wet growth has occurred. It can be seen from Figs. 6.42 and 6.43 that the surface of a hailstone can contain fairly large lobes. Lobe-like growth appears to be more pronounced when the accreted droplets are small and growth is near the wet limit. The development of lobes may be due to the fact that any small bumps on a hailstone will be areas of enhanced collection efficiency for droplets.

(c) Growth by aggregation. The third mechanism by which ice particles grow in clouds is by colliding and aggregating with one another. Ice particles can collide with each other, provided their terminal fall speeds are different. The terminal fall speed of an unrimed column-like ice crystal increases as the length of the crystal increases; for example, the fall speeds of needles 1 and 2 mm in length are about 0.5 and 0.7 m s⁻¹, respectively. In contrast, unrimed plate-like ice crystals have terminal fall speeds that are virtually independent of their diameter for the following reason. The thickness of a plate-like crystal is essentially independent of its diameter, therefore, its mass varies linearly with its cross-sectional area. Because the drag force acting on a plate-like crystal also varies as the cross-sectional area of the crystal, the terminal fall speed, which is determined by the balance between the drag force and the weight of the crystal, can be expressed as:

\[ \text{Fall speed} = \sqrt{\frac{2 \cdot \text{Mass}}{\text{Drag force}}} \]

where the drag force is proportional to the cross-sectional area of the crystal. Thus, the terminal fall speed of a plate-like crystal depends on its diameter, while that of a column-like crystal depends on its length.

Hailstones represent an extreme case of the growth of ice particles by riming. They form in vigorous convective clouds that have high liquid water contents. The largest hailstone reported in the United States (Nebraska) was 13.8 cm in diameter and weighed about 0.7 kg. However, hailstones about 1 cm in diameter are much more common. If a hailstone collects supercooled droplets at a very high rate, its surface temperature rises to 0°C and some of the water it collects remains unfrozen. The surface of the hailstone then becomes covered with a layer of water and the hailstone is said to grow wet. Under these conditions some of the water may be shed in the wake of the hailstone, but some of the water may be incorporated into a water-ice mesh to form what is known as spongy hail.

If a thin section is cut from a hailstone and viewed in transmitted light, it is often seen to consist of alternate dark and light layers (Fig. 6.42). The dark layers are opaque ice containing numerous small air bubbles, and the light layers are clear (bubble-free) ice. Clear ice is more likely to form when the hailstone is growing wet. Detailed examination of the orientation of the individual crystals within a hailstone (which can be seen when the hailstone is...
viewed between crossed polarizing filters; see inset to Fig. 6.42) can also reveal whether wet growth has occurred. It can be seen from Figs. 6.42 and 6.43 that the surface of a hailstone can contain fairly large lobes. Lobe-like growth appears to be more pronounced when the accreted droplets are small and growth is near the wet limit. The development of lobes may be due to the fact that any small bumps on a hailstone will be areas of enhanced collection efficiencies for droplets.

The third mechanism by which ice particles grow in clouds is by colliding and aggregating with one another. Ice particles can collide with each other, provided their terminal fall speeds are different. The terminal fall speed of an unrimed column-like ice crystal increases as the length of the crystal increases; for example, the fall speeds of needles 1 and 2 mm in length are about 0.5 and 0.7 m s$^{-1}$, respectively. In contrast, unrimed plate-like ice crystals have terminal fall speeds that are virtually independent of their diameter for the following reason. The thickness of a plate-like crystal is essentially independent of its diameter, therefore, its mass varies linearly with its cross-sectional area. Because the drag force acting on a plate-like crystal also varies as the cross-sectional area of the crystal, the terminal fall speed, which is determined...
6.5 Microphysics of Cold Clouds

Apart from this dependence upon habit, the probability of two colliding crystals adhering increases with increasing temperature, with adhesion being particularly likely above about 
\[ T > T_c \text{ (in } \text{K}) \]

at which temperatures ice surfaces become quite "sticky." Some examples of ice particle aggregates are shown in Fig. 6.44.

6.5.4 Formation of Precipitation in Cold Clouds

As early as 1789 Franklin suggested that "much of what is rain, when it arrives at the surface of the Earth, might have been snow, when it began its descent...

