Divergence and Deformation

Atmos 5110 Synoptic–Dynamic Meteorology I

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Suggested reading: These notes

Deformation and divergence are two fundamental components of fluid flow and widely used and considered in meteorological analysis and forecasting.

Divergence

Defined mathematically as the dot product of the del operator with the velocity vector

$$\text{Divergence} = \nabla \cdot \vec{U} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$$

Or, in pressure coordinates

$$\text{Divergence} = \nabla \cdot \vec{U} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial p}$$

In many synoptic situations, we consider only the horizontal divergence

$$\nabla_p \cdot \vec{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$$

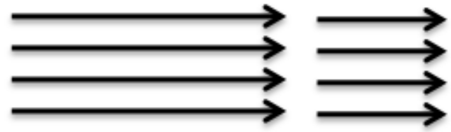
Where $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$ is evaluated on a pressure surface.

Examples (dropping p subscript hereafter for convenience):

Divergent Straight Flow

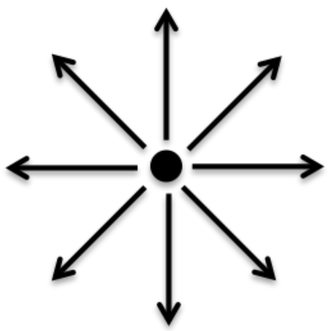
$$\frac{\partial u}{\partial x} > 0;\ \frac{\partial v}{\partial y} = 0\ \Rightarrow\ \nabla \cdot \vec{V} > 0$$

Convergent Straight Flow

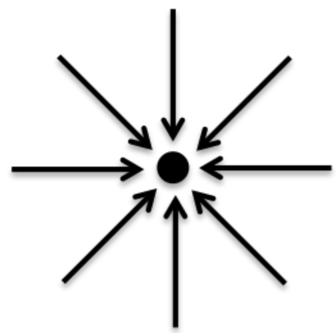


$$\frac{\partial u}{\partial x} < 0;\ \frac{\partial v}{\partial y} = 0\ \Rightarrow\ \nabla \cdot \vec{V} < 0$$

Pure Divergence



Pure Convergence

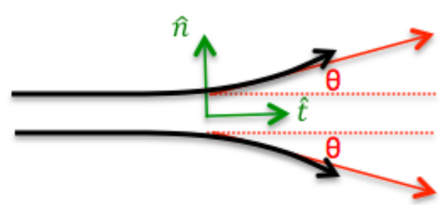


Divergence in natural coordinates

$$\nabla_p \cdot \vec{V} = \frac{\partial V}{\partial S} + V\frac{\partial \theta}{\partial n}$$

Stretching Term

Diffluence Term

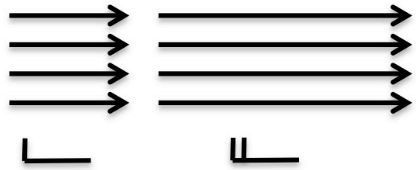


$\frac{\partial V}{\partial S} \rightarrow$ Along flow changes in wind speed

$\frac{\partial \theta}{\partial n} \rightarrow$ Divergence (or convergence) of the streamlines normal to (across) the flow

Examples

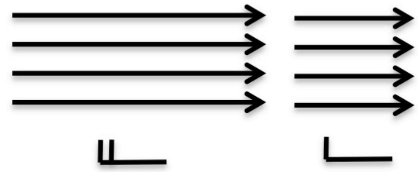
Divergent Straight Flow



$$\frac{\partial V}{\partial S} > 0;\ V\frac{\partial \theta}{\partial n} = 0$$

$$\Rightarrow \nabla \cdot \vec{V} > 0 \ \Rightarrow Divergence$$

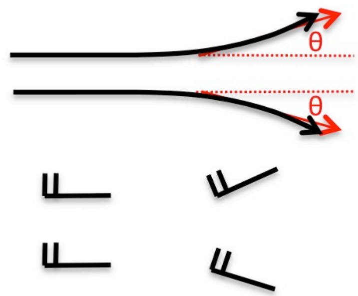
Convergent Straight Flow



$$\frac{\partial V}{\partial S} < 0;\ V\frac{\partial \theta}{\partial n} = 0$$

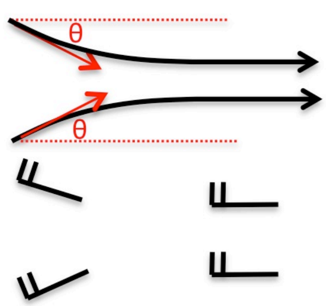
$$\Rightarrow \nabla \cdot \vec{V} < 0 \ \Rightarrow Convergence$$

No along-flow divergence, but flow is diffluent



$$\frac{\partial V}{\partial S} = 0;\ V\frac{\partial \theta}{\partial n} > 0 \ \Rightarrow \nabla \cdot \vec{V} > 0 \ \Rightarrow Divergence$$

No along-flow divergence, but flow is confluent



$$\frac{\partial V}{\partial S} = 0;\ V\frac{\partial \theta}{\partial n} < 0 \ \Rightarrow \nabla \cdot \vec{V} < 0 \ \Rightarrow Convergence$$