This idea was not developed until the early part of the last century when Wegener, in 1911, stated that ice particles would grow preferentially by deposition from the vapor phase in a mixed cloud. Subsequently, Bergeron, in 1933, and Findeisen, in 1938, developed this idea in a more

The second factor that influences growth by aggregation is whether two ice particles adhere when they collide. The probability of adhesion is determined primarily by two factors: the types of ice particles and the temperature. Intricate crystals, such as dendrites, tend to adhere to one another because they become entwined on collision, whereas two solid plates...
Cloud Microphysics

quantitative manner and indicated the importance of ice nuclei in the formation of crystals. Because Findeisen carried out his field studies in northwestern Europe, he was led to believe that all rain originates as ice. However, as shown in Section 6.4.2, rain can also form in warm clouds by the collision-coalescence mechanism.

We will now consider the growth of ice particles to precipitation size in a little more detail. Application of (6.36) to the case of a hexagonal plate growing by deposition from the vapor phase in air saturated with respect to water at \( H_{11002} \) shows that the plate can obtain a mass of \( H_{11011} \) in half an hour (see Exercise 6.27). Thereafter, its mass growth rate decreases significantly. On melting, a 7-g ice crystal would form a small drizzle drop about 130 m in radius, which could reach the ground, provided that the updraft velocity of the air were less than the terminal fall speed of the crystal (about 0.3 m s\(^{-1}\)) and the drop survived evaporation as it descended through the subcloud layer. Calculations such as this indicate that the growth of ice crystals by deposition of vapor is not sufficiently fast to produce large raindrops.

Unlike growth by deposition, the growth rates of an ice particle by riming and aggregation increase as the ice particle increases in size. A simple calculation shows that a plate-like ice crystal, 1 mm in diameter, falling through a cloud with a liquid content of 0.5 g m\(^{-3}\), could develop into a spherical graupel particle about 0.5 mm in radius in a few minutes (see Exercise 6.28). A graupel particle of this size, with a density of 100 kg m\(^{-3}\), has a terminal fall speed of about 1 m s\(^{-1}\) and would melt into a drop about 230 m in radius. The radius of a snowflake can increase from 0.5 mm to 0.5 cm in \( H_{11011} \) due to aggregation with ice crystals, provided that the ice content of the cloud is about 1 g m\(^{-3}\) (see Exercise 6.29). An aggregated snow crystal with a radius of 0.5 cm has a mass of about 3 mg and a terminal fall speed of about 1 ms\(^{-1}\). Upon melting, a snow crystal of this mass would form a drop about 1 mm in radius. We conclude from these calculations that the growth of ice crystals, first by deposition from the vapor phase in mixed clouds and then by riming and/or aggregation, can produce precipitation-sized particles in reasonable time periods (say about 40 min).

The role of the ice phase in producing precipitation in cold clouds is demonstrated by radar observations. For example, Fig. 6.45 shows a radar screen (on which the intensity of radar echoes reflected from atmospheric targets are displayed) while the radar antenna was pointing vertically upward and clouds drifted over the radar. The horizontal band (in brown) just above a height of 2 km was produced by the melting of ice particles. This is referred to as the "bright band." The radar reflectivity is high around the melting level because, while melting, ice particles become coated with a film of water that increases their radar reflectivity greatly. When the crystals have melted completely, they collapse into droplets and their terminal fall speeds increase so that the concentration of
Because there are no supercooled droplets present, ice crystals (i.e., it is glaciated). The ice crystals in a cloud is overseeded it is converted completely into falling through air at.

For example, a pellet of dry ice 1 cm in diameter to form in its wake by homogeneous nucleation.

The seeding of natural clouds with silver iodide was first tried as part of Project Cirrus on 21 December 1947. Pieces of burning charcoal impregnated with silver iodide were dropped from an aircraft into a supercooled cloud or fog (Fig. 6.47). This technique is used for clearing supercooled fogs at several international airports.

Examination of crystallographic tables revealed that silver iodide fulfilled this requirement. Subsequent laboratory tests showed that silver iodide could act as an ice nucleus at temperatures as high as 10°C. The cloud was...