Divergence/Convergence vs. Diffluence/Confluence

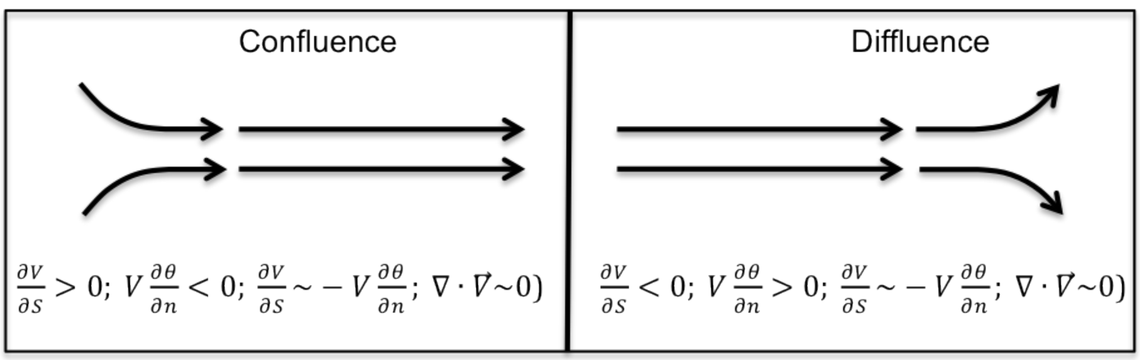
It is possible to have nondivergent flow (i.e., $\nabla \cdot \vec{V} = 0$) even when it "looks" divergent or convergent if

$$\frac{\partial V}{\partial S} = -V\frac{\partial \theta}{\partial n}$$

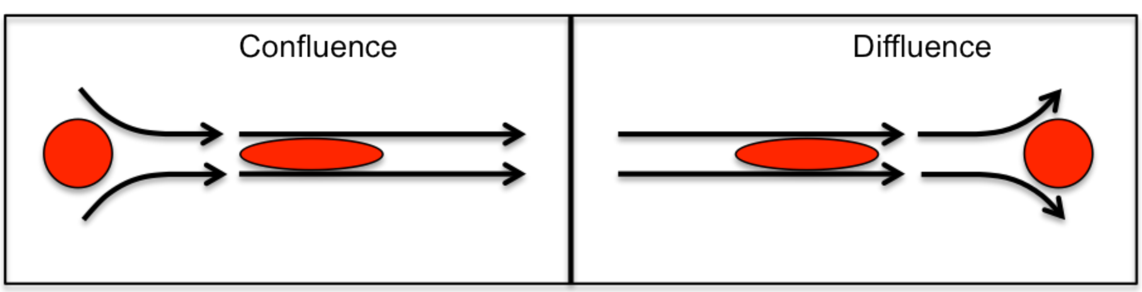
This means that the along-flow divergence is balanced by cross-flow convergence (or vice versa).

On the large-scale, this is the norm. The total divergence is usually a small difference between two terms of nearly equal magnitude but opposite sign.

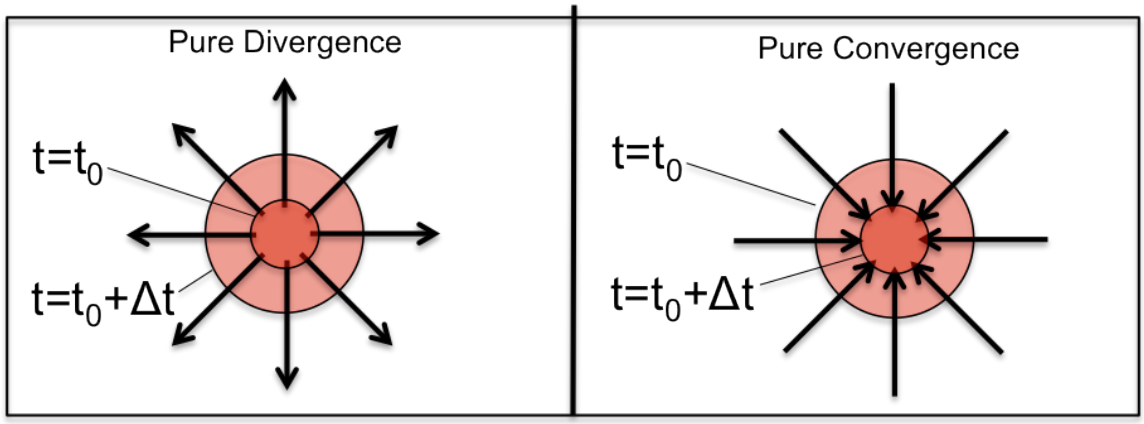
$\Longrightarrow$ On the large scale, we typically refer to flow patterns as diffluent and confluent rather than divergent and convergent since it is very difficult to assess divergence by eye.



Confluence and diffluence are examples of deformation, the changing of the shape of a fluid element by the flow field



In a fluid pattern with pure deformation, the shape of the fluid element changes, but not the area. In a pattern with pure divergence (or convergence), the area of the fluid element changes, but not the shape.



Flow patterns that change the shape of fluid columns that change the shape of fluid columns are called deformation zones.

Class activity

Using the IDV Supernational bundle, examine the water vapor loop and see if you can identify areas of upper-level diffluence, upper-level confluence, and deformation. After doing this, add 300-mb wind vectors and see if the analyzed flow is consistent with your analysis. Then, add a contour analysis of 300-mb divergence and evaluate the correspondence (or lack thereof) between divergence and diffluence, as well as convergence and confluence.